كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثانية

أستاذ المادة : ميمون ابراهيم اسماعيل

اسم المادة بالغة العربية :التفاضل المتقدم

اسم المادة باللغة الإنكليزية : **Advance Calculus**

اسم المحاضرة الرابعة باللغة العربية: تدوير المحاور

اسم المحاضرة الرابعة باللغة الإنكليزية: Rotating coordinate Axes

Equations For Rotating coordinate Axes

The equations for the rotations we use are derived in the following way. In the notation of Figure (1) which shows in anticlockwise rotation about the origin through an angle ($\propto$),

$\cos(\propto +\theta) = \frac{x}{\overrightarrow{op}}$, $\sin(\propto +\theta) = \frac{y}{\overrightarrow{op}}$

x $= \overrightarrow{op}\cos(\propto)\cos(\theta) - \overrightarrow{op}\sin(\propto)\sin(\theta)$.

y $= \overrightarrow{op}\sin(\propto)\cos(\theta) + \overrightarrow{op}\cos(\propto)\sin(\theta)$.

Note that $x' = \overrightarrow{op}\cos(\theta)$ *and* $y' = \overrightarrow{op}\sin(\theta)$

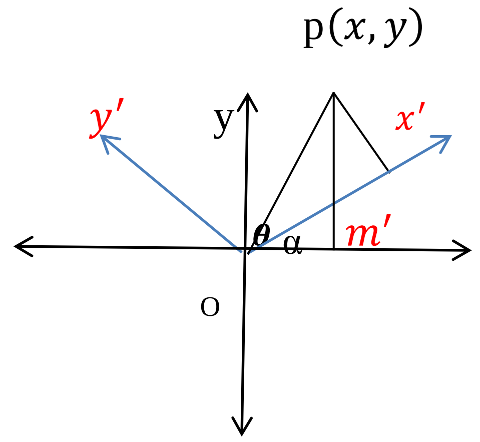

Figure (1)

Thus,

x $= x'\cos(\propto) - y'\sin(\propto)$.

y $= x'\sin(\propto) + y'\cos(\propto)$.

Now, if we apply equations above to the equation $\boldsymbol{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$,

We obtain a equation $\boldsymbol{A'x^2 + B'xy + C'y^2 + D'x + E'y + F' = 0}$, where:

$A' = Acos^2\alpha + Bcos\alpha\ sin\alpha + Csin^2\alpha$

$B' = Bcos2\alpha + (C - A)sin2\alpha$

$C' = Asin^2\alpha - Bcos\alpha\ sin\alpha + Ccos^2\alpha$

$D' = Dcos\alpha + E\ sin\alpha$

$E' = E\ cos\alpha - Dsin\alpha$

$F' = F$.

To find ($\propto$), we put $B' = 0$ and solve the resulting equation,

$Bcos2\alpha + (C - A)sin2\alpha = 0$, so $\cot 2\alpha = \frac{A-C}{B}$ *or* $\tan 2\alpha = \frac{B}{A-C}$

Example : Prove that the equation $x^2 + y^2 = r^2$ does not change for any angle α in the rotation.

Solution:

For any angle $\alpha \rightarrow$ x = $x^{'}\cos(\propto) - y^{'}\sin(\propto) and \;\; y = x^{'}\sin(\propto) + y^{'}\cos(\propto)$

$$\left(x^{'}\cos(\propto) - y^{'}\sin(\propto)\right)^2 + \left(x^{'}\sin(\propto) + y^{'}\cos(\propto)\right)^2 = r^2$$

$$\boldsymbol{x'^2 + y'^2 = r^2}$$