UNIVERSITY OF ANBAR

الكلية : التربية للعلوم الصرفة

القسم او الفرع : الرياضيات

المرحلة: الثالثة

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اسم المادة باللغة العربية : نظرية احتمالية 1

اسم المادة باللغة الإنكليزية : Probability Theory 1

اسم المحاضرة الثانية عشر باللغة العربية: اختبارات عن موضوع احتمالية 1

اسم المحاضرة الثانية عشر باللغة الإنكليزية : <u>Tests of the Probability Theory 1</u>

Q1: Let X is a random variable has probability mass function

$$f(x) = \begin{cases} \frac{c4^x}{x!}, & \text{for } x = 0,1,\dots \\ 0 & \text{O.w.} \end{cases}$$

Find 1)The constant value of c. 2)$P(0 < x < 1)$.

Q2: Let $p(A)=\frac{1}{3}$,$p(B)=\frac{1}{4}$, $\rho(A\cap B)=\frac{1}{6}$, find

$p(A^c)$, $p(A^c \cup B)$, $p(A \cup B^c)$, $p(A \cap B^c)$, $p(A^c \cup B^c)$.

Q3 : If X is a continuous random variable has the probability density function:

$f(x)= cx(1-x^2)$ $0 \le x \le 2$

Find : 1-The value of c . 2)$P(-1 < x < 1)$.

Q4 : If X is a continuous random variable has the cumulative distribution function :

$$F(x) = \frac{x}{1+x} \qquad x \geq 0$$

Find the probability density function of X.

Q5: (15 marks)

Calculate the coefficient of correlation between the values of X and Y, also find two regression equation form the following tables.

X	**2**	**3**	**4**	**6**	**9**
Y	**1.8**	**2.7**	**3.2**	**4.8**	**7.0**

Q6: The joint pmf of (x,y) is given below

$$f(x,y)= \begin{cases} cxy^2 & \text{for } x=1,2,3 \text{ and } y=1,2 \text{ and } c > 0 \\ 0 & \text{o. w} \end{cases}$$

obtain the constant c. b) Are X and Y independent.

Q7: Let X is a random variable with the probability mass function as follows:

$$f(x) = p[X = x] = \begin{cases} 2^{-x} & \text{for } x = 1,2,\dots. \\ 0 & \text{other wise} \end{cases}$$

Find The moment generating function of X.

Q8 : If X is a continuous random variable has the probability density function:

$$f(x)= cx(1-x^2) \qquad 0 \le x \le 2$$

Find : 1-fixed value of c .

2- The expectation value and the variance.

Q9 : Let X is a random variable with the function :

$$f(x) = \frac{1}{6}\sqrt{x} \qquad x= 1, 4 , 9$$

Is f(x) probability mass functions? If yes, find the moment generating function of X.

Q10:

a) **Show that** $\binom{n}{r}=\binom{n}{n-r}$**. Hence or otherwise show that if** $\binom{27}{r}=\binom{27}{n-3}$ **the value of** $\binom{r}{12}=455$.

b) **For any r.v. X and Y prove that** $E\left[E\left(X/y\right)\right]=E(x)$.

Q11:

The joint pmf of (X,Y) is given by $p[X=x]=\begin{cases} cxy^2 & \{\text{for } x=1,2,3 \text{ and } y=1,2 \\ 0 & \text{otherwise} \end{cases}$

Obtain unknown constant c, Express the joint pmf of (X,Y) in the tabular form, and obtain the probabilities of the events $A_1=\{X=1\}$, $A_2=\{X+Y<4\}$, $A_3=\{X=1/X+Y<4\}$

Q12: A bivariate r.v. (X,Y) has the joint pmf as follows:

$$f(x,y)=\begin{cases} \dfrac{e^{-2m} m^{x+y}}{x!y!} & \begin{cases}\text{for } x=0,1,2,\dots; \\ \text{for } y=0,1,2,\dots.\end{cases} \\ 0 & \text{otherwise} \end{cases}$$

i. **Obtain marginal distribution of X and of Y.**

ii. Show that X and Y are independent r.v.s.

iii. Find the conditional distribution of X given Y=y and of Y given X=x.

Q13:

A tetrahedron (four sided dice with outcomes 1,2,3,4) is rolled twice, let X be the sum of two outcomes. Obtain pmf of X?

Q14: The pmf of X is given by

$$p[X=x]=\begin{cases}\binom{10}{x}(0.3)^x\,(0.7)^{10-x} & \text{for } x=0,1,2,\ldots \\ 0 & \text{otherwise}\end{cases}$$

Obtain the skewedness and kurtosis coefficients and type of this distribution.

Q15: Let X, and Y are be two independent r.v. have p.d.f. for each as follows

$f(x)=12x^2(1-x),\ \text{for } 0\le x\le 1$ and $f(y)=2y,\ \text{for } 0\le y\le 1$ **, find** $E\left(\frac{y}{x^2}+\frac{x}{y}\right)$**?**

Q16: Show that $\binom{n}{r}=\binom{n-2}{r-2}+2\binom{n-2}{r-1}+\binom{n-2}{r}$.

Q16: The probability distribution of r.v. $p[X=x,Y=y]=\begin{cases} cxy & \{\text{for } x=1,2,3,4 \text{ and } y=1,2,3,4 \\ 0 & \text{otherwise}\end{cases}$ **Find the constant c, and the** $p[X\le 3, 3\le y\le 4]$, $\mathrm{Var}(x)$, $\mathrm{Var}(y)$, $\mathrm{Cov}(x,y)$, and $\rho_{x,y}$.

Q17: Let A_i $(i=1,2,3)$ **are independent events define on** Ω**. Can we conclude that** $A_1\cup A_2$ **is independent** A_3**? If yes, prove that?**