UNIVERSITY OF ANBAR

الكلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : نظرية احتمالية 1

اسم المادة باللغة الإنكليزية : Probability Theory 1

اسم المحاضرة السادسة باللغة العربية: الاستقلالية للمتغير العشوائي

اسم المحاضرة السادسة باللغة الإنكليزية :   Independent random variable

Joints p.m.f

Let X and y are two random variable let $x_j (j = 1, 2, ..)$ and $y_k (k = 1, 2, ..)$ denote the values of X and y respectively the function $f(x_j, y_k)$defind for all ordered pairs (x_j, y_k) j ,k =1,2,..

be the relation $f(x_j, y_k) = P[x = x_j, y = y_k]$satisfy

$$f(x_j, y_k) \geq 0 \quad \text{and} \quad \sum_J \sum_k f(x_j, y_k) = 1$$

is called the joint pmf of (x,y).

Example: The joint pmf of (x,y) is given by $f(x_j, y_k) = cxy^2$ for X=1,2,3 and for y=1,2

a) Obtain unknown constant C

b) Express the joint pmf of (x,y)in the tabular form

c) Obtain the probabilities of the events

$A_1 = [x = 1]$ $\qquad$ $A_2 = \{x + y < 4\}$ $\qquad$ $A_3 = \{x = 1 \,/\, x + y < 4\}$.

Solution:

a) **f (x,y) is a joint pmf of (x,y) hence $f(x_j, y_k) \geq 0$ and $\sum_J \sum_k f(x_j, y_k) = 1$**

C > 0 $\quad C \sum_1^3 \sum_1^2 xy^2 = 1 \qquad \Rightarrow C = \frac{1}{30}$

b) **the join pmf of (x,y) in a tabular form is as follows**

Y → / x ↓	1	2	total
1	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{5}{30}$
2	$\frac{2}{30}$	$\frac{8}{30}$	$\frac{10}{30}$
3	$\frac{3}{30}$	$\frac{12}{30}$	$\frac{15}{30}$
Total	$\frac{6}{30}$	$\frac{24}{30}$	1

c) using the above table

$P(A_1) = P[x=1] = P[x=1, y=1] + P[x=1, y=2] = \frac{5}{30}$

$P(A_2) = P[x+y<4] = P[x=1, y=1] + P[x=1, y=2] + P[x=2, y=1] = \frac{7}{30}$

$P(A_3) = P[x=1 \,/\, x+y<4] = \frac{P[x=1]}{P[x+y<4]} = \frac{\frac{5}{30}}{\frac{7}{30}} = \frac{5}{7}$

Example: suppose that f(x,y,z) = $\begin{cases} e^{-x-y-z} & \text{if } x \geq 0 \quad ,y \geq 0 \quad ,z \geq 0 \\ 0 & \text{other wise} \end{cases}$

Is p. d. f for x,y and z , find CDF ???

Solution: **F(X,Y,Z)=P[**$X \leq x, Y \leq y, Z \leq z$**]**

$$\int_0^x \int_0^y \int_0^z f(x,y,z)\,dx\,dy\,dz$$

$\int_0^x e^{-x}dx \int_0^y e^{-y}dy \int_0^z e^{-z}dz = e^{-x} \int_0^x \;.\, e^{-y} \int_0^y \;.\, e^{-z} \int_0^z = (1-e^{-x})(1-e^{-y})(1-e^{-z})$

Def: let x and y be two r.v. with joint pmf or pdf $f(x_j, y_k) = P[x = x_j, y = y_k]$ **for** $x_j (j = 1, 2, ..)$ **and** $y_k (k = 1, 2, ..)$**the function**

$f_1(x_j) = \sum_j \int_j f(x_j, y_k)$ *defind for all values* x_j *of the r . v X is called the marginal*

Pmf or pdf of X similarly the function

$f_2(y_k) = \sum_k \int_k f(x_j, y_k)\, f(x_j, y_k)$ **defind for all value** y_k **of the r.v Y B called the marginal pmf or pdf of Y**

Comments

<u>1</u> $f_1(x_j) \geq 0$ *moreover* $\sum_j f_1(x_j) = \sum_j [\sum_k f(x_j, y_k)] = 1$

بنفس الطريقة فية المتغيرات العشوائية المستمرة باستخدام التكامل بدل الΣ

2- $f_1(x_j) = p[x = x_j] = \sum_k [x = x_j, y = y_k]$

=p$[x = x_j, y = y_1] + p[x = x_j, y = y_2] + \cdots$

بنفس الطريقة لإيجاد $f_2(y_k)$**.**