كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الرابعة

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اسم المحاضرة الأولى باللغة الإنكليزية Indecomposable module

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. Indecomposable module

Definition

A module M is called indecomposable if $0 \neq M$ and cannot be written as a direct sum of two non-zero submodules. In other words, $M = N \oplus K$ where $0 \neq N$ and $\neq K$. The dual of indecomposable is, of course, decomposable

Remarks and Examples

1. $Z_6 = <2> \oplus <3> \simeq Z_2 \oplus Z_3$.

2.Indecomposable module is a weaker than simple module and maybe called irreducible module. simple means "no proper submodule" .

3. A direct sum of indecomposables is called completely decomposable.

4. completely decomposable is a weaker than semi-simple, which is a direct sum of simple modules.

5. A direct sum decomposition of a module into indecomposable modules is called an indecomposable decomposition.

6.A vector space is indecomposable module if and only if it has one dimension.

7. Every vector space is completely decomposable (indeed, semi-simple), with infinitely many summands if the dimension is infinite.

8. Primary decomposition is a decomposition into indecomposable modules, so every finitely-generated module over a PID is completely decomposable module

9. Over the integers $\mathbf{Z}$, modules are abelian groups, so finitely-generated abelian group is indecomposable if and only if it is isomorphic to $\mathbf{Z}$ or to a factor group of the form $Z./P^{n}Z$ for some prime number p and some positive integer n.

10. Every finitely-generated abelian group is a direct sum of (finitely many) indecomposable abelian groups.

Theorem

Every simple module is indecomposable. The converse is not true in general.

Proof.

Definition

Any left principal indecomposable module of a ring R is a left submodule of R, so that is a direct summand of R and finally is an indecomposable module.

Examples

1. Projective module is principal indecomposable module.

2. Cyclic module is principal indecomposable module.