جامعة الانبار

كلية العلوم -قسم الجيولوجيا التطبيقية

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اسم المادة بالعربي: الطرق الكهربائية

عنوان المحاضرة بالإنكليزي:Theoretical background

Theoretical background

Introduction

The DC resistivity method is one of the simplest geophysical techniques used to measure earth conductivity, but it is still employed extensively because of its easy using and relatively easy interpretation. The most common application is groundwater exploration, but it is also used in geothermal, environmental, and engineering studies. The measurement of the earth's resistivity is very similar in concept to the laboratory resistivity measurement of rock samples. A DC electric current is passed through the ground via a pair of current electrodes and a resulting potential difference is measured between a second pair of potential electrodes

Theory

It was Ohm who, through many experiments using wires of various dimensions, voltaic cells and thermocouples came up with the relationship between current and voltage. That is:

$$\Delta V = IR \quad \Rightarrow \quad R = \frac{\Delta V}{I} \text{ ..... (1)}$$

where (ΔV volts) is the potential difference between two points in a conductor, (I amperes) is the current flow and (R ohms) is the constant of proportionality called resistance.

For bulk materials the resistance of a conducting object is found to be directly proportional to the length (L) of the object and inversely proportional to its cross-sectional area (A).

The constant of proportionality in this case is called resistivity (ρ) of the conductor, that is:

$$R = \frac{\rho L}{A} \text{ ..... (2)}$$

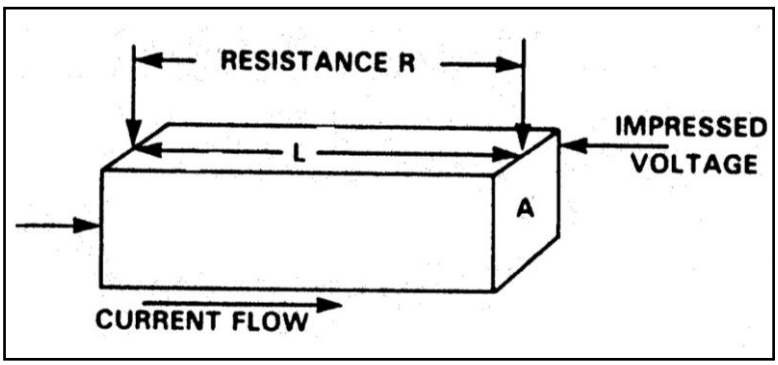

Figure (2.1) Schematic defining variables in equation 2.

This relationship holds for earth materials as well as simple circuits. Substituting the value of (R) in equation (1) we get:

$$\frac{\Delta V}{I} = \frac{\rho L}{A} \quad \Rightarrow \quad \rho I L = \Delta V A \ldots\ldots (3)$$

The resistivity (ρ) depends on the property of the material and is a geometrically- independent quantity that describes a material's ability to transmit electrical current. The value of (ρ) is measured in ohm-meter (Ωm).

For a half space solution we consider a single current electrode, a point source of current, on the surface of a homogeneous-isotropic half space, injecting a current (I) into the Earth. The flow of electric current will be radially symmetric in the half space. We balance the current flowing into the earth at the electrode with the total current flow out of a hemispherical surface as in figure (2.2).

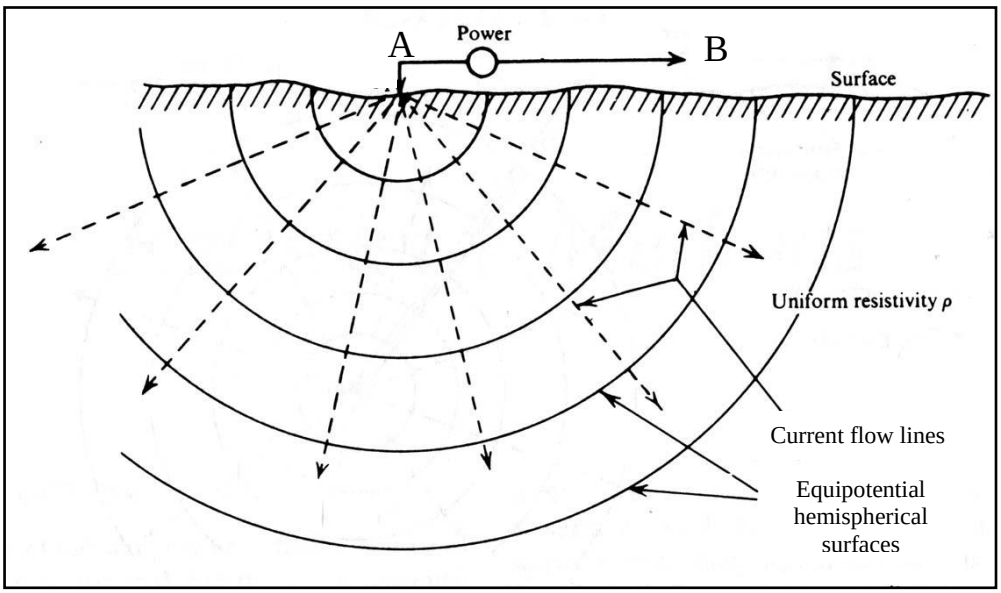

Figure (2.2) Point source of current at the surface of a homogeneous medium.

Because of the radial symmetry of current flow, the current will be constant at a distance (r) from the current electrode, so the total current flow across the hemispherical surface with cross sectional area of ($2\pi r^2$), therefore the equation (3) will be:

$$\rho I r = \Delta V 2\pi r^2 \ldots\ldots (4)$$

where (r) is the outward normal to the hemisphere.

From equation (4) we obtain the potential (ΔV) from point current source at distance (r) as:

$$\Delta V_r = \frac{\rho I}{2\pi r} \ldots\ldots (5)$$

Now for the general four electrodes array as in figure (2.3), the potential at electrode (M) is simply the sum of the effects of the two current electrodes (A) and (B):

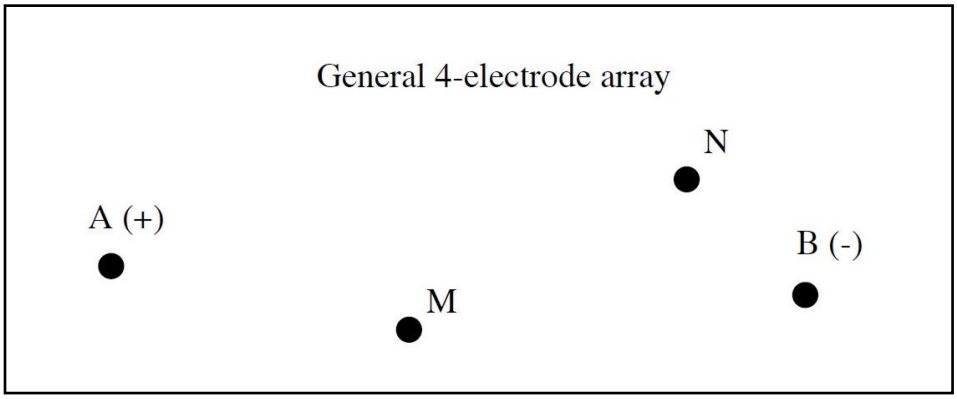

Figure (2.3) General 4 electrode array. A and B are current electrodes, M and N are potential electrodes.

$$\Delta V_M^{AB} = \frac{I\rho}{2\pi}\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{BM}}\right) \ldots\ldots (6)$$

and similarly the potential at N is:

$$\Delta V_N^{AB} = \frac{I\rho}{2\pi}\left(\frac{1}{\overline{AN}} - \frac{1}{\overline{BN}}\right) \ldots\ldots (7)$$

so the potential difference measured across ($\overline{MN}$) is:

$$\Delta V_{MN}^{AB} = \Delta V_M^{AB} - \Delta V_N^{AB} = \frac{I\rho}{2\pi}\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{BM}} - \frac{1}{\overline{AN}} + \frac{1}{\overline{BN}}\right) \ldots\ldots (8)$$

equation (8) will yield the resistivity of anisotropic earth:

$$\rho = \frac{2\pi}{\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{BM}} - \frac{1}{\overline{AN}} + \frac{1}{\overline{BN}}\right)} \frac{\Delta V}{I} \ldots\ldots (9)$$

where (K) the geometric factor :

$$K = \frac{1}{\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{BM}} - \frac{1}{\overline{AN}} + \frac{1}{\overline{BN}}\right)} \ldots\ldots (10)$$

$$\rho = 2\pi K \frac{\Delta V}{I} \ldots\ldots (11)$$

If the Earth is not a homogeneous-isotropic halfspace the above expression would not yield the true resistivity of the Earth. The resistivity will change if we use another electrode arrangement or changing the measurement positions, so the quotient ($\frac{\Delta V}{I}$) will not be directly proportional to (K) as in

an isotropic earth, and the value of (ρ) found by substituting the measured ($\frac{\Delta V}{I}$) and the correct (K) into equation (11), is called the apparent resistivity (ρ_a):

$$\rho_a = 2\pi K \frac{\Delta V}{I} \ldots\ldots (12)$$

The full theory of DC resistivity is set out in geophysical textbooks such as (Keller and Frischknecht, 1966; Battacharya and Petra, 1968; Kunetz, 1966).

Earth resistance spacing

The resistance (R_{AM}) between two electrodes (A and M) is specifically dependent (for any ground surface) on the electrode location (x,y).

Alternatively, adopting the mean coordinate position x, y and letting the bearing of A → M be θ and the separation of the electrode be AM as in figure (2.4) then (Habberjam, 1979):

$$R_{AM} = f\left(xy\overline{AM}\theta\right) \ldots\ldots (13)$$

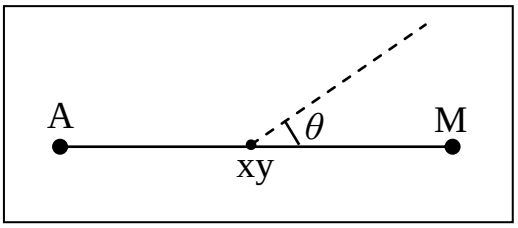

Figure (2.4) Coordinates for the 2 electrodes configuration, A and M modified from (Habberjam, 1979).

In plane, a uniform semi infinite medium (homogeneous and isotropic) there are many plane isoresistance surfaces parallel to the surface (r = 0), the apparent depth of (R_{AM}) (r) = 0, the earth resistance between the electrodes A and M will be:

$$R_{AM} = \frac{\Delta V_{AM}}{I}$$

For a specified value of the orientation θ, values of R_{AM} can be plotted in a three dimensional space and series of such planes as in figure (2.5), another factor will affect the value of (R_{AM}) is the apparent depth of (R_{AM}) (r).

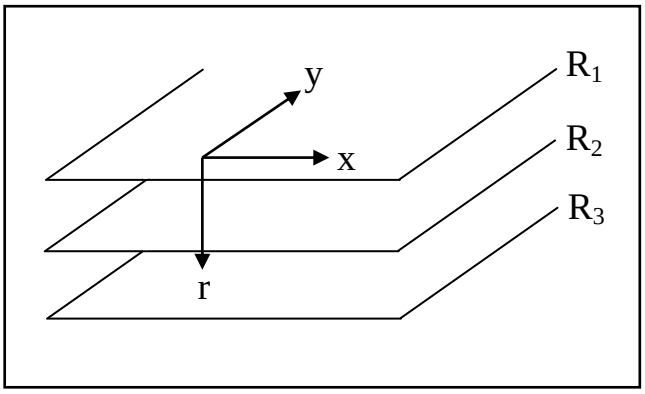

Figure (2.5) A 2-electrode resistance space for a uniform semi – infinite subsurface modified from (Habberjam, 1979).

Habberjam (1979) show that for 4 electrode collinear arrays, each set of 2 electrodes provides an operator which samples the 2 electrode resistances at the locations equal to the distance between any two current and potential electrodes. The sum of these resistances yields the 4 electrode resistance value as shown in figure (2.6).

$$R = R_{AM} - R_{AN} - R_{BM} + R_{BN} \ldots\ldots (14)$$

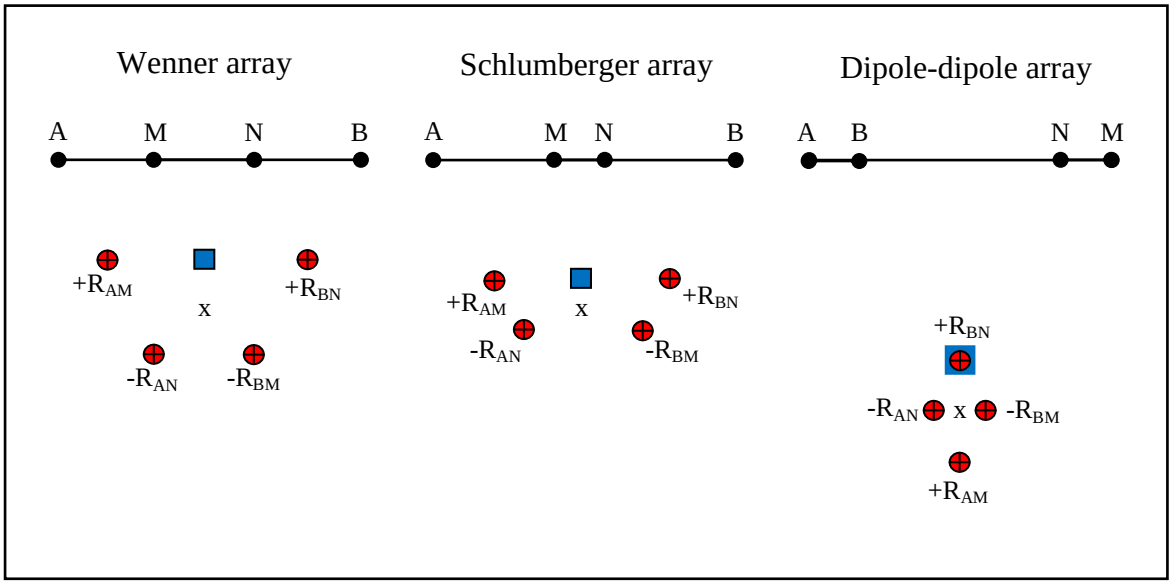

Figure (2.6) Space operators for Wenner, Schlumberger and Dipole-dipole arrays (red circles) modified from (Habberjam, 1979). The four electrode assignment locations as blue squares (at the minimum operator spacing) and x (at the mean operator spacing).

In figure (2.6), red circles indicate the 2 electrode resistance locations and spacing for each configuration. If the convention of assigning a spacing according to the minimum spacing of the 2 electrode components adopted (minimum operator spacing), the four electrode resistance can be plotted at the position shown by the squares, and if the convention of assigning a spacing according to the mean spacing of the 2 electrode components adopted (mean operator spacing), the four electrode resistance can be plotted at the position shown by the x.

Reviewing the operators of figure (2.6), it can be seen that they all sample the 2 electrode space at widely separated points. In particular, on array involves six points which embrace the three configurations and are responsible for the appropriate additive rule. Where a configuration involves widely different 2 electrode spacings, the smaller will normally contribute the major part of the four electrode measurement.

In the illustrated arrays in figure (2.6), Wenner array has a particularly favorable geometric factor (K) value so that signal amplitude decays at the same rate as for the 2 electrode system and further the signal can usefully be checked using the tripotential rule. A drawback of this array is that the sample points are somewhat widespread and this again may blur details which would be clearer on a 2 electrode space.

In Schlumberger array the sampled resistances are still close together. The sampled differences are added together so that the (K) value is much more favorable. If higher ($\frac{\overline{AM}}{\overline{MN}}$) ratios are used, however, signal magnitude is again reduced. The lateral separation of the two resistance differences implies that variations which would be clear on the two electrode array are being averaged out. The potential base ($\overline{MN}$) is also small compared with ($\overline{AB}$) and this is advantageous when telluric noise becomes serious.

Dipole-dipole array provides the closest sampling of 2 electrode space, this closeness also implies that the sampled four electrode resistance value will be small (the K factor is high). On the other hand, the current base ($\overline{AB}$) is shorter so that it is easier to pass larger signals.

Al-Ani (1998 in arabic) show that there is a relationship between the mean operator spacing and the depth function ($\frac{\overline{AB}}{2}$) for Schlumberger array as:

Mean operator spacing = $\frac{\overline{AB}}{2}$

And there is relationship between minimum operator spacing and depth function (a) for Wenner array as:

Minimum operator spacing = a = $\overline{AM}$

He also show (for Schlumberger array) that when the ratio ($\frac{\overline{MN}}{\overline{AB}}$) increase the difference between the mean operator spacing and the minimum operator spacing increases, so two components of the measured resistance on the surface (R_{AM} and R_{BN}) will be closer to the surface, and because they have the same apparent depth of the minimum operator spacing, the weight of these components in the measured resistivity on the surface becomes higher than the weight of the other components (R_{AN} and R_{BM}), the apparent depth of the measured resistivity value will decrease and then the depth of investigation will decrease accordingly.

Electrical properties of rocks

For the resistivity method to be successful, a number of conditions must fall in place. The fundamental condition that needs to be fulfilled for motivating the use of the method is contrast in the physical property between the subsurface materials that is to be delineated. Therefore it is important to know the basics behind the electrical properties of the investigated materials.

The resistivity of natural soils and rocks varies within very wide ranges as in table (2.1), and this difference in resistivity is the foundation of resistivity survey.

Table (2.1) Resistivity ranges of earth materials modified from (Telford, 1976; Palacky, 1987).

Earth Material	Resistivity range (Ωm)
Sedimentary Rocks	
Shale	$10 - 10^3$
Sandstone	$1 - 10^8$
Conglomerate	$10^3 - 10^4$
Limestone	$50 - 10^7$
Dolomite	$10^2 - 10^4$
Unconsolidated sediments	
Gravel	$10^2 - 10^4$
Sand	$10^2 - 10^4$
Clay	$1 - 10^2$
Marl	$1 - 10^2$
Ground water	
Fresh water	$0.1 - 10^3$
Brackish water	0.3 - 1
Sea water	0.2
Super saline brine	0.05 - 0.2

It is, however, essential to be aware v2e of the large overlaps in resistivity between the different types of earth materials. As a result measured

resistivities should never be interpreted directly to a certain material category without additional knowledge of the specific situation. Electrical conduction in geological materials is mainly electrolytic. The most common soil and rock forming minerals are insulators in the dry state, and thus the amount of wet and the properties of the water largely determine the resistivity.

For a rock mass this means that fractures, faults and shear zones constitute the dominating current paths, whereas the solid rock normally is considered as an electric insulator. As an exception, rocks with metallic content may have significant conduction through the crystalline structure.

Soils, on the other hand, are porous media consisting of a solid skeleton of particles, or grains, and pores in between. The grains are considered electrical insulators and the conduction is concentrated to the pore space that is typically filled or partly filled with water. Therefore, resistivities of soils are strongly influenced by the amount of water, which is determined by the porosity and the degree of saturation. Also the resistivity of the water, to a great extent governed by the ion content, and the connectivity of the pore spaces are important parameters. Another important factor influencing soil resistivities is the presence of clay minerals, since these minerals bind water molecules and ions and thereby facilitate electrical conduction. Clay particles coating the surfaces of the larger mineral particles may have a dominating effect on the bulk resistivity of a predominantly coarse grained soil, creating so called surface conduction (Ward, 1990; Revil and Glover, 1997). Therefore, in the different models that have been used for describing resistivity of soils, there has been two categories depending on if the soil has clay content or not.

Survey Design

Survey design should be based on the problem definition (i.e., the aim of the survey). In general, the four electrodes A, B, M and N can be placed at arbitrary locations on the surface. However, a variety of specific electrode arrangements are commonly employed. Each layout offers advantages in equipment handling or in measurement instrumentation.

The survey design is based on two bases: Choosing the electrode configuration and choosing the measuring technique.

Electrode Configurations

There are numerous configurations or arrangements for placing the current and potential electrodes for surveying. The three most appropriate for geologic investigations will be discussed as below.

Schlumberger array

It is the most commonly used arrangement, and was developed by Conrad Schlumberger. Like most of the standard arrays it is collinear and symmetrical as in figure (2.7). The particular feature of the Schlumberger array is that the potential electrode spacing (MN) is very much smaller (about 1/5 to 1/6) than the current electrode spacing (Roy, 1972).

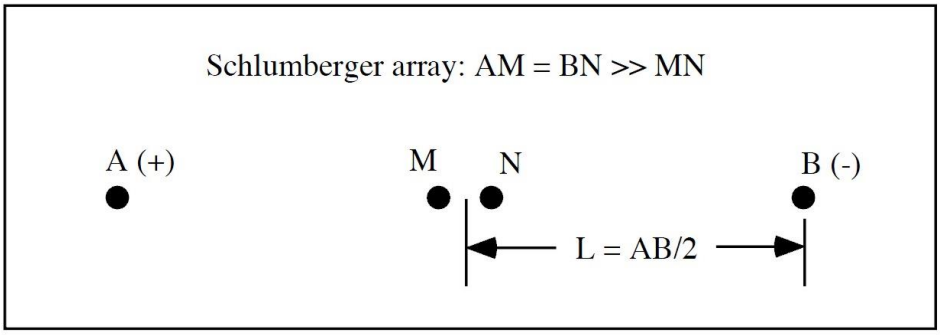

Figure (2.7) The Schlumberger array. MN is small compared with AB and the array is symmetrical and collinear.

To compute the expression for apparent resistivity for this array we firstly note that for a symmetrical array $\overline{AM} = \overline{BN}$ and $\overline{AN} = \overline{BM}$ so that:

$$\frac{1}{K} = \left(\frac{2}{\overline{AM}} - \frac{2}{\overline{BM}} \right) \ldots\ldots (15)$$

now for the Schlumberger array we write

$$\overline{AM} = \overline{BN} = \frac{\overline{AB}}{2} - \frac{\overline{MN}}{2} = \frac{\left(\overline{AB} - \overline{MN}\right)}{2}$$

$$\overline{AN} = \overline{BM} = \frac{\overline{AB}}{2} + \frac{\overline{MN}}{2} = \frac{\left(\overline{AB} + \overline{MN}\right)}{2}$$

so

$$\frac{1}{K} = \frac{4}{\overline{AB} - \overline{MN}} - \frac{4}{\overline{AB} + \overline{MN}}$$

$$K = \frac{\left(\overline{AB} - \overline{MN}\right)\left(\overline{AB} + \overline{MN}\right)}{4\overline{MN}} \ldots\ldots (16)$$

yielding an expression for apparent resistivity:

$$\rho_a = 2\pi \frac{\Delta V}{I} \frac{\left(\overline{AB} - \overline{MN}\right)\left(\overline{AB} + \overline{MN}\right)}{4\overline{MN}} \ldots\ldots (17)$$

Because $\overline{MN} \Rightarrow 0$ we may set $\overline{AB} \pm \overline{MN} \approx \overline{AB}$ and also write $E \approx \frac{\Delta V}{\overline{MN}}$ so

$$\rho_a = \frac{\pi E}{I}\left(\frac{\overline{AB}}{2}\right)^2 \ldots\ldots (18)$$

Where (E) the electrical field.

This equation is representing the theoretical case because it is based on $\overline{MN} \rightarrow 0$, but in the field the distance ($\overline{MN}$) is more than zero, so the equation (13) must be used to calculate (ρ_a) value because it represents the practical case.

Wenner array

This array was developed by Frank Wenner in the U.S. at about the same time that Schlumberger developed his techniques in France. Wenner worked in the national standards laboratory on material properties, and realized that he could apply the same four point method used in the laboratory in the field to measure bulk Earth resistivity. Consequently, the array that bears his name has equal spacing between all electrodes as in figure (2.8).

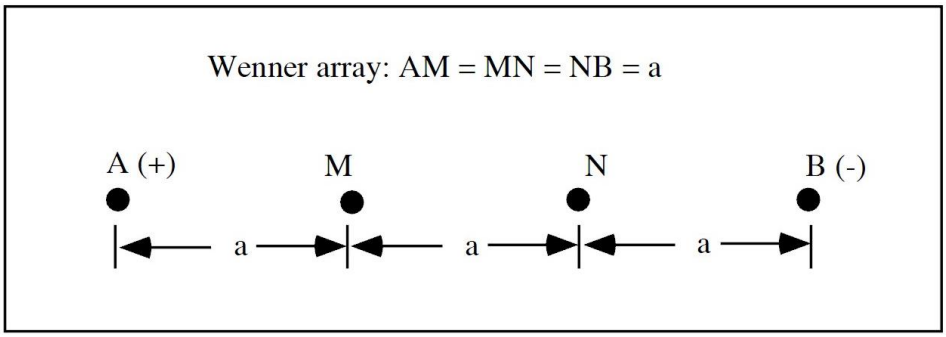

Figure (2.8) The Wenner array. The array is symmetrical and collinear but now the electrodes are equally spaced.

From our expression for a symmetric array we have very simply that:

$$\frac{1}{K} = \frac{2}{a} - \frac{2}{2a} = \frac{1}{a}$$

so that $K = a$ and

$$\rho_a = 2\pi a \frac{\Delta V}{I} \ldots\ldots (19)$$

Advantages of the Schlumberger array over the Wenner array include the following (Zohdy *et al.*, 1974):

1. Sounding curves provide slightly greater probing depth and resolving power than Wenner soundings for equal AB electrode spacing.

2. Less manpower and time is required for making soundings than for a Wenner array.
3. When wide electrodes spacing are used, stray currents in industrial areas and telluric currents are more likely to affect measurements with the Werner array.
4. The Schlumberger array is more sensitive in measuring lateral variations in resistivity.
5. The Wenner array is more susceptible to drifting or unstable potential differences created by driving electrodes into the ground.
6. Schlumberger sounding curves can be more readily smoothed.

Schlumberger arrays are generally considered the most suitable configuration for vertical electrical sounding of a quasi-layered earth (Oldenburg, 1978), whereas the Wenner and dipole-dipole configurations are commonly employed for mapping lateral variations in electrical resistivity (Telford *et al*., 1990).

Dipole-dipole arrays:

These arrays are originally developed in the Soviet Union in the 1950s; they have certain advantages over the Schlumberger array for deep soundings because relatively short AB and MN lines reduce field measurement times. In addition, fewer problems are associated with current leakage and inductive coupling than for Schlumberger soundings. A detailed description of these configurations has been given by Al'pin (1950).

We will explain the linear kind of dipole-dipole group which is the polar or axial dipole-dipole array as illustrated in figure (2.9).

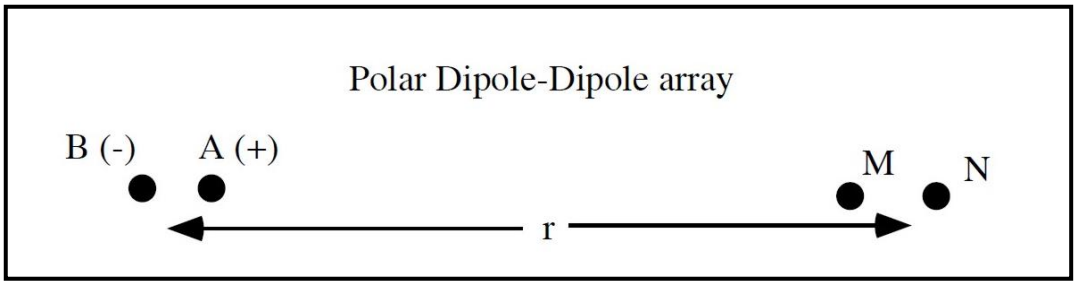

Figure (2.9) The polar dipole - dipole array. Both the current electrode pair and potential electrode pair form dipoles which are separated by a distance which is large compared to the dipole length.

For the polar dipole - dipole array we have:

$$\rho_a = \frac{\pi r^3}{\overline{AB}.\overline{MN}} \frac{\Delta V}{I} \quad \ldots\ldots (20)$$

References

AL-Menshed, F.H, 2011. Evaluation of resistivity method in delineation ground water hydrocarbon contamination southwest of Karbala city. Unpublished, PhD thesis. University of Baghdad, college of science.