الكلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : نظرية احتمالية 1

اسم المادة باللغة الإنكليزية : **Probability Theory 1**

اسم المحاضرة الثامنة باللغة العربية: تطبيقات التوقع الرياضي

اسم المحاضرة الثامنة باللغة الإنكليزية : **Applications of Mathematical Expectation**

Applications of the Mathematical Expectation تطبيقات التوقع الرياضي

EX / A biased die the p.m.f of x is

X	1	2	3	4	5	6	total
P[x=x]	$\frac{1}{21}$	$\frac{2}{21}$	$\frac{3}{21}$	$\frac{4}{21}$	$\frac{5}{21}$	$\frac{6}{21}$	1

Find E(x)

Sol ; $E(x)=\sum_{x=1}^{6} x\, p[x=x] = \frac{1^2+2^2+3^2+4^2+5^2+6^2}{21} = \frac{91}{21}$

Note that the $E(x)=\frac{91}{21}$ is not one of the possible values of x in fact its need not be so

E(X) is defend mean of probabilities distribution and denote M be such that $M = E(X) = \int_{-\infty}^{\infty} \boldsymbol{Xf(x)\,dx} = \sum_x \boldsymbol{Xf(x)}$

if X and y are independent random variable having finite expectation then E(XY)=E(X).E(Y)

proof : exercise

$$E(XY)=\sum_j \sum_k x_j y_k f(x_j, y_k)$$

$$=\sum_j \sum_k x_j y_k f(x_j).F(y_k)$$

$$=\sum_j x_j\ f(x_j)\ \sum_k Y_k\ f(Y_k)$$

$$\sum_j x_j\ f(x_j)\ \{E(y)\}\ by\ \text{def}\ of\ E(y)$$

= {E(x) E(y)}

Since the E(Y) is constant

Comment $E\left(\prod_{j=1}^{n} x_j\right) = \prod_{j=1}^{n} E(x_j) \ \ if\ E(x_j) IS\ FINITE$

Example : if f(x) = 2(1-x) 0< x< i is p.d.f

0 o.w

for x find E($3x^2 + 5x$)

Sol:

E($3x^2 + 5x$)= 3E(x^2)+5E(X)

Or $=\int_0^1 (3x^2 + 5x)\big(2(1-x)\big)dx$

$=\int_0^1 (6x^2 + 10x - 3x^3 - 10x^3)dx$

$=\frac{13}{6}$

Example: if f(x)= $\frac{x}{6}$ x=1,2,3 is p.m.f of x find

0 o .w

Example: if f(x,y)= x+y 0<x<1 , 0<y<1

0 o.w joint is p.d.f for (x,y)

find E(X),E(Y),E(X y^2)

E(X)=$\int_0^1 \int_0^1 X(X+Y)dx\,dy = \frac{7}{12}$

$$E(y)=\int_0^1\int_0^1 y(X+Y)dx\,dy = \frac{7}{12}$$

$$E(X\,y^2)=\int_0^1\int_0^1 X\,y^2(X+Y)dx\,dy = \frac{17}{72}$$

4) The mathematical expectation of $(x-M)^2$ is defined of **variance of probability distribution** and denote by σ^2 search that

$$\text{var (x)=v(x)=}\ \sigma^2 = E(x-M)^2 = \int_{-\infty}^{\infty}(x-M)^2 f(x)dx$$

$$= \sum_x (x-M)^2 f(x)$$

$$Var(x)=E(x^2)-[E(X)]^2 \geq 0$$

$$E(x^2) \geq [E(X)]^2$$

5) The mathematical expectation of e^{tx} is defined of **moment generating function (m.g.f)** and denote by$\left(M_x(t)=M(t)\right)$ such that

$$M_x(t)=E(e^{tx})=\int_{-\infty}^{\infty} e^{tx} f(x)\,dx = \sum_x e^{tx} f(x)$$

Comment

)The variance is always a non- negative number 1

2)The positive square root of the variance is called standard deviation (s.d) and usually denoted by σ^2

3)The variance and s.d are the measure of spread or dispersion of the distribution

if a random variable X takes values near to E(X) with a large probability, X will have (con cent rated) distribution otherwise it will have a large variance

4) variance is zero iff r.v X assumes only one constant values.