كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الرابعة

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اسم المادة بالغة العربية :مقاسات

اسم المادة باللغة الإنكليزية : **MODULES**

اسم الحاضرة الأولى باللغة العربية: المقاس الجزئي الاعظمي

اسم المحاضرة الأولى باللغة الإنكليزية: **Maximal submodule**

محتوى المحاضرة الرابعة

(2.1): Let M be an R-module, a proper submodule N of M, is called Large-maximal (L-maximal) submodule of M if there exists a submodule K of M such that $N < K \leq M$, then K is essential submodule of M ($K \leq e\ M$). An ideal I is called L-maximal ideal if there exists an ideal J of R such that $I < J \leq R$, then J is essential ideal of R ($J \leq e\ R$).

Remarks and Examples (2.2):

1- Every maximal submodule is L-maximal submodule. Thus in $Z4$ as Z-module: $\{\overline{0},\overline{2}\}$ is maximal and L-maximal submodule since $\{\overline{0},\overline{2}\}<Z4\leq Z4$ and $Z4\leq eZ4$.

2- The converse of (1) is not true, as the following example: In Z as Z-module, 4Z is L-maximal submodule since $4Z<2Z\leq Z$ and $2Z\leq eZ$ but 4Z is not maximal submodule since $2Z\neq Z$.

3- A submodule of L-maximal submodule need not be L-maximal submodule , as the following example: In $Z36$ as Z-module,$4Z36$ is L-maximal since $4Z36<2Z36\leq Z36$ and $2Z36\leq eZ36$ but $12Z36$ not L-maximal since $12Z36<4Z36\leq Z36$ and $4Z36$ is not essential in $Z36$.

4- If M is a uniform module, then every submodule of M is L-maximal. For examples: the Z- modules Q and $ZP\infty$.

5- The converse of (4) is not true, as the following example: In $Z6$ as Z-module, $2Z6$ and $3Z6$ are maximal hence L-maximal submodule by (1) , but $Z6$ is not uniform module.

6- Every essential submodule of M is L-maximal submodule of M.

Proof: Let N be a proper submodule of M such that $N \leq eM$ and let $N<K\leq M$, then we have $N \leq eK\leq eM$ by [1], so $K \leq eM$ hence N is L-maximal submodule of M.

13- Every integral domain is uniform module, and then every submodule is L-maximal submodule by (4).

14- Every simple module is uniform module, and then every submodule is L-maximal submodule by (4).

Proposition (2.3): Let N and K are proper submodules of M such that $N\leq K$, if N is L-maximal submodule of M then K is L-maximal submodule of M .