كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الرابعة

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اسم المادة بالغة العربية :مقاسات

اسم المادة باللغة الإنكليزية : **MODULES**

اسم الحاضرة الأولى باللغة العربية: المقاس الحر

اسم المحاضرة الأولى باللغة الإنكليزية: Free Module

محتوى المحاضرة الثامنة

Free Modules Fix a ring R.

Definition 7.1. A free module is one which is isomorphic to a direct sum of copies of R. That is, M is a free module if M ∼= M i∈I R (=: R I if I = {1, 2, . . . , n} .) Proposition 7.1. Let R be a commutative ring, and G a group. Let RG = M g∈G gR be the free right (or left) R-module with "basis" the elements of G. The R-module structure on RG extends to an R-algebra structure with multiplication induced by group multiplication, that is, □ □ X g∈G grg □ □ X h∈G hsh ! := X l∈G l □ □ X gh=l rgsh □ □ , and unit map ι : R → RG; r 7→ 1r. This is called the group algebra.

Proof. Generating Submodules Let M be a module, I an index set. Recall there is a group isomorphism (from the universal property) HomR M i∈I R, M! ∼= Y i∈I HomR (R, M) prop5.2 ∼= Y i∈I M. We then ask, what is the homomorphism in HomR RI , M corresponding to (mi)i∈I ∈ Q i∈I M? The answer is called the universal property for free modules, and is (mi)i∈I : (ri)i∈I 7→ X i∈I miri . Such an expression of element of M is called a (right) R-linear combination of the mi 's.

Example 7.1. We see HomR (R m, Rn) ∼= Ym i=1 R n = (R n) m = Mnm(R). Homomorphisms corresponding to n × m matrices are given by left multiplication. Example 7.2. Let R be a commutative ring, and G a group. Let H be a subgroup of G.

Exercise. Show RH is an R-subalgebra of RG. Hence RG is also a (right) RH-module. In fact, it is a free RH-module. Why? For each left coset C of H in G, we pick a representative gC so C = gCH. The universal property of free modules gives a homomorphism (gC)C∈G/H : M C∈G/H (RH) −→ RG (aC)C∈G/H 7→ X C∈G/H gCaC. Exercise. This is clearly bijective so gives an isomorphism.

Proposition 7.2. Let M be a (right) R-module, and L a subset. The submodule generated by L is the set of all R-linear combinations of elements of L. It is a submodule of M, and is denoted X l∈L lR.

Proof. XlR is a submodule as it is the image of the R-linear map (l)l∈L : M l∈L R −→ M given by the universal property of free module