UNIVERSITY OF ANBAR

كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : Mathematical Analysis

اسم الحاضرة االحاديةعشر باللغة العربية: اختبار المتسلسلات

اسم المحاضرة الحاديةعشرباللغة الإنكليزية :Test of Series

(1) Given an example for two divergent Sories but their Sum is Convergent Series.
Sol:

let $\sum_{n=1}^{\infty} = \sum_{n=1}^{\infty} \frac{1}{n}$
and $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} -\frac{1}{\mathrm{n}}$
the $\sum_{ns1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n}\right) = \sum_{n=1}^{\infty} 0 = 0$ **con**

and $\langle a_n + b_n \rangle \longrightarrow 0$

Series test اختبار المتسلسلات

(1) Comparison test:

Theorem: If $0 \leq a_n \leq b_n \; \forall \; n \in N$, then

(1) $\sum_{n=1}^{\infty}$ **bn convergent, then** $\sum_{n=1}^{\infty}$ **an Canvergont**
(2) $\sum_{n=1}^{\infty} a_n$ **divergent, then** $\sum_{n\leq 1}^{\infty}$ **bn divergent**

`of partial sums of $\sum_{n_1 1}^{\infty} b_n$ **sina** $0 \leqslant$ **an** $\leqslant b_n$**, then**
$S_n = a_1 + a_2 + a_3 + \cdots + a_n$
$\leqslant b_1 + b_2 + b_3 + \cdots + b_n$
$= \mathrm{t_n}$
but $\sum_{n=1}^{\infty} b_n$ **Convergent, thei** $\langle t_n \rangle \Rightarrow$ **t as** $n \to \infty$ $b_n \geqslant 0 \Rightarrow \langle t_n \rangle$ **increasing seq and** $t_n \leq t, \forall_n$ **and** $S_n \leqslant t_n, \forall n$**, the** $S_n \leqslant t$**,** $\mathrm{S_0}$ $\langle s_n \rangle$ **is bounded**

$\to \langle S_n \rangle$ **is bounded and increasing (mono ton)** $\Rightarrow \langle \mathrm{s_n} \rangle$ **Convergent sequence**

$\Rightarrow \sum_{ns}^{\infty}$ **an Cowlergint.**

(2) Suppose $\sum_{n=1}^{\infty}$ bn Converyont

by (1), $\sum_{n=1}^{\infty}$ an Convergent and $\rightarrow C!$, so $\sum_{n,1}^{\infty} b_n$ divergent

P – Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}, \qquad p > 0.$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \begin{cases} \text{Canerge} & \text{if } p > 1 \\ \text{diveng} & \text{if } < p \leqslant 1 \end{cases}$$

Examples:

(1) $\sum_{n=1}^{\infty} \frac{1}{5n^3} = \frac{1}{5}\sum_{n=1}^{\infty} \frac{1}{n^3}, p = 3 > 1$

then P - Series $\rightarrow p = 3\rangle 1$, sa Caniergent

(2) $\sum_n \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{t}}}, p^{-1}\frac{1}{2} < 1$

Then p series, $P = \frac{1}{2} < 1, d$ inevgent

Theorem: let Lan and $\sum b_n$ be posifive term Series s.t $\lim_{n\rightarrow\infty} \frac{a_n}{b_n} L \neq 0$ then

Example: D)$\sum^{\infty} n^3 - 1n = 04n^5 - 3n^2 + 3$

$a_n = \frac{n^3-1}{4a^5-3n^2+3} \geqslant 0,$ choose bns $\frac{1}{n^2}$ to Compave $\sum$ thans $\sum \frac{1}{n^2}$ Convergant (p-sories $p = 2 > 1$)

$$\lim_{n\to\infty}\frac{a_n}{b_n} = 1\text{in}\ \frac{n^3-1}{4n5-3n^2+3} \div \frac{1}{n^2}$$

$$= \lim_{n\to\infty} = \frac{n^5-n^2}{4n^5-3n^2+3} = \lim_{n\to\infty}\frac{\frac{n^5}{n5}-\frac{n^2}{n^5}}{4\frac{n^5}{n^5}-3\frac{n^2}{n^5}+\frac{3}{n^5}}$$

$$= L_{n\to\infty}\frac{1-\frac{1}{n^3}}{4-\frac{3}{n^3}+\frac{3}{n^5}} = \text{Lim}_{n\to\infty}\ \frac{1}{4} = \frac{1}{4} = 0$$

by theorem above $\sum_{n=0}^{\infty}$ an Convargant.

$$\left(2\sum_{n=0}^{\infty}\frac{2n+1}{n^2+2n+1}\right.$$

(3) Ratio test

1 - If $b < 1 \Rightarrow \sum$ an Convergent.

2 if $h > 1 \Rightarrow 2$ an divergent.

3 -if $b = 1 \implies$ no infurmations.