كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثانية

أستاذ المادة : ميمون ابراهيم اسماعيل

اسم المادة بالغة العربية :التفاضل المتقدم

اسم المادة باللغة الإنكليزية : **Advance Calculus**

اسم المحاضرة الثالثة باللغة العربية: القطوع المخروطية (القطع الزائد)

اسم المحاضرة الثالثة باللغة الإنكليزية: **Conic sections** (Hyperbola)

4) Hyperbola is the set of points in a plane whose distance from two fixed points (foci) in the plane have a constant difference.

Table of standard-form

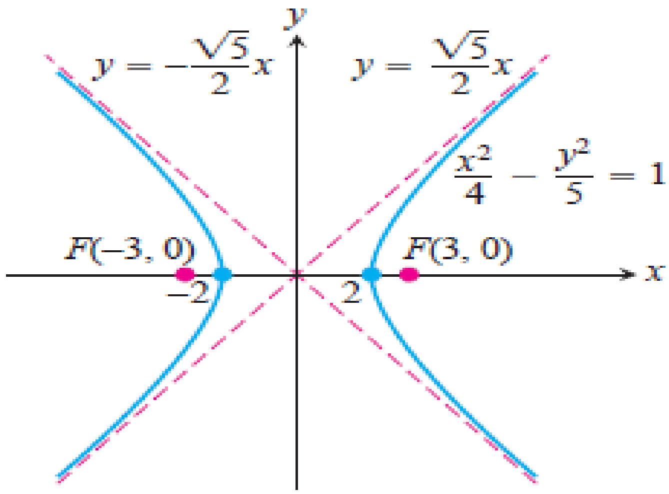
	Equation	Foci	Vertices	Minor axis	center
1	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	$F_{1,2}(\mp c,0)$ $\mathbf{c^2=a^2+b^2}$	$A_{1,2}(\mp a,0)$	$B_{1,2}(0,\mp b)$	(0,0)
	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\boldsymbol{F_{1,2}(h\mp c,k)}$	$A_{1,2}(h\mp a,k)$	$\boldsymbol{B_{1,2}(h,k\mp b)}$	(h,k)
2	$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$	$F_{1,2}(0,\mp c)$ $\mathbf{c^2=a^2+b^2}$	$A_{1,2}(0,\mp a)$	$B_{1,2}(\mp b,0)$	(0,0)
	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$	$\boldsymbol{F_{1,2}(h,k\mp c)}$	$A_{1,2}(h,k\mp a)$	$\boldsymbol{B_{1,2}(h\mp b,k)}$	(h,k)

4) Examples: Find the center, vertices, and foci of the hyperbola and sketch the hyperbola: (1) $\boldsymbol{\frac{x^2}{4}-\frac{y^2}{5}=1}$ (2) $\boldsymbol{16x^2+160x-9y^2+18y-185=0}$.
3)$5x^2-4y^2+20x+8y=4$.

Solution:

1) Center : (0,0) vertices: $A_{1,2}(\mp a,0)=A_{1,2}(\mp 2,0)$

Foci: $c^2=a^2+b^2 \Rightarrow c=\mp\sqrt{\mathbf{4+5}} \Rightarrow c=\mp\sqrt{9} \Rightarrow F_{1,2}(\mp c,0)=F_{1,2}(\mp 3,0)$



2) $\mathbf{16x^2 + 160x - 9y^2 + 18y - 185 = 0.}$

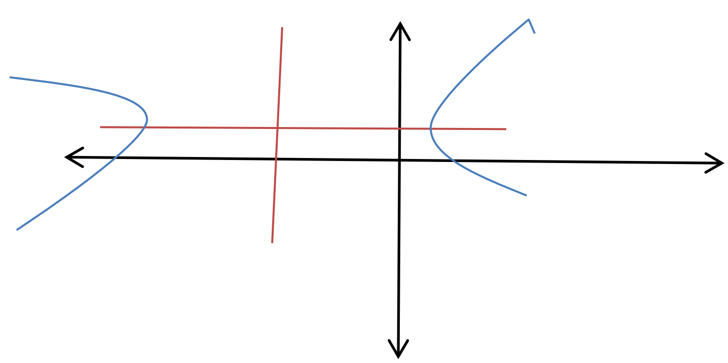
$$\mathbf{16(x^2+10x) - 9(y^2 - 2y) = 185}$$

$$\mathbf{16(x+5)^2 - 9(y-1)^2 = 185 + 400 - 9}$$

$$\mathbf{16(x+5)^2 - 9(y-1)^2} = 576$$

$$\mathbf{\frac{(x+5)^2}{36} - \frac{(y-1)^2}{64} = 1,}$$

$$\mathbf{C(h,k) = c(-5,1), F_{1,2}(-5 \pm 10, 1), V_{1,2}(-5 \pm 6, 1),}$$



3) $5x^2 + 20x - 4y^2 + 8y = 4 \rightarrow 5(x^2 + 4x) - 4(y^2 + 2y) = 4$

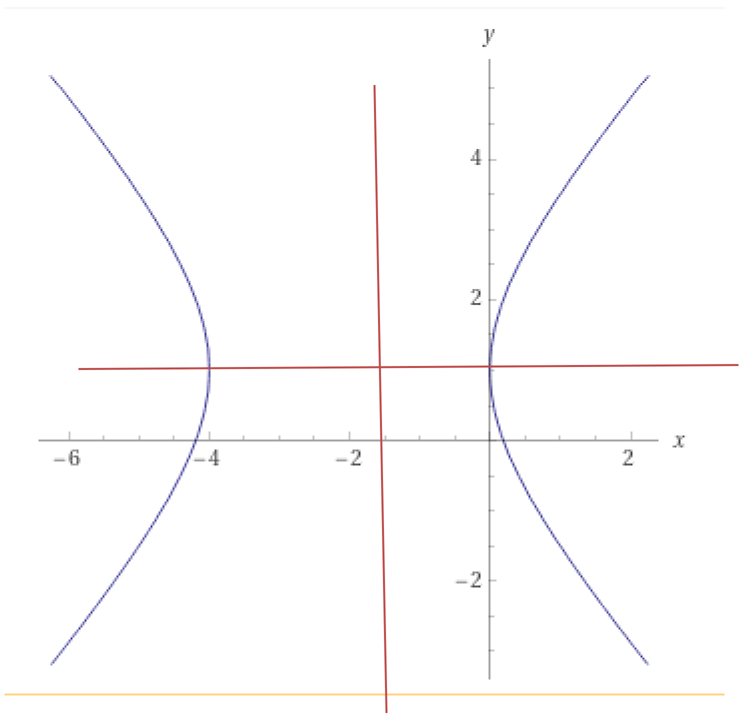
$5(x^2 + 4x + 4 - 4) - 4(y^2 + 2y + 1 - 1) = 4$

$$5(\boldsymbol{x}+\mathbf{2})^2-4(\boldsymbol{y}-\mathbf{1})^2 \ = 20$$

$$\frac{(\boldsymbol{x}+\mathbf{2})^2}{\mathbf{4}}-\frac{(\boldsymbol{y}-\mathbf{1})^2}{\mathbf{5}}=\mathbf{1}\ ,\ \frac{(\boldsymbol{x}-\boldsymbol{h})^2}{\boldsymbol{a}^2}-\frac{(\boldsymbol{y}-\boldsymbol{k})^2}{\boldsymbol{b}^2}=\mathbf{1}$$

$(\boldsymbol{h},\boldsymbol{k})=(-\mathbf{2},\mathbf{1}),\boldsymbol{F}_{\mathbf{1},\mathbf{2}}(\boldsymbol{h}\mp\boldsymbol{c},\boldsymbol{k})=\boldsymbol{F}_{\mathbf{1},\mathbf{2}}(-\mathbf{2}\mp\mathbf{3},\mathbf{1})$

$\boldsymbol{A}_{\mathbf{1},\mathbf{2}}(\boldsymbol{h}\mp\boldsymbol{a},\boldsymbol{k})=\boldsymbol{A}_{\mathbf{1},\mathbf{2}}(-\mathbf{2}\mp\mathbf{2},\mathbf{1}),\ \boldsymbol{B}_{\mathbf{1},\mathbf{2}}(\boldsymbol{h},\boldsymbol{k}\mp\boldsymbol{b})=\boldsymbol{B}_{\mathbf{1},\mathbf{2}}(-\mathbf{2},\mathbf{1}\mp\sqrt{\mathbf{5}})$



Note that eccentricity is $\frac{\boldsymbol{c}}{\boldsymbol{a}}$ $(e=\frac{c}{a})$ where $0<e<1$.