



الكلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : نظرية احتمالية 1

اسم المادة باللغة الإنكليزية : **Probability Theory 1**

اسم المحاضرة الحادي عشر باللغة العربية: التباين المشترك وبعض خواصه

اسم المحاضرة الحادي عشر باللغة الإنكليزية : **The Co – Variance and its Properties**

## Some properties of the Variance بعض خواص التباين

### **Theorem:**

$$\text{Var}(ax+b)=a^2 \text{ var}(x)$$

where a,b are constant

Proof/

We assume that  $u=E(x)$  is finite now by definition ,

$$\begin{aligned}\text{Var}(ax+b) &= E[ax+b-E(Ax+B)]^2 \\ &= E(ax+b -E(ax)-b)^2 \\ &= E(a(x-u))^2 = a^2 E(x-u)^2 = a^2 \text{var}(x)\end{aligned}$$

That is variance is independent of change of origin but not of scales

**Comment:** If to take mean as origin and the s. d. as unit hence we consider the variable

$Z = \frac{x-E(x)}{\sigma_x}$  it is called the normalized variable corresponding to X, and

$$E(Z)=0 , \text{Var}(Z)=1$$

To obtain  $V(X+Y)$  we need to define the covariance of x and y

$$\text{Var}(X+Y)= \text{Var}(x) +\text{Var}(Y)+ 2\text{COV}(X,Y))$$

**Definition:** Let x and y be two r.v on the same sample space if  $E(x),E(y),$ and  $E(xy),$ are finite then the covariance of x and y denote by  $(\text{COV}(X,Y))$  is defined as

$$\begin{aligned}\text{COV}(X,Y) &= E(\{X-E(X)\} \{Y-E(Y)\} ) \\ &= E(XY)-E(X)E(Y)\end{aligned}$$

**Comment:**

- 1) The notion of covariance was introduced by sir Francis Galton in 1885 1
- 2) If  $x$  and  $y$  are independent the  $\text{cov}(x,y)=0$ . However the reverse is not true 2
- 3) Covariance measure of the lagres of linear dependent between two r.v.s
- 4) a)  $\text{Var}(ax-by) = a^2\text{Var}(x) + b^2\text{Var}(y) - 2ab \text{Cov}(x,y)$   
 b)  $\text{Cov}(a+x, b+y) = \text{Cov}(x,y)$   
 c)  $\text{Cov}(x, ax+b) = a \text{Var}(x)$
- 5)  $\text{var}(x \pm y) = \text{var}(x) + \text{var}(y) \pm 2\text{cov}(x,y)$
- 6) If  $x$  and  $y$  are independent then  $\text{Cov}(x,y)=0$  hence6

$$\text{Var}(x+y) = \text{Var}(x) + \text{Var}(y)$$