щуЯ : щу٦Оэ٥F анЯЛЯ٥ щТОЋ٥F
ЧFуЎFтО٥F: ИОУ٥F мF бЃЧ٥F
щЛ٦FО٥F :щЯсОг٥F
СэК СгјвСŦFв.þ.а.F : ЩþFг٥F МFэЂY
ЧFЂFЧв: щу٦ОЛ٥F щП٥F٦ ЩþFг٥F бЂF
MODULES : щтЭуЯЫжшF щПЯ٥F٦ ЩþFг٥F бЂF
сАFЧЂъF ÈFЧг٥F :щу٦ОЛ٥F щПЯ٥F٦ п٥мцF ЩОЎFјг٥F бЂF
Projective Module: щтЭуЯЫжшF щПЯ٥F٦ п٥мцF ЩОЎFјг٥F бЂF

щзвFх٥F ЩОЎFјг٥F онэјв

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2. Projective Module

Definition

An R-module P is called projective if P ⊕ Q is a free R-module where Q is any module over R

Remark

Let M be an R-module and P ⊆ M is a submodule. Then P it is not necessarily free when M=P ⊕ N. For example, suppose R is the quotient module (R = Z/6Z), so Z/6Z ≅ Z/2Z ⊕Z/3Z such that Z/2Z and Z/3Z are Z/6Z-modules with an isomorphism of Z/6Z-modules. See Z/2Z and Z/3Z are non-free modules isomorphic to direct summands of the free module Z/6Z.

Lemma

Every free R-module is projective.

Example

Z/2Z and Z/3Z are non-free projective Z/6Z-modules.

Remark

If R is a PID, Let M be a free module over principal ideal domain with M has a finite rank, and $N \subseteq M$ is a submodule. So, N is a free module and rank $N \leq$ rank M.

Corollary

Every finitely generated projective module over principal ideal domain is free.

Proof. Suppose P is a finitely generated projective R-module. Then an onto mapping h: $R^n \to P$; $n > 0$. Hence there is an isomorphism $P \oplus Ker(f) \cong R^n$. Now we can identify P with a submodule of R^n, and by Remark 44.1 P is a free module.

Definition

An A-module P is projective if for every surjective mapping h: $M_1 \to M_2$ and another mapping g: $P \to M_2$;

$\exists\, h : P \to M_1 \ni g = f\, h$,

Proposition

For a projective A-module P, the following are equivalent:

(a) P is finitely generated.

(b) P is a direct summand of a free module of finite rank.

(c) There exists an exact sequence $A^n \to A^n \to P \to 0$ for some n. P here is finitely presented.

Proof. (a) ⇒ (b). Let P be generated by k elements. then there is a onto mapping π from A^n into P. So, $\oplus$ Q ≅A n with Q ≅ Ker π.

(b) ⇒ (c). Let $P \oplus Q \cong A^n$ for some n and some A-module Q. Let g: $A^n \to A^n$ be the composition of $A^n \to Q$ with $Q \to A^n$. Then Im(g) = {(0, q) ∈ P ⊕ Q: q ∈ Q}, and also the kernel of the projection $A^n \to P$. So, $A^n \to A^n \to P \to 0$ is an exact sequence.

(c) ⇒ (a). Immediate

Lemma

I. Every direct summand of a projective module is projective.

ii. Every direct summand of a free module is projective.

iii. Let M be a module and P a projective module. If P is a quotient of M then P is a direct summand of M. iv. Every projective module is a direct summand of a free module

Lemma

Let P and P ′ be projective indecomposable modules, at least one of which is finitely generated. If some nonzero module is a quotient of both P and P ′ then P ≅ P′.

Proof. Suppose that P is finitely generated, and that there exist a nonzero module M and epimorphisms $\pi: P \twoheadrightarrow M$, $\pi': P' \twoheadrightarrow M$. Since P and P ′ are projective, there exist homomorphisms