الكلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : نظرية احتمالية 1

اسم المادة باللغة الإنكليزية : **Probability Theory 1**

اسم المحاضرة الثانية باللغة العربية: التباديل والتوافيق

اسم المحاضرة الثانية باللغة الإنكليزية : **Combinations and Permutation**

Permutation التباديل

Definition: A permutation of n dissimilar (distInct) element (objects) takes r at a time (r ≤ n)

{n(n-1)(n-2)…(n(r-1))} denoted by p(n,r) other notation Is. n_{P_r} $n! = n \times (n-1) \times (n-2) \dots \times 2 \times 1$

And 0!=1

We can write p(n,r)=n(n-1)(n-2)….[n-(r-1)]

$$= \frac{n(n-1)(n-2)\dots(n-r+1)(n-r)(n-r-1)\dots(3)(2)(1)}{(n-r)(n-r-1)\dots(3)(2)(1)}$$

Example: Find r if p(n,r)=24× $\binom{n}{r}$

Solution: p(n,r)=24× $\frac{P(n,r)}{r!}$ $\Rightarrow r! = 24 = 4 \times 3 \times 2 \times 1 \Rightarrow r = 4$

Example: Find n if $\binom{n}{6} \div \binom{n-3}{3} = 91 \div 4$

Example: Interpret ,$\binom{n}{r} = \binom{n}{n-r}$ Henle or otherwise show that

If $\binom{27}{r} = \binom{27}{r-3}$ $\textit{the value of} \binom{r}{12} = 455$

Solution: $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n!}{r!(n-r)!} = \binom{n}{n-r}$ *Hence* *r+(r-3)=27* $\Rightarrow r = 15 \Rightarrow \binom{15}{12} = 455$

Some standard results

$$\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$$

L . H . S $= \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!} = \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$

$$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n+1}{r(n-r+1)}\right] = \frac{(n+1)!}{r!(n-r+1)!}$$

$$= \binom{n+1}{r} = R\ H\ S$$

The manager of the guest house informs that he has only one triple , two clouble and three single rooms available

How many ways all the ten participants can be assigneel to the available rooms?

Solution: p(10,3,2,2,1,1,1)=$\frac{10!}{3!2!2!1!1!1!} = 151200$

Combinations التوافيق

Def: the number of distinct ways In which subsets of r elements can be selected from a set of n elements

We divide p(n,r) by $r!$ Denoted by n_{C_r} or C(n,r) or $\binom{n}{r}$

Hence $\Rightarrow \binom{n}{r} = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!r!}$ where n ϵI^+ , r=0,1,…..,n

Comment: remember that In a permutation order Is taken into account In combination order is not taken Into account

Theorem: (Binomial coefficients): If n is a positive integer and x,y $\epsilon\mathbb{R}$, the set of real number , then

$$(x,y)^n = \sum_{r=0}^{n}\binom{n}{r} x^{n-r}\, y^r = \; x^n + \frac{n}{1!}x^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + y^n$$

Example :write clown the fifth term in $\left(2x - \frac{1}{\sqrt{x}}\right)^{10}$

Solution:

put n=10 , r=4 , x=2x and y=$\frac{-1}{\sqrt{x}}$ in the equation $(x+y)^n$ $\Rightarrow T_5 = 13440x^4$

Example : Using the binomial theorem , find values of $(0)^4$and 99^4

Solution : $|0|^4 = (100+1)^4 = 100^4 + 4\times 100^3 + 6\times 100^2 + 4\times 100 + 1\times 100^0$

= 104,060,401

$$99^4 = (100-1)^4 = 100^4 - 4\times 100^3 + 6\times 100^2 - 4\times 100 + 1\times 100^0$$
$$= 96{,}059{,}601$$

Example : Expand $(1-2y^3)^4$, Report Coefficient y^9

$$(1-2y^3)^4 = 1 + 4(-2y^3) + \frac{(4)(3)(-2y^3)^2}{(2)(1)} + \frac{(4)(3)(2)(-2y^3)^3}{(3)(2)(1)} + (-2y^3)^4$$

= 1 -8$y^3 + 24y^6 - 32y^9 + 16y^{12}$

Pascals Triangle for $\binom{n}{r}$

n	r = 0,1,2,…..,n
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1
6	1 6 15 20 15 6 1
7	1 7 21 35 35 21 7 1
8	1 8 28 56 70 56 28 8 1
9	1 9 36 84 126 126 84 36 9 1
10	1 10 45 120 210 252 210 120 45 10 1

B) show that $\binom{m}{n} = \binom{m-1}{n-1} + \binom{m-1}{n}$, $0 \leq n \leq m$

Proofe L H S $= \binom{m-1}{n-1} + \binom{m-2}{n-1} + \binom{m-3}{n-1} + \binom{m-3}{n}$

The find result is $\binom{m}{n} = \sum_k \binom{k}{n-1}$ for k=n-1 to m-1

Example : get $\binom{7}{4}$ (using pascals tringle)

Solution : $\binom{7}{4} = \binom{6}{3} + \binom{5}{3} + \binom{4}{3} + \binom{3}{3} = 20 + 10 + 4 + 1 = 35$

Exercises:

) Prove that 1

$a)$ $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$

$b)$ $\binom{n}{r} = \binom{n-2}{r-2} + 2\binom{n-2}{r-1} + \binom{n-2}{r}$

) Find n if 2

i) P(n,2)=72

P(n,3)=p(n-1,3) + 3p(7,2) ii)

Obtain

$i)$ $5^{th} Term\ in\ the\ expanses\ of$ $\left(x + \frac{1}{x}\right)^8$

ii) find middle term of $\left(\frac{x}{y} - \frac{y}{x}\right)^8$

multinomial coefficients

Given $x_i(i = 1,2,\cdots,k)\ \epsilon R, n \in I^+$

$$(x, +x_2 + \cdots + x_k)^n = \sum \frac{n!}{n_1!\, n_2! \ldots nk!} (x_1)^{n_1} (x_2)^{n_2} \ldots (x_k)^{n_k}$$

Example : find the coefficient of

i) $a^2b^3c^4d$ *in the expansile of* $(a-b-c+d)^{10}$

ii) a^2b^5d *in the expansile of* $(a+b-c-d)^8$

iii) a^3b^3c *in the expansile of* $(2a+b+3c)^7$

iv) $x^2y^3z^4$ *in the expansile of* $(ax-by+cz)^9$

v) $x^4y^6z^3$ *in the expansile of* $(2x^2-3y^2+z)^8$

solution :

i) $n_1 = 2\ , n_2 = 3, n_3 = 4, n_4 = 1 \Rightarrow hance\ the\ coefficient\ of\ \ a^2b^3c^4d$

$is\ \ -12600\ \ similarly$

ii) -168

iii) 3360

iv) -1260 $a^2b^3c^4$

v) -12 (7!)