كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة الثانيةعشر باللغة العربية: التقارب المطلق والتقارب المشروط

اسم المحاضرة الثانية عشر باللغة الإنكليزية :**Absolutly and Conditional Convergence**

Examples:

(1) $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

(2) $\sum_{n<0}^{\infty} \frac{n}{3^n}, \cdots$ Convergant.

(3) $\sum_{n\leq 0}^{2} n^2$

let $a_n = n^2, a_n + (n+1)^2$

$$L_{n\to\infty} \frac{a_n + 1}{a_n} = 1\text{mim}_{n\to\infty} \frac{(n+1)^2}{n^2} = \frac{1, m\frac{n^2+2n+1}{n^2}}{}$$

$$\lim_{n\to\infty} \frac{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}}{\frac{n^2}{n2}} = \text{Lim}_{n\to\infty} \frac{1 + \frac{2}{n} + \frac{1}{n}2}{=} 1$$

$\therefore b = 1, bw + \sum_{n=1}^{\infty} n^2$ is divergent

(4)$\sum_{ni0}^{\infty} \frac{1}{n^2}$

lel ans $\frac{1}{n^2}, -$ avit $= \frac{1}{(n+1)^2}$

$$= \text{Lim}_{n^2+2n+1} = \lim_{n\to\infty} \left(\frac{1}{1+0}\right) = \text{Lim}_{n\to\infty} \mid= 1$$

So $b = 1$

but $\sum \frac{1}{n^2}$ is Canventi, 5 ina $p = 2 > 1$

Theorem: let $\sum_{n+1}^{m}$ be a series, $ax > 0, \forall n$, if $\exists b \in R$ sit $n\sqrt{a_n} = b$

1 - if $b < 1 \Rightarrow$ \suman Comergent

2 - if $b > 1 \rightarrow$ enp di

3 if be1 $\Rightarrow$ no is Por matios.

Examples: Is the Pollowing Sarics Gurmergent?

$(1)\ \sum \frac{5n}{2(3)^n}$

let $a_{in} = \frac{5n}{2(3)} > 0$

$$\lim_n \sqrt{\frac{5n}{2(3)^n}} = L \cdot m - \sqrt{\frac{5}{2}} \cdot \frac{\sqrt[n]{n}}{\sqrt[3]{3^n}} = \text{Lim}_n \sqrt{\frac{5}{2}} \cdot \frac{\sqrt{n}}{3} = 1 \cdot \frac{1}{3} = \frac{1}{3}$$

$$= 1 \cdot \frac{1}{3} = \frac{1}{3}$$

$$b = \frac{1}{3} \langle 1 \Rightarrow \sum_{n=0}^{\infty} a_n \text{ Convevgent.}$$

$$\text{Excersices: } \sum_{ns0}^{n} 2^2$$

Definition: The number e

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

Remark:

The Series $\sum_{n=0}^{\infty} \frac{1}{n!}$ is Convergent Serice.

$$s_n = 1 + \frac{1}{1!} + \frac{1}{2t} + \frac{1}{sT} + \cdots + \frac{l}{n!}$$

$$= 1 + 1 + \frac{1}{2 \times 1} + \frac{1}{3 \times 2 \times 1} + \frac{1}{4 \times 3 \times 2x_1} + \frac{1}{n!}$$

$$= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \cdots + \frac{1}{n!}$$

$$< 1 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^{(n-1)}}$$

$$= 1 + 1 + \frac{1}{2} + \frac{1}{t^2} + \frac{1}{2^3} + \frac{11 + -1}{x^{n-1}}$$

$$\frac{1/2}{1/2} = 1 \Rightarrow \text{ sn } \langle t + 1 + 1 = 3$$

$\therefore S_n \langle 3 \Rightarrow \langle S_n \rangle$ **boumided and incveasing** $\Rightarrow \langle \text{sn} \rangle$ **Canvevge**

Exercises

prove that $e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)$

Example:

prove that e is irrational number.

Proof: **Suppose e is rational number** $\Rightarrow 3m, n > 0$ **sit**

$e = \dfrac{m}{n}.$

$\because e = \sum_{n=0}^{\infty} \dfrac{1}{n!} \Rightarrow S_n = 1 + 1 + \dfrac{1}{2!} + \dfrac{1}{3!} + \infty + \dfrac{1}{n!}$

$e - 5n = \dfrac{1}{(n+1)!} + \dfrac{1}{(n+2)!} + \dfrac{1}{(n+3)!}$

$= \dfrac{1}{(n+1)!} + \dfrac{1}{(n+2)(n+1)!} + \dfrac{1}{(n+3)(n+2)(n+1)!} +$

$= \dfrac{1x}{(n+1)!}\left[i + \dfrac{1}{(n+2)} + \dfrac{1}{(n+3)(n+2)} + \infty\right]$

$< \dfrac{1}{(n+1)!}\left[1 + \dfrac{1}{(n+1)} + \dfrac{1}{(n+1)^2} +\right]$

$= \dfrac{1}{(n+1)!}, \dfrac{n+1}{n} = \dfrac{1}{n+1)n!} \cdot \dfrac{n+1}{n} = \dfrac{1}{n \cdot n!}$

$(n!)e \in N$ **since** $nl_1 = n!\frac{m}{n} = n(n-1)!\frac{m}{n}$

$$= (n-1)!\, m \in N$$

and n!s $n = n!\left(1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}\right)$

$$= n! + n! + \frac{n1}{2!} + \frac{n!}{3!} + \cdots - 1$$

Since $n \geqslant 1 \Rightarrow 3$ natwal number $(e - 5n)n!$
sil $0 < e - 5_n < \frac{1}{n} < 1$ by (1) $-C!$
e is inpational amariber

- Alternating Series aj 23 4 a al

$$\sum_{n=1}^{\infty} (-1)^{n-1} a_n = a_1 - a_2 + a_3 - 44 +$$

$$\text{or } {}_{n=1}(-1)^{n+1} a_n = a_1 - a_2 + a_3 - a_4 + \cdots$$

Theorem: (Alternating Series test)

The series $\sum_{n=1}^{\infty} (-2)^{n-1} a_n +$ is Convergent if
(1) $a_n > 0, v_n$
(e) $a_{n+1} \leqslant 0, vn$
(3) Lim an $= 0$

Example: Is the fallowing series are Convergent.
(1) $\sum_{n\leqslant 1}^{\infty} \frac{(-1)^n}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$

$a_n = \frac{1}{n} > 0,\ an + 1 = \frac{1}{n+1} < \frac{1}{x} = a_n,\ \lim_{n\to\infty} \frac{1}{n} = 0 = \sum_n^n \frac{(-1)^n}{n}$ Gonvergent.
8
$\left(2 \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} ?\right.$
Absolule and Comditional Convergencen)
Pejintion (Absetutely Covergant)
A series $\sum$ an is Called absolutely convergena is the associated series $\sum$ lanl Garvergent.
Definition: (Conditionally Convergent). A series

$\sum a_n$ **is guled Condilionally Convergenl if the ssеciated series** $\sum_a$ **Covengentbut** $\sum$ **lanl divengent**

(1) lel $\sum a_n = \sum_{n=0}^{\infty} \frac{(-1)^4}{2^n}$

$\Rightarrow$ **Σlan** $|= \sum_{n=0}^{\infty} | \frac{(-y)^n}{2^n} |= \sum_{n=0}^{\infty} \frac{1}{2^n}$, **Geometric saries**

(2) let $\sum ax = \sum_{n=0}^{\infty} \frac{(-1)^2}{n+1}$

i $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ **not alosolutely Cowengent).**

$\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}, a_n = \frac{l}{n+1}, a_{n_7 1} = \frac{1}{n+2} < \frac{1}{n+1} =$ **cm**

$\therefore \sum \frac{\sum(-1)^n}{n+1}$ **Conditiontity convergent Theorem** $\Rightarrow \langle 5n \rangle$ **is Cauthy seq.**

If $\langle t_n \rangle$ **is a seq if partial sions of \sum an** $\Rightarrow t_n = a_1 + a_2 + \cdots + a_n$ **and** $\Rightarrow \langle$ **tn** $>$ **canchy ser.** $\Rightarrow \langle t_n \rangle$ **convergent** $\Rightarrow$ **2an Convengant**

If Eang 5 bn Convergent series.
Is $\sum a_n \cdot \sum b_n - (a_n + a_n + c) \cdot (l_1 + b_i + = a_1(b_1 + b_t + y) + a_2(b_1 + b_{2x}) + \cdots$
Camergent?

Definition: **(Cavshy produch of Series) let** $\sum_{n=1}^{\infty}, \sum_{n}^{\infty} b_n$ **be twa Suries and** $C_{n=}\sum_{ks}^{n} a_k b_{n-k} = a_{10} bn + a_1 b_{n-1} + \cdots + a_n b_0$

Example: $\sum_{n=0}^{\infty} a_n + \sum_{n>0}^{\infty} b_n$ **not Convergent**

$$= 1 - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}}\right)$$

(power Series A series of form $\sum_{n=0}^{\infty} a_n x^n = a_0 + ax + a_2 x^2 + 93x^3 + \cdots$

where $x \in R$ **is Called power scriesin** x

Exc: shew that \sumx+rak $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ **is Convergent.**