



الكلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : نظرية احتمالية 1

اسم المادة باللغة الإنكليزية : **Probability Theory 1**

اسم المحاضرة الخامسة باللغة العربية: الدالة الاحتمالية المشتركة

اسم المحاضرة الخامسة باللغة الإنكليزية : **Joint Probability Function**

**Joints p.m.f**

Let X and y are two random variable let  $x_j(j = 1, 2, \dots)$  and  $y_k(k = 1, 2, \dots)$  denote the values of X and y respectively the function  $f(x_j, y_k)$  defined for all ordered pairs  $(x_j, y_k)$   $j, k = 1, 2, \dots$

be the relation  $f(x_j, y_k) = P[x = x_j, y = y_k]$  satisfy

$$f(x_j, y_k) \geq 0 \quad \text{and} \quad \sum_j \sum_k f(x_j, y_k) = 1$$

is called the joint pmf of (x,y).

**EX:** The joint pmf of (x,y) is given by  $f(x_j, y_k) = cxy^2$  for  $X=1,2,3$  and for  $y=1,2$

a) Obtain unknown constant C

b) Express the joint pmf of (x,y) in the tabular form

c) Obtain the probabilities of the events

$$A_1 = [x = 1]$$

$$A_2 = \{x + y < 4\}$$

$$A_3 = \{x = 1 / x + y < 4\}$$

a)  $f(x,y)$  is a joint pmf of (x,y) hence  $f(x_j, y_k) \geq 0$  and  $\sum_j \sum_k f(x_j, y_k) = 1$

$$c > 0 \quad c \sum_1^3 \sum_1^2 xy^2 = 1 \quad \Rightarrow c = \frac{1}{30}$$

b) the joint pmf of (x,y) in a tabular form is as follows

Y $\longrightarrow$	1	2	total
X $\downarrow$			
1	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{5}{30}$
2	$\frac{2}{30}$	$\frac{8}{30}$	$\frac{10}{30}$
3	$\frac{3}{30}$	$\frac{12}{30}$	$\frac{15}{30}$
Total	$\frac{6}{30}$	$\frac{24}{30}$	1

c) using the above table

$$P(A_1) = P[x = 1] = P[x = 1, y = 1] + P[x = 1, y = 2] = \frac{5}{30}$$

$$P(A_2) = P[x + y < 4] = P[x = 1, y = 1] + P[x = 1, y = 2] + P[x = 2, y = 1] = \frac{7}{30}$$

$$P(A_3) = P[x = 1 / x + y < 4] = \frac{P[x=1]}{P[x+y<4]} = \frac{\frac{5}{30}}{\frac{7}{30}} = \frac{5}{7}$$

**EX:** suppose that  $f(x,y,z) = \begin{cases} e^{-x-y-z} & \text{if } x \geq 0, y \geq 0, z \geq 0 \\ 0 & \text{other wise} \end{cases}$

Is p. d. f for x,y and z , find CDF ???

SOL/

$$F(X,Y,Z) = P[X \leq x, Y \leq y, Z \leq z]$$

$$\int_0^x \int_0^y \int_0^z f(x,y,z) dx dy dz$$

$$\int_0^x e^{-x} dx \int_0^y e^{-y} dy \int_0^z e^{-z} dz = e^{-x} \int_0^x \cdot e^{-y} \int_0^y \cdot e^{-z} \int_0^z = (1 - e^{-x})(1 - e^{-y})(1 - e^{-z})$$

Def: let x and y be two r.v. with joint pmf or pdf  $f(x_j, y_k) = P[x = x_j, y = y_k]$  for  $x_j (j = 1, 2, \dots)$  and  $y_k (k = 1, 2, \dots)$  the function

$$f_1(x_j) = \sum_k \int_j f(x_j, y_k) \text{ defind for all values } x_j \text{ of the r. v } X \text{ is called the marginal}$$

Pmf or pdf of X similarly the function

$$f_2(y_k) = \sum_k \int_k f(x_j, y_k) f(x_j, y_k) \text{ defind for all value } y_k \text{ of the r.v } Y \text{ B called the marginal pmf or pdf of Y}$$

Comments

$$\underline{1} \ f_1(x_j) \geq 0 \text{ moreover } \sum_j f_1(x_j) = \sum_j [\sum_k f(x_j, y_k)] = 1$$

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$$\underline{2} \ f_1(x_j) = p[x = x_j] = \sum_k [x = x_j, y = y_k]$$

$$=p[x = x_j, y = y_1] + p[x = x_j, y = y_2] + \dots$$

بنفس الطريقة لإيجاد  $f_2(y_k)$