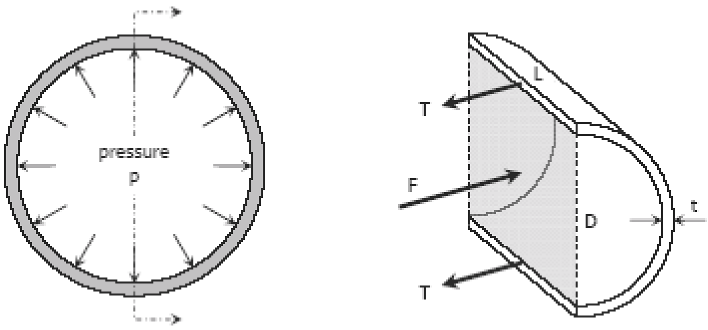
# *Thin-Walled Pressure Vessels*

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

**TANGENTIAL STRESS**

(Circumferential Stress)

Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



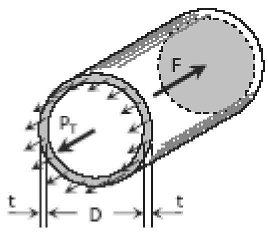

$$F = pA = pDL$$
$$T = \sigma_t A_{wall} = \sigma_t tL$$
$$[\ \Sigma F_H = 0\ ]$$
$$F = 2T$$
$$pDL = 2(\sigma_t tL)$$
$$\sigma_t = \frac{pD}{2t}$$

If there exist an external pressure $p_o$ and an internal pressure $p_i$, the formula may be expressed as:

$$\sigma_t = \frac{(p_i - p_o)D}{2t}$$

**LONGITUDINAL STRESS, $\sigma_L$**

Consider the free body diagram in the transverse section of the tank:




The total force acting at the rear of the tank F must equal to the total longitudinal stress on the wall $P_T = \sigma_L A_{wall}$. Since t is so small compared to D, the area of the wall is close to $\pi Dt$

$$F = pA = p\frac{\pi}{4}D^2$$

$$P_T = \sigma_L \pi D t$$

$$[\ \Sigma F_H = 0\ ]$$

$$P_T = F$$

$$\sigma_L \pi D t = p\frac{\pi}{4}D^2$$

$$\sigma_L = \frac{pD}{4t}$$

If there exist an external pressure $p_o$ and an internal pressure $p_i$, the formula may be expressed as:

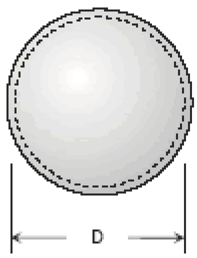
$$\sigma_L = \frac{(p_i - p_o)D}{4t}$$

It can be observed that the tangential stress is twice that of the longitudinal stress.

$$\sigma_t = 2\ \sigma_L$$

**SPHERICAL SHELL**

If a spherical tank of diameter D and thickness t contains gas under a pressure of p, the stress at the wall can be expressed as:



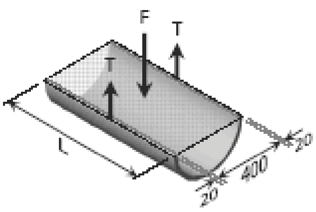

$$\sigma_L = \frac{(p_i - p_o)D}{4t}$$

SOLVED PROBLEMS IN  THIN WALLED PREASSURE VESSELS

**Problem 133**

A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m$^2$. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m$^2$? (c) If the internal pressure were increased until the vessel burst, sketch the type of fracture that would occur.

**Solution 133**



Longitudinal Section

(a) Tangential stress (longitudinal section):

$F = 2T$

$pDL = 2(\sigma_t\, tL)$

$\sigma_t = \dfrac{pD}{2t} = \dfrac{4.5(400)}{2(20)}$

$\sigma_t = \mathbf{45\ MPa}$

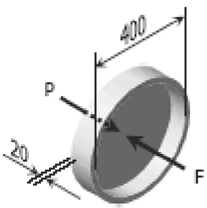
Longitudinal Stress (transverse section):

$F = P$

$\frac{1}{4}\pi D^2 p = \sigma_l\,(\pi D t)$

$\sigma_l = \dfrac{pD}{4t} = \dfrac{4.5(400)}{4(20)}$

$\sigma_l = \mathbf{22.5\ MPa}$



Transverse Section

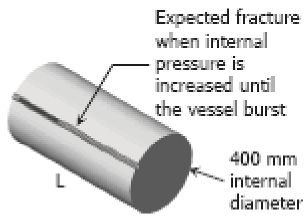
(b) From (a), $\sigma_t = \dfrac{pD}{2t}$ and $\sigma_l = \dfrac{pD}{4t}$ thus, $\sigma_t = 2\sigma_l$, this shows that tangential stress is the critical.

$\sigma_t = \dfrac{pD}{2t}$

$120 = \dfrac{p(400)}{2(20)}$

$P = \mathbf{12\ MPa}$

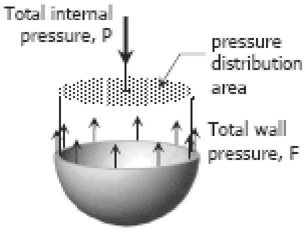
(c) The bursting force will cause a stress on the longitudinal section that is twice to that of the transverse section. Thus, fracture is expected as shown.



**Problem 134**

The wall thickness of a 4-ft-diameter spherical tank is 5/16 in. Calculate the allowable internal pressure if the stress is limited to 8000 psi.

**Solution 134**




Total internal pressure:

$$P = p\left(\tfrac{1}{4}\pi D^2\right)$$

Resisting wall:

$$F = P$$

$$\sigma A = p\left(\tfrac{1}{4}\pi D^2\right)$$

$$\sigma(\pi D t) = p\left(\tfrac{1}{4}\pi D^2\right)$$

$$\sigma = \frac{pD}{4t}$$

$$8000 = \frac{p(4\times 12)}{4(\frac{5}{16})}$$

$$p = \mathbf{208.33\ psi}$$

**Problem 135**

Calculate the minimum wall thickness for a cylindrical vessel that is to carry a gas at a pressure of 1400 psi. The diameter of the vessel is 2 ft, and the stress is limited to 12 ksi.

**Solution 135**

The critical stress is the tangential stress

$$\sigma_t = \frac{pD}{2t}$$

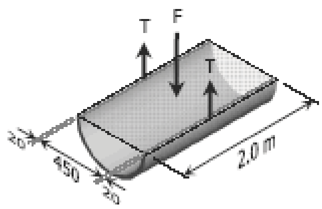
$$12\,000 = \frac{1400(2\times 12)}{2t}$$

$$t = \mathbf{1.4\ in}$$

**Problem 136**

A cylindrical pressure vessel is fabricated from steel plating that has a thickness of 20 mm. The diameter of the pressure vessel is 450 mm and its length is 2.0 m. Determine the maximum internal pressure that can be applied if the longitudinal stress is limited to 140 MPa, and the circumferential stress is limited to 60 MPa.

**Solution 136**




Based on circumferential stress (tangential):

$\Sigma F_V = 0$

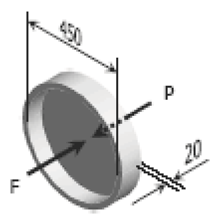
$F = 2T$

$p(DL) = 2(\sigma_t Lt)$

$\sigma_t = \dfrac{pD}{2t}$

$60 = \dfrac{p(450)}{2(20)}$

$p = 5.33\text{ MPa}$




Based on longitudinal stress:

$\Sigma F_H = 0$

$F = P$

$p\left(\frac{1}{4}\pi D^2\right) = \sigma_l(\pi Dt)$

$\sigma_l = \dfrac{pD}{4t}$

$140 = \dfrac{p(450)}{4(20)}$

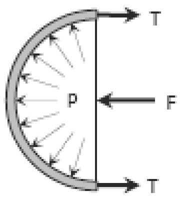
$p = 24.89\text{ MPa}$

Use $p$ = **5.33 MPa**

**Problem 137**

A water tank, 22 ft in diameter, is made from steel plates that are ½ in. thick. Find the maximum height to which the tank may be filled if the circumferential stress is limited to 6000 psi. The specific weight of water is 62.4 lb/ft$^3$.

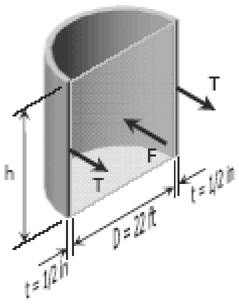
**Solution 137**




$\sigma_t = 6000$ psi

$$\sigma_t = \frac{6000\text{ lb}}{\text{in}^2}\left(\frac{12\text{ in}}{\text{ft}}\right)^2$$

$\sigma_t = 864\,000$ lb/ft$^2$




Assuming pressure distribution to be uniform:

$p = \gamma h = 62.4h$

$F = pA = 62.4h(Dh)$

$F = 62.4(22)h^2$

$F = 1372.8h^2$

$T = \sigma_t A_t = 864\,000(th)$

$T = 864\,000\left(\frac{1}{2} \times \frac{1}{12}\right) h$

$T = 36\,000h$

$\Sigma F = 0$

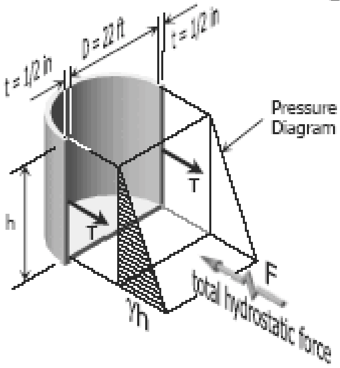
$F = 2T$

$1372.8h^2 = 2(36\,000h)$

$h = \mathbf{52.45\ ft}$

**Comment:**

Given a free surface of water, the actual pressure distribution on the vessel is not uniform. It varies linearly from 0 at the free surface to $\gamma h$ at the bottom (see figure below). Using this actual pressure distribution, the total hydrostatic pressure is reduced by 50%. This reduction of force will take our design into critical situation; giving us a maximum height of 200% more than the $h$ above.



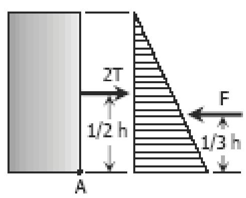

Based on actual pressure distribution:

Total hydrostatic force, $F$:

$F$ = volume of pressure diagram

$F = \frac{1}{2}(\gamma h^2)D = \frac{1}{2}(62.4h^2)(22)$

$F = 686.4h^2$




$\Sigma M_A = 0$

$2T\left(\frac{1}{2}h\right) - F\left(\frac{1}{3}h\right) = 0$

$T = \frac{1}{3}F$

$\sigma_t(ht) = \frac{1}{3}(686.4h^2)$

$$h = \frac{3\sigma_t t}{686.4} = \frac{3(864\,000)\left(\frac{1}{2} \times \frac{1}{12}\right)}{686.4}$$

$h = 157.34$ ft