كلية : التربية للعلوم الصرفة

القسم او الفرع :الرياضيات

المرحلة: الثانية

أستاذ المادة : ميمون ابراهيم اسماعيل

اسم المادة بالغة العربية :التفاضل المتقدم

اسم المادة باللغة الإنكليزية : Advance Calculus

اسم المحاضرة العاشرة باللغة العربية: الرسم في الاحداثيات القطبية (الاشكال القلبية)

اسم المحاضرة العاشرة باللغة الإنكليزية: Graphing in Polar Coordinates (Limaçons)

Limaçons has formed as $r = a \pm b\ cos\theta$ or $r = a \pm b\ sin\theta\ where\ a, b \in R - \{0\}$:

- The polar equation is a *Cardioid curve* if $a = b$
- The polar equation is a *dimpled curve* if $a > b$
- The polar equation is a *inner* loop *curve* if $a < b$

Examples:

1) Graph the Curve $r = 2 + 2\cos\theta$

Solution:

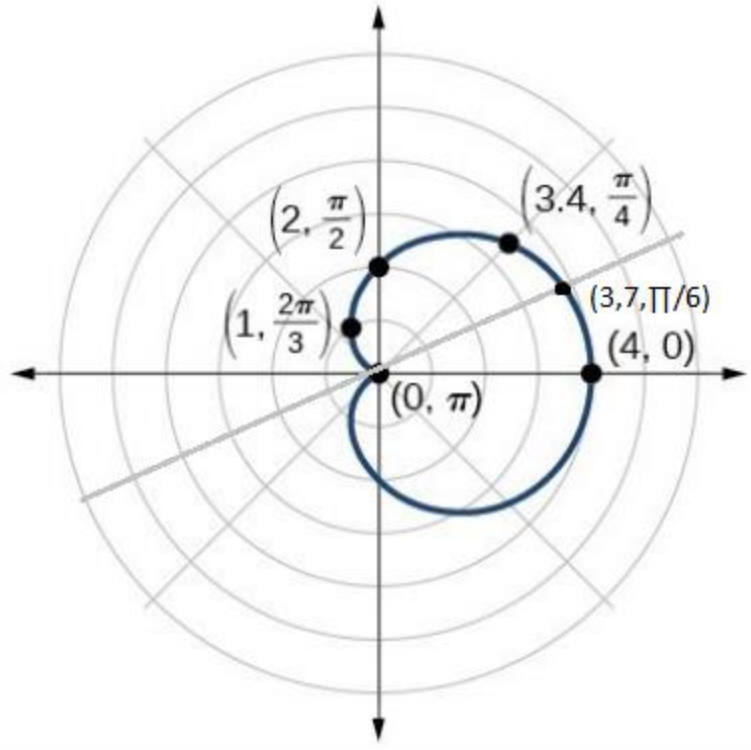
- The curve is symmetric about the *x*-axis because (r, θ) on the graph then

$$r = 2 + 2\cos(-\theta) \rightarrow r = 2 + 2\cos\theta \rightarrow (r, -\theta) \text{ on the graph}$$

There is not symmetric about the *y*-axis and the origin point

θ	r	(r, θ)
0	4	(4,0)
$\frac{\pi}{6}$	$2+\frac{2\sqrt{3}}{2}$	$\left(3.7, \frac{\pi}{6}\right)$
$\frac{\pi}{4}$	$2+\frac{2}{\sqrt{2}}$	$\left(3.4, \frac{\pi}{4}\right)$
$\frac{\pi}{3}$	$2+\frac{2}{2}$	$\left(3, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	2+0	$\left(2, \frac{\pi}{2}\right)$
$\frac{2\pi}{3}$	$2-\frac{2}{2}$	$\left(1, \frac{2\pi}{3}\right)$
π	2-2	$(0, \pi)$

$$r = 2 + 2\cos\theta$$



2) Graph the Curve $r = 2 + 2\sin\theta$

Solution:

●The curve is symmetric about the *y*-axis because (r, θ) on the graph then

$r = 2 + 2\sin(\pi - \theta) \rightarrow r = 2 + 2\sin\pi\cos\theta - 2\sin\theta\cos\pi \rightarrow$

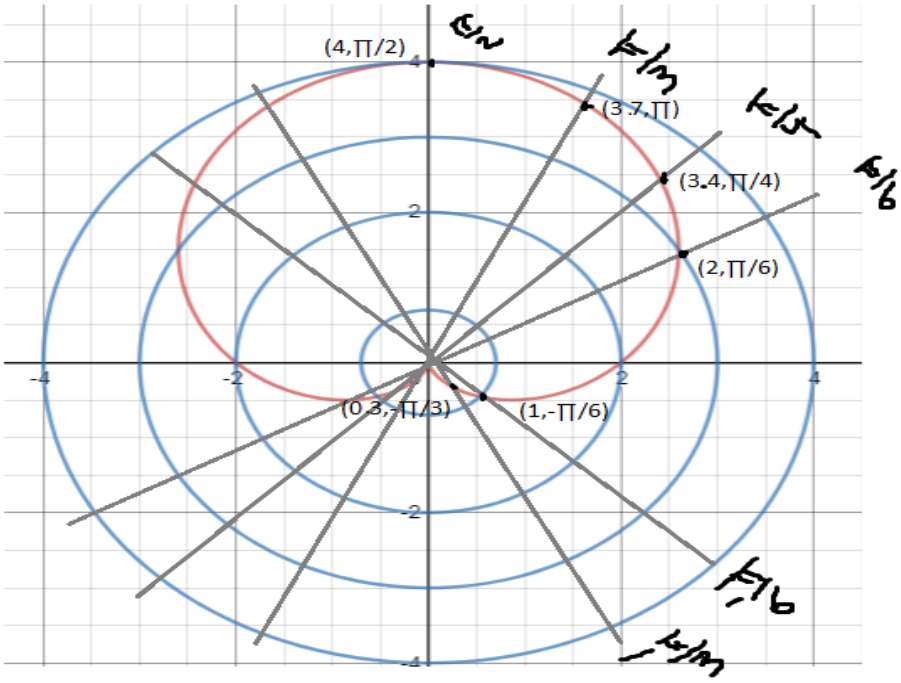
$r = 2 + 2\sin\theta$ So $(r, \pi - \theta)$ on the graph

There is not symmetric about the *x*-axis and the origin point

θ	r	(r, θ)
0	2	(2,0)
$\frac{\pi}{6}$	$2+\frac{2}{2}$	$\left(3, \frac{\pi}{6}\right)$
$\frac{-\pi}{6}$	$2-\frac{2}{2}$	$\left(1, \frac{\pi}{6}\right)$
$\frac{\pi}{4}$	$2+\frac{2}{\sqrt{2}}$	$\left(3.4, \frac{\pi}{4}\right)$

$\frac{\pi}{3}$	$2+\frac{2\sqrt{3}}{2}$	$\left(3.7,\frac{\pi}{3}\right)$
$-\frac{\pi}{3}$	$2-\frac{2\sqrt{3}}{2}$	$\left(0.3,-\frac{\pi}{3}\right)$
$\frac{\pi}{2}$	2+2	$\left(4,\frac{\pi}{2}\right)$

$$r = 2 + 2\sin\theta$$



3) Graph the Curve $r = 4 + 3\sin\theta$

Solution:

●The curve is symmetric about the *y*-axis because (r,θ) on the graph then

$r = 4 + 3\sin(\pi-\theta) \rightarrow r = 4 + 3\sin\pi\cos\theta - 3\sin\theta\cos\pi \rightarrow$

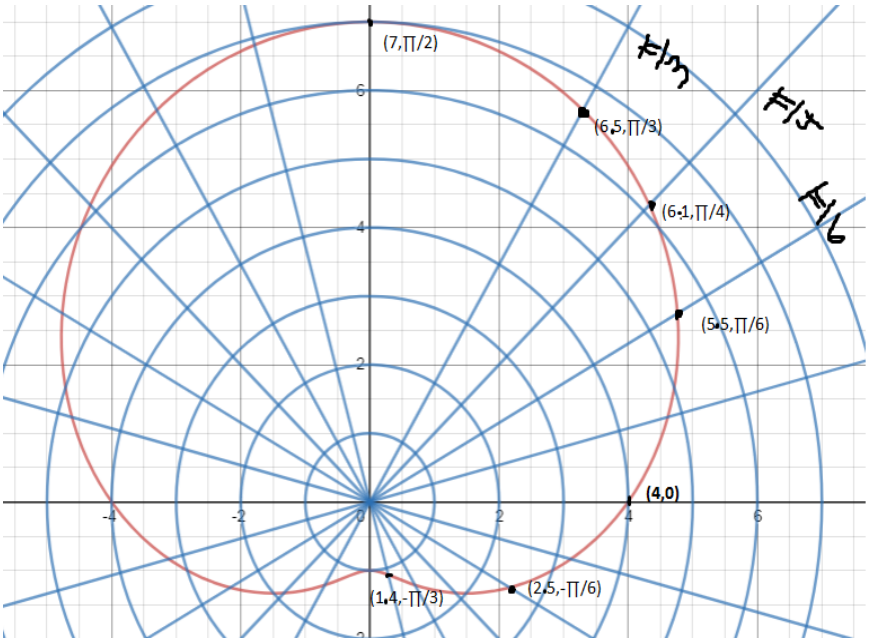
$r = 4 + 3\sin\theta$ So $(r,\pi-\theta)$ on the graph

There is not symmetric about the *x*-axis and the origin point

θ	r	(r,θ)
0	4	(4,0)

$\frac{\pi}{6}$	$4+\frac{3}{2}$	$\left(5.5,\frac{\pi}{6}\right)$
$\frac{-\pi}{6}$	$4-\frac{3}{2}$	$\left(2.5,\frac{\pi}{6}\right)$
$\frac{\pi}{4}$	$4+\frac{3}{\sqrt{2}}$	$\left(6.12,\frac{\pi}{4}\right)$
$\frac{\pi}{3}$	$4+\frac{3\sqrt{3}}{2}$	$\left(6.5,\frac{\pi}{3}\right)$
$-\frac{\pi}{3}$	$4-\frac{3\sqrt{3}}{2}$	$\left(1.4,-\frac{\pi}{3}\right)$
$\frac{\pi}{2}$	4+3	$\left(7,\frac{\pi}{2}\right)$

$$r = 4 + 3\sin\theta$$



4) Graph the Curve $r = 2 + 5\cos\theta$

Solution:

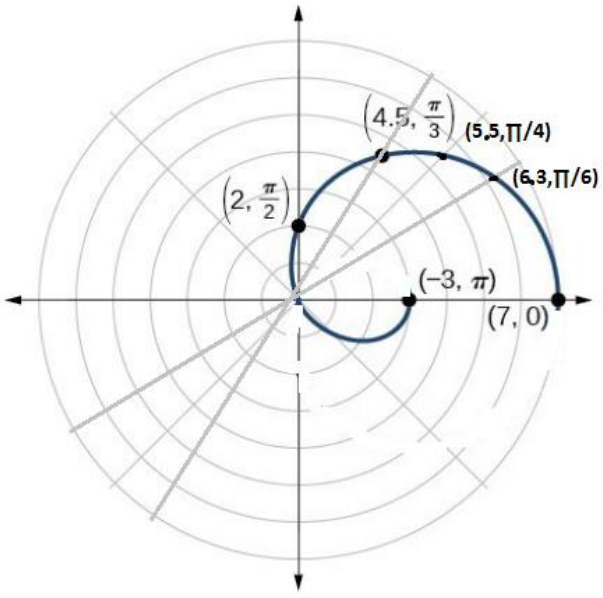
●The curve is symmetric about the *x*-axis because (r,θ) on the graph then

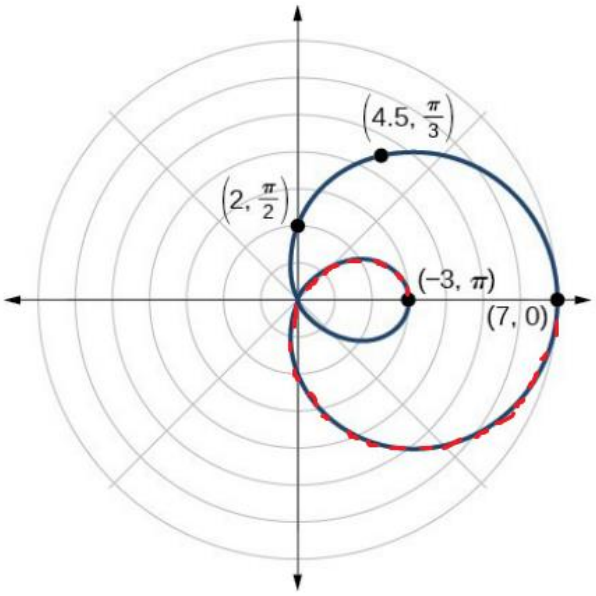
$$r = 2 + 5\cos(-\theta) \rightarrow \quad r = 2 + 5\cos\theta \rightarrow (r,-\theta) \text{ on the graph}$$

There is not symmetric about the *y*-axis and the origin point

θ	r	(r,θ)
0	7	(7,0)
$\frac{\pi}{6}$	$2+\frac{5\sqrt{3}}{2}$	$\left(6.3,\frac{\pi}{6}\right)$
$\frac{\pi}{4}$	$2+\frac{5}{\sqrt{2}}$	$\left(5.5,\frac{\pi}{4}\right)$
$\frac{\pi}{3}$	$2+\frac{5}{2}$	$\left(4.5,\frac{\pi}{3}\right)$
$\frac{\pi}{2}$	2+0	$\left(2,\frac{\pi}{2}\right)$
$\frac{2\pi}{3}$	$2-\frac{5}{2}$	$\left(0.5,\frac{2\pi}{3}\right)$
π	2-5	$(-3,\pi)$

$$r = 2 + 5\cos\theta$$





because The curve is symmetric about the x-axis