



كلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الثانية

أستاذ المادة : ميمون ابراهيم اسماعيل

اسم المادة بالغة العربية : التفاضل المتقدم

اسم المادة باللغة الإنكليزية : Advance Calculus

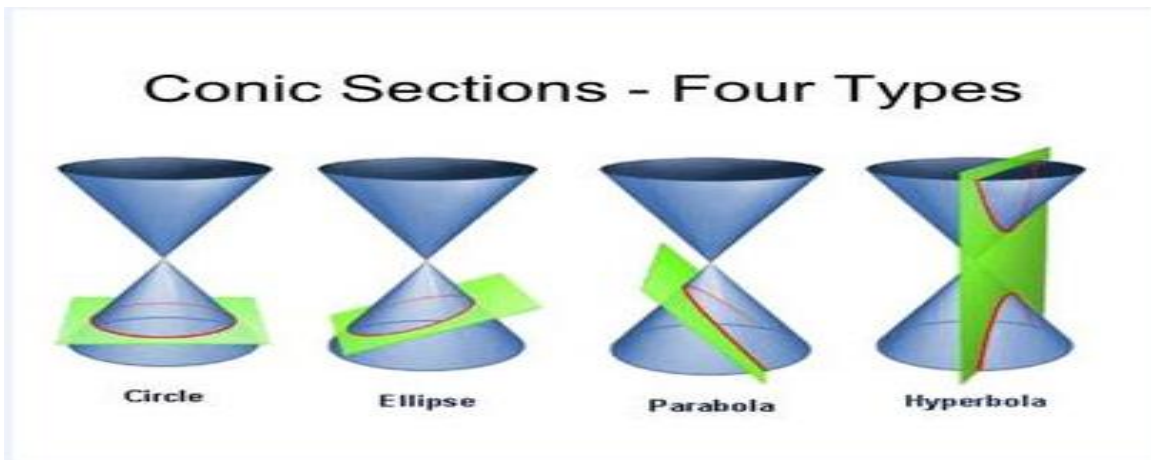
اسم المحاضرة الأولى باللغة العربية: القطوع المخروطية

اسم المحاضرة الأولى باللغة الإنكليزية: Conic sections

In this chapter we examine the Cartesian graph of any equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0,$$

In which A,B, and C are not all zero, and show that it is nearly always a conic section. Also, we will give geometric definitions of a circle, parabola, ellipse, and hyperbola and derive their standard equations.



1) **The circle:** the set of points in a plane whose distance from some fixed center point is a constant radius value. If the center (h,k) and the radius is r , the standard equation for the circle is $(x - h)^2 + (y - k)^2 = r^2$.

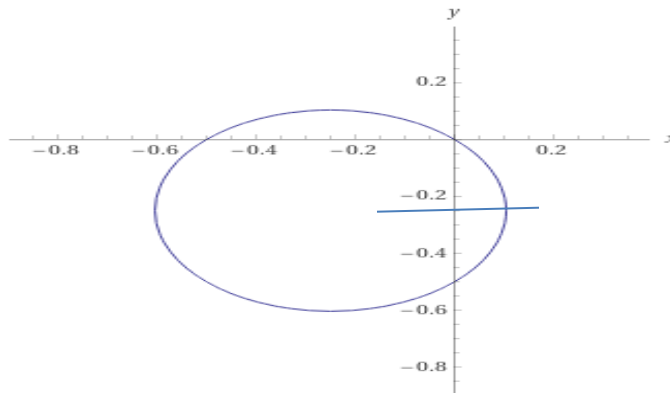
Example: Discussed and sketch the following equation

$$2x^2 + 2y^2 + x + y = 0.$$

Solution:

$$x^2 + \frac{x}{2} + y^2 + \frac{y}{2} = 0 \rightarrow \left(x + \frac{1}{4}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{1}{8}$$

$$(x - h)^2 + (y - k)^2 = R^2 \rightarrow (h, k) = \left(\frac{-1}{4}, \frac{-1}{4}\right), R = \frac{1}{2\sqrt{2}}$$



2) A **parabola** is the set of points in a plane that are equidistant from given fixed point (**focus**) and fixed line (**directrix**) in the plane.

Table of standard-form

	Equation	Focus	Directrix	Vertex	Opens
1	$y^2 = 4px$	$(p,0)$	$X=-p$	$(0,0)$	To the right
	$(y - k)^2 = 4p(x - h)$	$(h+p,k)$	$X=h-p$	(h,k)	To the right
2	$y^2 = -4px$	$(-p,0)$	$X=p$	$(0,0)$	To the left
	$(y - k)^2 = -4p(x - h)$	$(h-p,k)$	$X=h+p$	(h,k)	To the left
3	$x^2 = 4py$	$(0,p)$	$y=-p$	$(0,0)$	Up

	$(x - h)^2 = 4p(y - k)$	$(h, k+p)$	$y = k - p$	(h, k)	Up
4	$x^2 = -4py$	$(0, -p)$	$y = p$	$(0, 0)$	Down
	$(x - h)^2 = -4p(y - k)$	$(h, k-p)$	$y = k + p$	(h, k)	Down

Examples: Find the focus, vertex, and directrix of the parabolas and sketch the parabola:

1) $x^2 + 2y = 0$

2) $y^2 + 2x + 4y + 6 = 0$

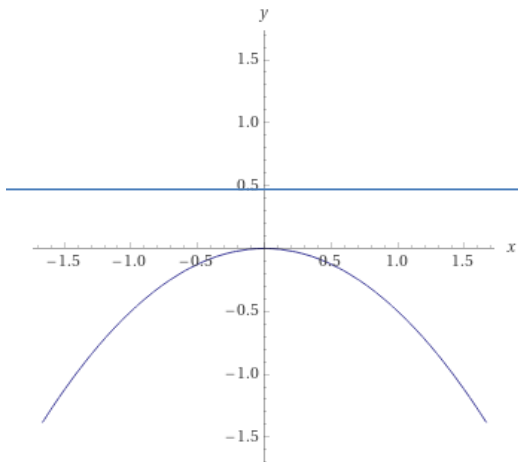
3) Using the definition of parabola to find standard equation of parabola which focus is $F(h, k + p)$ and the directrix is $y = k - p$

Solution:

1) We find the value of p in the standard equation: $x^2 = -4py$

$$x^2 = -4py \Rightarrow x^2 = -2y \text{ so } 4p = 2 \Rightarrow p = \frac{1}{2}$$

Then, the focus $(0, -\frac{1}{2})$, vertex $(0, 0)$, and directrix $y = \frac{1}{2}$



2)

$$y^2 + 2x + 4y + 6 = 0$$

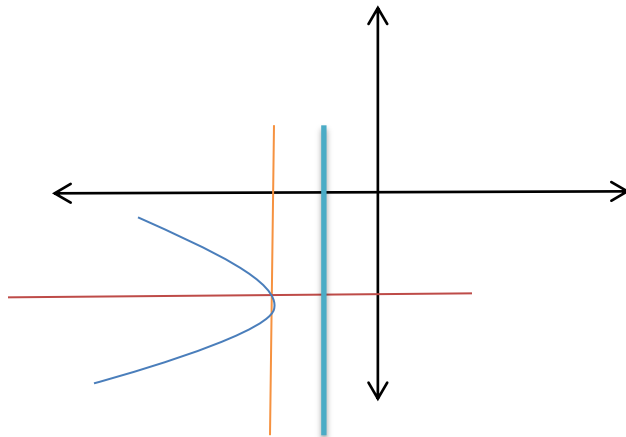
$$y^2 + 4y + 4 - 4 = -2x - 6$$

$$y^2 + 4y + 4 = -2x - 6 + 4$$

$$y^2 + 4y + 4 = -2x - 2$$

$$(y + 2)^2 = -2(x + 1)$$

$$V(-1, -2), F\left(-1 - \frac{1}{2}, -2\right), \text{ so } x = h + p \rightarrow x = -1 + \frac{1}{2} = -\frac{1}{2}$$



3) H.W