



كلية : التربية للعلوم الصرفة

القسم او الفرع : الفيزياء

المرحلة: الثالثة

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اسم المادة باللغة العربية : نظرية المجاميع

اسم المادة باللغة الإنجليزية : Set Theory

اسم الحاضرة السابعة باللغة العربية: امثلة على سلاسل فورييه

اسم المحاضرة السابعة باللغة الإنجليزية : Examples of Fourier Series

Lecture 7

Example:(2) Compute the Fourier Series of the odd square wave function of period $2L$

$$f(t) = \begin{cases} -1 & -L \leq t < 0 \\ 1 & 0 \leq t < L \end{cases}$$

Solution:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{L}\right)t + b_n \sin\left(\frac{n\pi}{L}\right)t \right)$$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(t) dt = 0$$

قيمة $a_0 = 0$ لأن الدالة $f(t)$ هي دالة فردية (معطاة في السؤال).

$$a_n = \frac{1}{L} \int_{-L}^{L} f(t) \cdot \cos\left(\frac{n\pi}{L}\right)t dt , \quad n = 1,2,3, \dots$$

$$a_n = 0$$

قيمة $a_n = 0$ لكل قيم $n = 1,2,3, \dots$ وذلك لأن $f(t) \cdot \cos\left(\frac{n\pi}{L}\right)t$ هي دالة فردية.

$$b_n = \frac{1}{L} \int_{-L}^{L} f(t) \cdot \sin\left(\frac{n\pi}{L}\right)t dt , \quad n = 1,2,3, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^0 f(t) \cdot \sin\left(\frac{n\pi}{\pi}\right) t dt + \frac{1}{L} \int_0^L f(t) \cdot \sin\left(\frac{n\pi}{\pi}\right) t dt$$

$$b_n = \frac{1}{L} \int_{-L}^0 (-1) \cdot \sin\left(\frac{n\pi}{\pi}\right) t dt + \frac{1}{L} \int_0^L (+1) \cdot \sin\left(\frac{n\pi}{\pi}\right) t dt$$

$$= \left[\frac{-1}{L} \cdot \frac{L}{n\pi} \cdot (-1) \cdot \cos\left(\frac{n\pi}{L}\right) t \right]_{-L}^0 - \left[\frac{1}{L} \cdot \frac{L}{n\pi} \cdot \cos\left(\frac{n\pi}{L}\right) t \right]_0^L$$

$$= \left[\frac{1}{n\pi} \cos\left(\frac{n\pi}{L}\right) t \right]_{-L}^0 - \left[\frac{1}{n\pi} \cos\left(\frac{n\pi}{L}\right) t \right]_0^L$$

$$= \frac{1}{n\pi} \left[\cos\left(\frac{n\pi}{L}\right)(0) - \cos\left(\frac{n\pi}{L}\right)(-L) \right] - \frac{1}{n\pi} \left[\cos\left(\frac{n\pi}{L}\right)(L) - \cos\left(\frac{n\pi}{L}\right)(0) \right]$$

$$= \frac{1}{n\pi} \left[1 - \cos\left(\frac{n\pi}{L}\right)(-L) \right] - \frac{1}{n\pi} \left[\cos\left(\frac{n\pi}{L}\right)(L) - 1 \right]$$

$$= \frac{1}{n\pi} [1 - \cos(-n\pi)] - \frac{1}{n\pi} [\cos(n\pi) - 1]$$

$$= \frac{1}{n\pi} [1 - \cos(n\pi)] - \frac{1}{n\pi} [\cos(n\pi) - 1]$$

$$= \frac{1}{n\pi} - \frac{\cos(n\pi)}{n\pi} - \frac{\cos(n\pi)}{n\pi} + \frac{1}{n\pi}$$

$$= \frac{2}{n\pi} - \frac{2}{n\pi} \cos(n\pi)$$

$$= \frac{2}{n\pi} (1 - \cos(n\pi))$$

$$b_n = \frac{2}{n\pi} (1 - (-1)^n)$$

such that, $(-1)^n = \begin{cases} -1 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases}$

if n is even $(-1)^n = 1$

$$\Rightarrow b_n = \frac{2}{n\pi} (1 - 1) = 0$$

if n is odd $(-1)^n = -1$

$$\Rightarrow b_n = \frac{2}{n\pi} (1 - (-1)) = \frac{2}{n\pi} (1 + 1) = \frac{4}{n\pi}$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{L}\right) t + b_n \sin\left(\frac{n\pi}{L}\right) t \right)$$

$$a_0 = 0 , \quad a_n = 0 , \quad b_n = \frac{4}{n\pi} , \quad n \text{ is odd}$$

$$f(t) = \frac{0}{2} + \sum_{n=1}^{\infty} \left((0) \cos\left(\frac{n\pi}{L}\right) t + b_n \sin\left(\frac{n\pi}{L}\right) t \right)$$

$$f(t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}\right) t , \quad n \text{ is odd}$$

$$f(t)=b_1\sin\left(\frac{\pi}{L}\right)t+b_3\sin\left(\frac{3\pi}{L}\right)t+b_5\sin\left(\frac{5\pi}{L}\right)t+\cdots$$

$$f(t)\sim \frac{4}{\pi}\sin\left(\frac{\pi}{L}\right)t+\frac{4}{3\pi}\sin\left(\frac{3\pi}{L}\right)t+\frac{4}{5\pi}\sin\left(\frac{5\pi}{L}\right)t+\cdots$$

$$f(t)\sim \frac{4}{\pi}\Big(\sin\left(\frac{\pi}{L}\right)t+\frac{1}{3}\sin\left(\frac{3\pi}{L}\right)t+\frac{1}{5}\sin\left(\frac{5\pi}{L}\right)t+\cdots\Big).$$