كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثانية

أستاذ المادة : ميمون ابراهيم اسماعيل

اسم المادة بالغة العربية :التفاضل المتقدم

اسم المادة باللغة الإنكليزية : **Advance Calculus**

اسم المحاضرة السابعة باللغة العربية: العلاقة بين الإحداثيات القطبية والكارتيرية

اسم المحاضرة السابعة باللغة الإنكليزية:

Relationship between Polar Coordinates and Cartesian Coordinates

Relationship between Polar Coordinates and Cartesian Coordinates
When we use both polar and Cartesian coordinates in a plane, we place the two origins together and take the initial polar ray as the positive *x*-axis. The ray becomes the positive *y*-axis (Figure 4). The two coordinate systems are then related by the following equations.

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad x^2 + y^2 = r^2$$

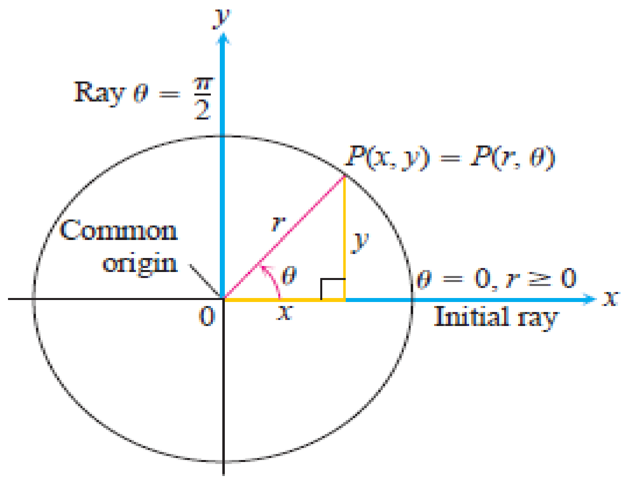


Figure 4

Example: Find a Cartesian equation and sketch it for

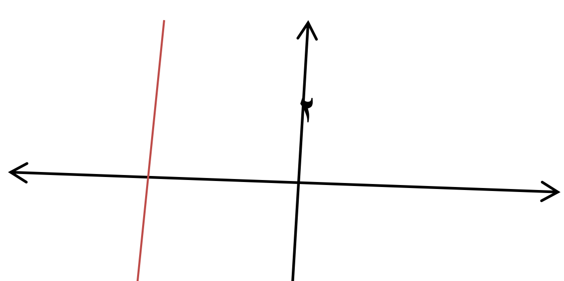
1) $r\ \cos(\theta + 7\pi) = 2$
2) $r = \tan\theta\ \sec\theta$
3) $r^2 = \sec 2\theta$

Solution:

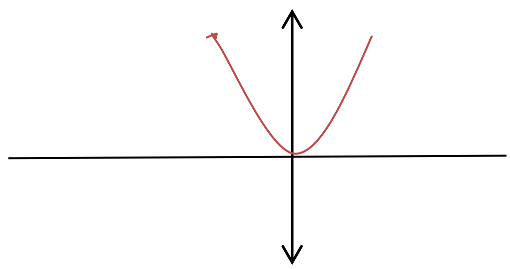
1)$r\ \cos(\theta + 7\pi) = 2 \rightarrow r\ \cos\theta\cos 7\pi - r\sin\theta\sin 7\pi = 2$

$$r\ \cos\theta\cos 7\pi - r\sin\theta\sin 7\pi = 2$$

$$-x = 2 \rightarrow x = -2 \cdot$$

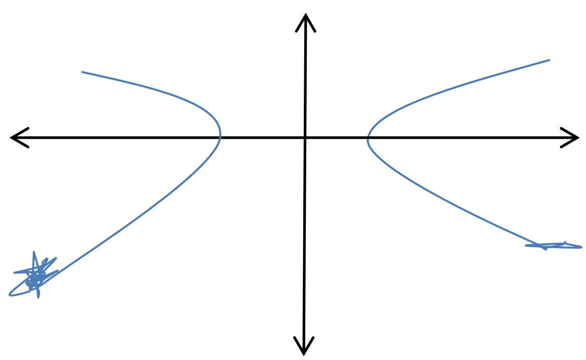


2) $r = \tan\theta \ \sec\theta \rightarrow r = \frac{\sin\theta}{\cos\theta}\frac{1}{\cos\theta} \rightarrow r\cos^2\theta = \sin\theta \rightarrow r^2\cos^2\theta = r\sin\theta \rightarrow x^2 = y$



3) $r^2 = \sec 2\theta \rightarrow r^2 = \frac{1}{\cos 2\theta} \rightarrow r^2 \cos 2\theta = 1 \rightarrow r^2\cos^2\theta - r^2\sin^2\theta \ x = 1$

$$x^2 - y^2 = 1$$

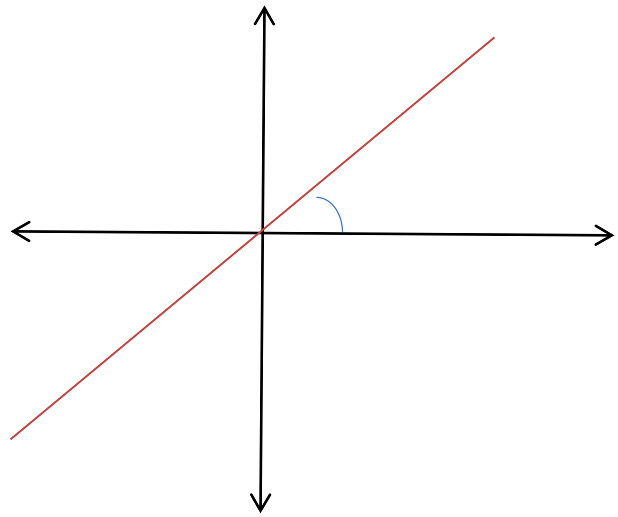


Example : Plot and transfer the points $A(1,1)$ and $B(-1,-\sqrt{3})$ to polar coordinates?

Solution: $r = \sqrt{x^2+y^2} \rightarrow r = \sqrt{1^2 + 1^2} \rightarrow r = \sqrt{2}$

$$\theta = \tan^{-1}\frac{y}{x} \rightarrow \theta = \tan^{-1}\frac{1}{1} \rightarrow \theta = \tan^{-1}1 \rightarrow \theta = \frac{\pi}{4}.$$

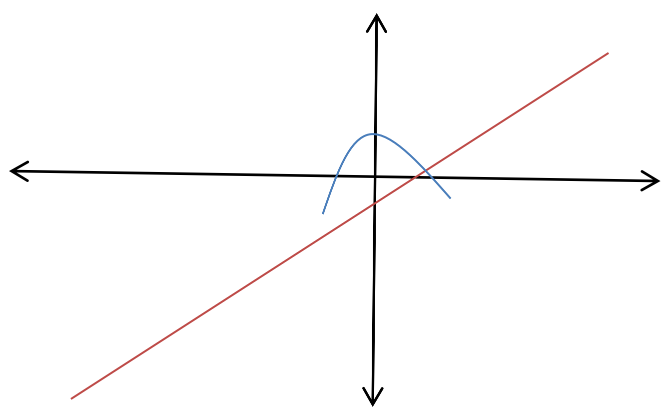
Thus, $A(x,y) \rightarrow A(r,\theta) \rightarrow A(1,1) \rightarrow A(\sqrt{2},\frac{\pi}{4}).$



$$r = \sqrt{x^2+y^2} \rightarrow r = \sqrt{(-1)^2 + (-\sqrt{3})^2} \rightarrow r = 2$$

$$\theta = \tan^{-1}\frac{y}{x} \rightarrow \theta = \tan^{-1}\frac{\sqrt{3}}{1} \rightarrow \theta = \tan^{-1}\sqrt{3} \rightarrow \theta = \pi + \frac{\pi}{3} \rightarrow \theta = \frac{4\pi}{3}.$$

Thus, $B(x,y) \rightarrow B(r,\theta) \rightarrow B(-1,-\sqrt{3}) \rightarrow B(2,\frac{4\pi}{3}).$



Example: Find the Cartesian coordinates of the following point $\left(13, \sin^{-1}(\frac{12}{13})\right)$

Solution: $x = r\cos\theta, y = r\sin\theta$,

$$\theta = \sin^{-1}\left(\frac{12}{13}\right) \rightarrow \sin\theta = \frac{12}{13} \rightarrow 12^2 + B^2 = 13^2 \rightarrow B^2 = 169 - 144 \rightarrow B^2 = 25 \rightarrow B = 5$$

$$x = r\cos\theta, y = r\sin\theta \rightarrow x = 13\frac{5}{13} = 5, y = 13\ \sin\sin^{-1}\left(\frac{12}{13}\right) \rightarrow$$

$$y = 13\frac{12}{13} = 12,\ or\ y = 13\frac{12}{13} = 12 \rightarrow \therefore\ (5,12)$$