كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الرابعة

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اسم المادة بالغة العربية :مقاسات

اسم المادة باللغة الإنكليزية : **MODULES**

اسم الحاضرة الأولى باللغة العربية: الزمرة والحلقة والمقاس

اسم المحاضرة الأولى باللغة الإنكليزية: **Group, ring and Module**

محتوى المحاضرة الأولى

Definition 1.2 (Group) Let G be a nonempty set with a binary operation that assigns to each ordered pair (a b,) of elements of G an element ab in G. We say G is a group under this operation if the following three properties are satisfied:

1. Associativity The operation is associative, that is (ab c a bc) = ( ) for all a b c G , , ∈ .

2. Identity There is an element e (called the identity) in G, such that ea ae a = = for all a G∈ .

3. Inverse For each element a G∈ , there is an element 1 a – in G such that 1 1 aa a a e – – = = .

Example 1.1 1. The set of integers , the set of rational numbers and the set of real numbers are all groups under ordinary addition. In each case the identity is 0 and the inverse of a is –a.

Definition 3.1 (Modules). Let R be a ring. An R-module is a set M together with two operations

+ : M_M !M and _ : R_M !M

(an “addition” in M and a “scalar multiplication” with elements of R) such that for all m;n 2 M and a;b 2 R we have:

(a) (M;+) is an Abelian group;

(b) (a+b) _m = a _m+b _m and a _ (m+n) = a _m+a _ n;

(c) (a _ b) _m = a _ (b _m);

(d) 1 _m = m.

We will also call M a module over R, or just a module if the base ring is clear from the context.

Example 3.2.

(a) For a field R, an R-module is by definition exactly the same as an R-vector space [G2, Definition 13.1].

(b) Of course, the zero set f0g is a module, which we often simply write as 0.

(c) For n 2 N>0 the set Rn = f(a1; : : : ;an) : a1; : : : ;an 2 Rg is an R-module with componentwise addition and scalar multiplication. More generally, for two R-modules M and N the product M_N with componentwise addition and scalar multiplication is an R-module again.

(d) A Z-module is just the same as an Abelian group. In fact, any Z-module is an Abelian group by definition 3.1 (a), and in any Abelian group (M;+) we can define a multiplication with integers in the usual way by (□1) _m := □m and a_m := m+_ _ _+m (a times) for a 2 N and m 2 M.

(e) Any R-algebra M is also an R-module by Remark 1.24, if we just forget about the possibility to multiply two elements of M.