UNIVERSITY OF ANBAR

كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثالثة

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اسم المادة بالغة العربية : التحليل الرياضي

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Sequences in Metric Space

Definition: Let S be any set a function f whose domain is the set N and the range is S is

Called a sequence in S.

i.e. $f: N \to S$, where $\forall\, n \in N, \exists\, x_n \in S\ s.t\ f(n) = x_n$

1. $< \frac{1}{5n} > = \frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \ldots$
2. $< \frac{1}{n+1} > = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$
3. $< 4 > = 4, 4, 4, \ldots$
4. $< n - 3 > = -2, -1, 0, 1, \ldots$

Definition: Let (X, d) be a metric space and $< X_n >$ be seq. in X, then $< X_n >$ is said to be converges to appoint in X, if $\forall\, \epsilon > 0, \exists k \in N\ s.t\ d?(X_n, x) < \epsilon, \forall n > k$. We write $X_n \to x$ or $\lim_{n\to\infty} X_n = x$, x is called A Limit point of $< X_n >$. If $\forall n > K$, does not Converge, them $< X_n >$ is called divergent Sequence.

Not that: K depend on ϵ only.

$\forall \epsilon > 0, \exists\, k \in N\ s.t\ d(X_n, x) < \epsilon, \forall n > k \implies X_n \in B(x, \epsilon)$.

Example: Let $< X_n > = < 1 >$ constant seq. show that $\lim_{n\to\infty} X_n = 1$

$< 1 >$ convergs to 1 since $\forall\, \epsilon > 0, \exists\, k \in N$

$s.t\ d(X_n, x) = |1 - 1| = 0 < \epsilon, \forall n > k$

Example: Let $< X_n >$ be a seq. defined by $X_n = \begin{cases} n\ if\ n \leq 50 \\ 3\ if\ n \geq 50 \end{cases}$ .show that $\lim_{n\to\infty} X_n = 3$

Solution:

$< X_n \geq 1, 2, 3, \dots, 50\ , 3\ , 3\ , 3\ , \dots$

$\forall \epsilon > 0\ , \exists k = 50\ s.t\ d(X, x) = |3 - 3| = 0 < \epsilon$

Example: Show that $\lim_{n\to} X_n = 2$, where $< X_n > = < \frac{2n-3}{n+1} >$

Solution:

$\forall\ \epsilon > 0$, to find $K \in N$ s.t $d(X_n, x) <\ \epsilon, \forall n > k$?

$$d(X_n, x) = \left|\frac{2n-3}{n+1} - 2\right| = \left|\frac{2n-3-2(n+1)}{n+1}\right|$$

$$= \left|\frac{2n-3-2n-2}{n+1}\right| = \left|\frac{-5}{n+1}\right| = \frac{5}{n+1}$$

$\forall \epsilon > 0$, by Arch. Property $\rightarrow \exists K \in N \ni$

$\forall k > 5 \rightarrow \frac{5}{\epsilon} < k.$

$\forall n > K \rightarrow n + 1 > k + 1\ and\ k + 1 > k\ , k > \frac{5}{\epsilon}$

$\Longrightarrow n + 1 > k + 1 > k > \frac{5}{\epsilon}$

$\frac{1}{n+1} < \frac{\epsilon}{5}\ , \forall n > k$

Exercise:

1. Let $< X_n > =< \frac{2}{\sqrt{n}} >$, show that $\lim_{n\to\infty} X_n = 0$
2. Let $< X_n > = < \frac{5n-4}{2-3n} >$, show that $\lim_{n\to\infty} X_n = -\frac{5}{3}$
3. Let $< X_n > = < \frac{2-7n}{1-5n} >$, show that $\lim_{n\to\infty} X_n = \frac{7}{5}$

Show that the following sequence are divergent

1. $< X_n > = < \sqrt{n} >$
2. $< X_n > = < (-1)^n >$
3. $< X_n > 3^n >$
4. $< X_n > = < \frac{n^2}{2n-1} >$

Theorem: If $< X_n >$ is convergent sequence in (X, d), then $< X_n >$ has a unique limit point.

Proof:

Suppose that $< X_n >$ has two limit points x and y with $x \neq y$ and $d(x, y) = \epsilon$

Since $X_n \to y \Longrightarrow \forall \epsilon > 0, \exists k_2 \in N \; s, t \; d(x, y) < \frac{\epsilon}{2}$

Let $k = \max\{k_1, k_2\}$

Since $d(x, y) \leq d(x, x_n) + d(x_n, y) < \frac{\epsilon}{2} + \frac{\epsilon}{2} = \epsilon$

$\Rightarrow d(x, y) < \epsilon, \forall \epsilon > 0$. This true only when $d(x, y) = 0 \Rightarrow x = y \to C!$

$\therefore < X_n >$ has a unique limit point.

Definition: A seq. $< X_n >$ is called bounded the set $\{X_n : n \in N\}$ is bounded i.e. $< x_n >$ is bounded if $\exists m > 0 \; s.t \; d(x_n, x_m) \leq M, \forall n, \forall m$