كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثانية

أستاذ المادة : ميمون ابراهيم اسماعيل

اسم المادة بالغة العربية :التفاضل المتقدم

اسم المادة باللغة الإنكليزية : **Advance Calculus**

اسم المحاضرة الخامسة باللغة العربية: تدوير المحاور(امثلة متنوعة)

اسم المحاضرة الخامسة باللغة الإنكليزية: Rotating coordinate Axes

Example: what kind of conic section is the curve $x^2 - \sqrt{3}xy + 2y^2 = 1$ after rotate the coordinate axes through an angle of $\frac{\pi}{6}$.

Solution:

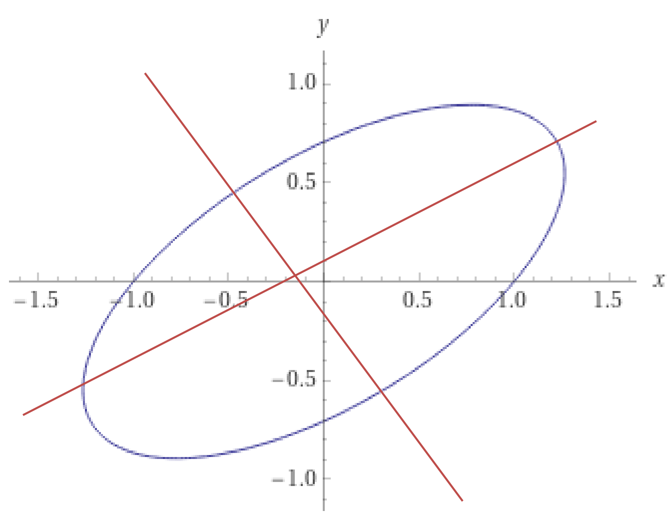
$$\propto = \frac{\pi}{6}$$

$$\mathrm{x} = x'\cos(\propto) - y'\sin(\propto) \rightarrow x = \frac{\sqrt{3}x' - y'}{2}.$$

$$\mathrm{y} = x'\sin(\propto) + y'\cos(\propto) \rightarrow y = \frac{x' + \sqrt{3}y'}{2}.$$

$$\left(\frac{\sqrt{3}x' - y'}{2}\right)^2 - \sqrt{3}\left(\frac{\sqrt{3}x' - y'}{2}\right)\left(\frac{x' + \sqrt{3}y'}{2}\right) + 2\left(\frac{x' + \sqrt{3}y'}{2}\right)^2 = 1$$

$$\frac{x'^2}{2} + \frac{y'^2}{\frac{2}{5}} = 1, A(\pm\sqrt{2}, 0), B\left(0, \frac{\pm\sqrt{2}}{\sqrt{5}}\right)$$



EXAMPLE: Decided whether the conic section with following equations represents a Parabola, an Ellipse, or Hyperbola.

i)$xy = 2$ (ii) $\boldsymbol{xy - \sqrt{2}\,x - 2 = 0}$

(iii) $4x^2 + 4xy + y^2 = 0$

Solution:

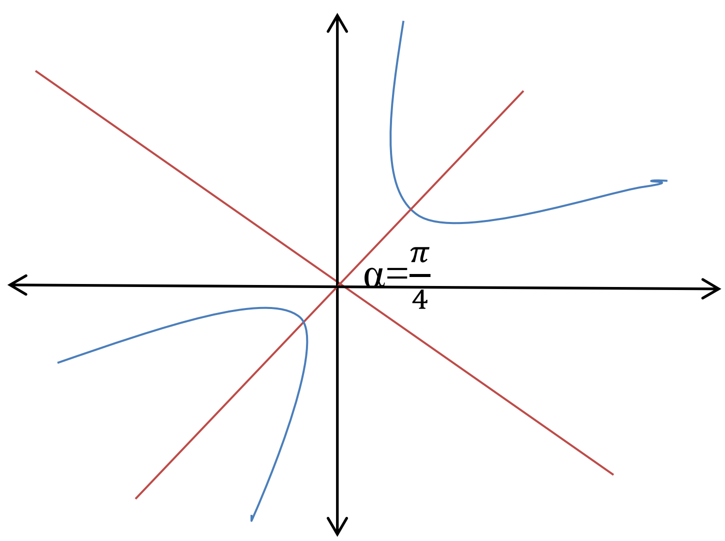
i) The equation $xy = 2$ has $A = 0, B = 1, and\ C = 0$. We substitute these values into $tan\,2\alpha = \frac{B}{A-C} \rightarrow\ tan\,2\alpha = \frac{1}{0-0} \rightarrow 2\alpha = tan^{-1}\frac{1}{0} \rightarrow 2\alpha = \frac{\pi}{2} \rightarrow \alpha = \frac{\pi}{4}$

Thus,

x $= x'\cos(\propto) - y'\sin(\propto) \rightarrow x = \frac{x' - y'}{\sqrt{2}}.$

y $= x'\sin(\propto) + y'\cos(\propto) \rightarrow y = \frac{x' + y'}{\sqrt{2}}.$

$$\boldsymbol{xy = 2 \rightarrow \left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right) = 2 \rightarrow x'^2 - y'^2 = 4 \rightarrow \frac{x'^2}{4} - \frac{y'^2}{4} = 1}$$



ii) $x = x'\cos\left(\frac{\pi}{4}\right) - y'\sin\left(\frac{\pi}{4}\right) \to x = \frac{x'-y'}{\sqrt{2}}.$

$y = x'\sin\left(\frac{\pi}{4}\right) + y'\cos\left(\frac{\pi}{4}\right) \to y = \frac{x'+y'}{\sqrt{2}}.$

$$\left(\frac{x'-y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right) - \sqrt{2}\left(\frac{x'-y'}{\sqrt{2}}\right) = 2 \Longrightarrow \frac{1}{2}(x'^2 - y'^2) - x' + y' = 2$$

$$x'^2 - y'^2 - 2x' + 2y' = 4 \Longrightarrow x'^2 - 2x' - y'^2 + 2y' = 4$$

$$(x'-1)^2 - (y'-1)^2 = 4 - 1 + 1 \Longrightarrow \frac{(x'-1)^2}{4} - \frac{(y'-1)^2}{4} = 1$$

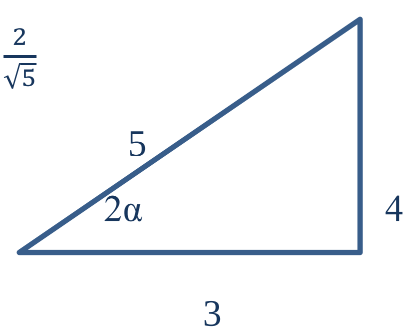
iii)The equation $4x^2 + 4xy + y^2 = 0$, has $A = 4, B = 4, and\ C = 1.$ we substitute these values into $tan\ 2\alpha = \frac{B}{A-C} \to tan\ 2\alpha = \frac{4}{4-1} \to tan\ 2\alpha = \frac{4}{3}$

$$sin^2\alpha = \frac{1-\cos 2\alpha}{2} \to \sin\alpha = \mp\sqrt{\frac{1-\cos 2\alpha}{2}} \to \sin\alpha = \sqrt{\frac{1-\cos 2\alpha}{2}}$$

$$cos^2\alpha = \frac{1+\cos 2\alpha}{2} \to \cos\alpha = \mp\sqrt{\frac{1+\cos 2\alpha}{2}} \to \cos\alpha = \sqrt{\frac{1+\cos 2\alpha}{2}}$$

As $0 < \alpha < \frac{\pi}{2}$ in quadrant I so $\cos 2\alpha = \frac{3}{5}$

$\sin\alpha = \sqrt{\frac{1-\frac{3}{5}}{2}} \to \sin\alpha = \frac{1}{\sqrt{5}}$ and $\cos\alpha = \sqrt{\frac{1+\frac{3}{5}}{2}} \to \cos\alpha = \frac{2}{\sqrt{5}}$



Thus,

$x = x'\cos(\propto) - y'\sin(\propto) \to x = \frac{2x'-y'}{\sqrt{5}}.$

$y = x'\sin(\propto) + y'\cos(\propto) \to y = \frac{x'+2y'}{\sqrt{5}}.$

$$4\left(\frac{2x'-y'}{\sqrt{5}}\right)^2+4\left(\frac{2x'-y'}{\sqrt{5}}\right)\left(\frac{x'+2y'}{\sqrt{5}}\right)+\left(\frac{2x'+y'}{\sqrt{5}}\right)^2=0$$

Example : what kind of conic section is the curve $\sqrt{x}+\sqrt{y}=1$ after rotate the coordinate axes through an angle of $\frac{\pi}{4}$.

$$x = x'\cos\left(\frac{\pi}{4}\right)-y'\sin\left(\frac{\pi}{4}\right)\rightarrow x=\frac{x'-y'}{\sqrt{2}}.$$

$$y = x'\sin\left(\frac{\pi}{4}\right)+y'\cos\left(\frac{\pi}{4}\right)\rightarrow y=\frac{x'+y'}{\sqrt{2}}.$$

$$\sqrt{\left(\frac{x'-y'}{\sqrt{2}}\right)}+\sqrt{\left(\frac{x'+y'}{\sqrt{2}}\right)}=1 \Longrightarrow \sqrt{\left(\frac{x'+y'}{\sqrt{2}}\right)}=1-\sqrt{\left(\frac{x'-y'}{\sqrt{2}}\right)} \Longrightarrow \frac{x'+y'}{\sqrt{2}}=1-2\sqrt{\left(\frac{x'-y'}{\sqrt{2}}\right)}+\frac{x'-y'}{\sqrt{2}}$$

$$2\sqrt{\left(\frac{x'-y'}{\sqrt{2}}\right)}=1-\frac{2y'}{\sqrt{2}} \Longrightarrow 4\left(\frac{x'-y'}{\sqrt{2}}\right)=1-\frac{4y'}{\sqrt{2}}+2y'^2 \Longrightarrow 2y'^2=\frac{4x'}{\sqrt{2}}-1$$

$$y'^2=\frac{2x'}{\sqrt{2}}-\frac{1}{2} \Longrightarrow y'^2=\sqrt{2}x'-\frac{1}{2} \Longrightarrow y'^2=\sqrt{2}\left(x'-\frac{1}{2\sqrt{2}}\right)$$

So, it is parabola equation