UNIVERSITY OF ANBAR

كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الرابعة

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اسم المادة بالغة العربية :مقاسات

اسم المادة باللغة الإنكليزية : MODULES

اسم الحاضرة الأولى باللغة العربية: المقاس الدوار

اسم المحاضرة الأولى باللغة الإنكليزية :Cyclic module

محتوى المحاضرة الخامسة

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1 Definition :- (Cyclic Module) المقاس الدائريLet (M ,+) be any R-module, then M is called cyclic module if x M. Such that =Rx={r. x : }=M, and x is called generators of module M . Example(1):- Let (Z , +) be a Z – module , then Z is a cyclic module since , 1 Z . s. t. , = Z . 1 ={ r. 1 : }={ r : } =Z

Example(2):- Let (,+)be a Z– module, then is a cyclic module , since 2 s .t , = 2Z ={ r. 2 : }={2. r : }= .

Example(3):- Let (, be a Z module . Find the generators of module . Solution :- To find the generators Therefore, the module generated

Proposition (6-3):- Let (M ,+) be an R-module , then if x M .Then = Rx is a sub module of module M .

Definition :- (Simple R-module) حلقة المقاس البسيطLet (M ,+) be an R-module , then M is called simple R-module if the only sub module of M is trivial sub module which are M and {0}.

Example:- The module (as Z module ,where p is a prime number is a simple Z module Since , the only sub module of is trivial sub module which are : and {0}. for example as Z module is a simple Z module as Z module is a simple Z module as Z module is a simple Z module as Z module is a simple Z module . Where p is prime.

Example The module (as Z module is not simple Z module , since the module has proper sub module which are : ( ) ( ) 3 Proposition (6-4):- Every simply R module is cyclic module. Proof:- Let (M, +) be a simply module < T.P. M is a cyclic module > Let s .t , , then by Proposition (5) we get is a sub module of module M . either , =M or ={0} If =M M is a cyclic module generated by x . If ={0} C! (since) Therefore, M is a cyclic module.

Example: Give an example of cyclic module but is not simple module Solution :- Let (,) be a Z-module , then is a cyclic module generated by 1 , 5 but is not simple Z-module , since the module has proper sub module

Definition: - (Quotient Module) ألقسمه مقاس Let be an R-module and N is a sub module of M. Then the set ⁄ is called quotient set of module such that for all x, y

Theorem . (Quotient module) If (R ,+,.)is a ring with identity and let be an R-module and N is a sub module of module M. Then (M is an R–module, which is called Quotient module .

Proposition (6-7):- If (R ,+,.)is a ring with identity and be an R-module, N is a submodule of M and (⁄ is R–module, then iff . iff .

Module Homomorphism

Definition :- (Module Homomorphism) Let and be two R-modules, then a function f: is called module homomorphism or R-homomorphism iff : 1- f(x+ y) = f(x) + f(y) , x , y + f(r. x) = r . f(x) , r R and x

Example . Let and be two R-modules, and let g: be a function s.t. , g(m) = 0 , . Show that g is module homomorphism . Solution :- (1) To show , **m m** + and , then = = + . (2) , Z To show , **r m r m** =r . 0 = r . g(m) g is module homomorphism .

Example . Let (Z , +) be a Z module and f: Z Z be a function such that . Is f module homomorphism ? 6 Solution :- (1) To show , **m** + **m** and , then = . + .m = + . (2) , Z To show , **r r** = (2 . r) . n = (r.2). n =r . (2.n) = r . f(n) f is module homomorphism .

Example . Let (Z , +) be a Ze -module and f: Z Z be a function s .t. , is f module homo ? Solution :- , To show + , and , then = . + .y)+3 and + =(= (2.x +2.y) + 6 by and we get , + f is not module homo.

Proposition - Let and be two R-modules and f: be a module homomorphism then 1-f( ) = 2- f( ) = f( ) , . 3- f (= f( ) f(y) , .

Theorem(6.9):- If and are two R modules , and if f : be a module homomorphism , then f(is a sub module of module . 7 Proof :- f(= { f(x) : } < T.P. f(is a sub module of >? Since , is R-module (i)Let f(x),f(y) ,where x , y , be a R-module and x , y x + y f(x + y) (بأخذ f للطرفيين) f(x) + f(y) (since f is module homo) (ii)Let and , x and x and is a R module. r . x f(r . x) (بأخذ f للطرفيين) r . f(x) (since f is module homo.) Therefore, by steps (i) and (ii) we get, (f(is a sub module of module

Theorem - If and are two R modules , and f : a a module homomorphism and onto function . Then

(1) If (N ,+) is a sub module of module . Then (f(N) , is also sub module of module .

(2) If (,) is sub module of module . Then ((K) , +) is also sub module of module .