كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

أستاذ المادة : أ.م.د. علاء محمود فرحان علي الجميلي

اسم المادة بالغة العربية : التحليل الرياضي

اسم المادة باللغة الإنكليزية : **Mathematical Analysis**

اسم الحاضرة الرابعة باللغة العربية: بعض الخواص والنظريات حول المجاميع المفتوحة

اسم المحاضرة الرابعة باللغة الإنكليزية: **Some Properties Open Sets**

Proposition: $[a,b],[a,b),[a,\infty)$ *and* $(-\infty,b]$ **are not open set**

Proof:

If S=[a,b] ,then S is not open set ?

Since, if $x=a \rightarrow \forall r>0, B(a,r)=(a-r,a+r) \not\subset [a,b]$

Proposition: **The intersection of any tow open set is open set i.e ((the intersection of any finite family of open set is open))**

Proof:

Let $A=\{S_k : S_k \text{ is open set } k=1,2,\dots,n\}$

$T.p \bigcap_{k=1}^{n} S_k$ **is open set**

Let $x \in \bigcap_{k=1}^{n} S_k \rightarrow x \in S_k, \forall k$ **, but** S_k **is open set** $\forall k$**, then** $\exists r_k > 0$ **s.t** $B(x,r_k) \subset S_k$

Let $r=\min\{r_1, r_2, \dots, r_n\}$

Then $B(x,r) \subset S_k, \forall k.$

$\therefore B(x,r) \subset \bigcap_{k=1}^{n} S_k$ **, therefore** $\bigcap_{k=1}^{\infty} S_k$ **is open set.**

Theorem: **the infinite intersection of open sets is not necessary open set.**

Ex: let $S_n=\left(x-\frac{1}{n}, x+\frac{1}{n}\right) \forall x \in R$**, open interval.**

$n=1 \rightarrow s_1=(x-1, x+1)$

$n=2 \rightarrow S_2=(x-\frac{1}{2}, x+\frac{1}{2})$

$n=3 \rightarrow S_3=(x-\frac{1}{3}, x+\frac{1}{3})$

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When $n \rightarrow \infty \bigcap_{k=1}^{\infty} S_k=\{x\}$ **is not open**

Theorem: the union of any family (finite or infinite) – (countable or uncountable) of open set is open

Proof:

Let $A = \{S_\alpha , S_\alpha \text{ is open set } \alpha \in \wedge\}$

T.P: $\bigcup_{\alpha\in\wedge} S_\alpha$ is open set

Let $x \in \bigcup_{\alpha\in\wedge} S_\alpha \rightarrow \exists \alpha \in\wedge \ s.t \ x \in S_\alpha$

Since S_α is open set $\rightarrow \exists \alpha > 0 \ s.t$

$B(x, r_\alpha) \subset S_\alpha$, then $x \in B(x, r_\alpha) \subset S_\alpha \subset \bigcup_{\alpha\in\wedge} S_\alpha$

This is true $\forall x \in \bigcup_{\alpha\in\wedge} S_\alpha$, therefore $\bigcup_{\alpha\in\wedge} S_\alpha$ is open set

Theorem: S is open iff S is the Union of balls

Definition: let X be anon-empty set and τ is a family of subsets of X, if τ satisfy the following

1- $\phi , X \in \tau$
2- If $G , H \in \tau \rightarrow G \cap H \in \tau$
3- If $\{G_\lambda\} \in \tau \rightarrow \bigcup_{\lambda\in\wedge} G_\lambda \in \tau$

Then, the order pair (X, τ) is called topological Space.

Theorem: every metric space is topological space.

Proof:

Let (X, d) be a metric space and $\tau =$ the family of all open subsets of X, then

1- ϕ , X open sets $\rightarrow \phi, X \in \tau$
2- $G_1 , G_2 \in \tau \rightarrow G_1, G_2$ are open sets
$\rightarrow G_1 \cap G_2 \in \tau$
3- If $G_\lambda \in \tau , \lambda \in\wedge \rightarrow \forall\lambda , G_\lambda$ open subset of X
$\rightarrow \bigcup_{\lambda\in\wedge} G_\lambda$ open set of
$\rightarrow \bigcup_{\lambda\in\wedge} G_\lambda \in \tau$
$\therefore (X, \tau)$ is a topological space

Definition: let d_1 and d_2 be two metric mapping in the set X, then d_1, d_2 are called Equivalent if every open set in (X, d_1) is open in (X, d_2) and Vice Versa

Definition: let (X, d) be a metric space and $S \subseteq X$, S is called closed set if S^c is open Set where $S^c = X - s$ (Complement of S)

Example:

1- $S = X$ is closed set. Solution: Since $S^c = X^c = \phi$ open set

2- $S = \phi$ is closed set

Solution: since $S^c = \phi^c = X$ is open set

3- $S = [a, b], [a, b), S = (-\infty, b]$ are closed set in R

Solution:

if $S = [a, b] \rightarrow S^c = (-\infty, a) \cup (b, \infty)$ open set $\rightarrow S$ is closed set

4- In R , $S = \{x\}$ is closed set

Since :$S^c = (-\infty, x) \cup (x, \infty) \rightarrow S^c$ is open, So S is closed set.

5- Any finite set in R is closed set

Solution: let $S = \{x_1, x_2, \dots, x_n\} \subseteq R$.

$S^c = (-\infty, x_1) \cup (x_1, x_2) \cup \dots \cup (x_{n-1}, x_n) \cup (x_n, \infty)$

So, S^c is open , then S is closed set

6- If $S = N$, $S = Z$, then S is Closed set

Solution: let $S = N$

then $S^c = (-\infty, 1) \cup (1, 2) \cup (2, 3) \dots (\cup_{n=4}^{\infty}(n, n+1))$

$\rightarrow S^c$ is open $\rightarrow$ S is closed

if $S = Z \rightarrow S^c = (\cup_{n=1}^{\infty}(-(n+1), -n)) \cup (-1, 0) \cup (0, 1) \cup (\cup_{n=1}^{\infty}(n, n+1))$

S^c is open, then S is closed

7- The Union of finite number of closed sets is closed.

Solution: let $A = \{S_i, ;\ S_i\ closed\ set\ in\ X, i = 1, 2, \dots, n\}$

T.P: $\cup_{i=1}^{n} S_i$ is closed set i.e. T.P $(\cup_{i=1}^{n} S_i)^c$ is open set. Since S_i is closed, $\forall i$ then S_i^c is open $\forall i$ and $\cap_{i=1}^{n} S_i^c$ is open

So, $(\cup_{i=1}^{n} S_i)^c$ is open $\qquad ((\cup_{i=1}^{n} S_i)^c = \cap_{i=1}^{n} S_i^c)$

therefore $\cup_{i=1}^{n} S_i$ is closed.