Statically Indeterminate Members

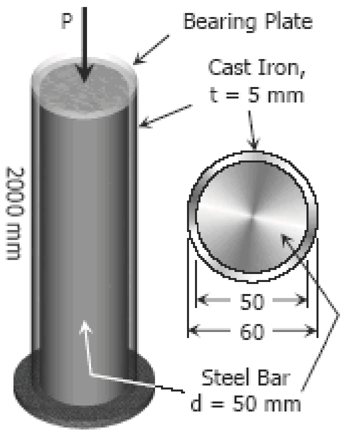
When the reactive forces or the internal resisting forces over a cross section exceed the number of independent equations of equilibrium, the structure is called **statically indeterminate**. These cases require the use of additional relations that depend on the elastic deformations in the members.

Solved Problems in Statically Indeterminate Members

Problem 233

A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.

Solution 233



$$\delta = \frac{PL}{AE}$$

$$\delta = \delta_{cast\ iron} = \delta_{steel} = 0.8\ \text{mm}$$

$$\delta_{cast\ iron} = \frac{P_{cast\ iron}(2000)}{[\frac{1}{4}\pi(60^2 - 50^2)](100\,000)} = 0.8$$

$$P_{cast\ iron} = 11\,000\pi\ \text{N}$$

$$\delta_{steel} = \frac{P_{steel}(2000)}{[\frac{1}{4}\pi(50^2)](200\,000)} = 0.8$$

$$P_{steel} = 50\,000\pi\ \text{N}$$

$$\Sigma F_V = 0$$

$$P = P_{cast\ iron} + P_{steel}$$

$$P = 11\,000\pi + 50\,000\pi$$

$$P = 61\,000\pi\ \text{N}$$

$$P = \mathbf{191.64\ kN}$$

Problem 234

A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use E_{co} = 14 GPa and E_{st} = 200 GPa.

Solution 234

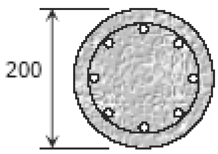
$$\delta_{co} = \delta_{st} = \delta$$

$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co} L}{14000} = \frac{\sigma_{st} L}{200000}$$

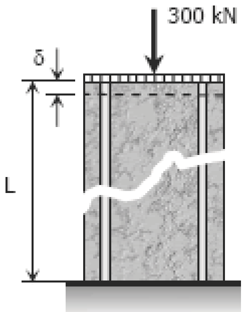
$$100\sigma_{co} = 7\sigma_s$$



When $\sigma_{st} = 120$ MPa

$$100\sigma_{co} = 7(120)$$

$$\sigma_{co} = 8.4 \text{ MPa} > 6 \text{ MPa (not ok!)}$$



When $\sigma_{co} = 6$ MPa

$$100(6) = 7\sigma_{st}$$

$$\sigma_{st} = 85.71 \text{ MPa} < 120 \text{ MPa (ok!)}$$

Use $\sigma_{co} = 6$ MPa and $\sigma_{st} = 85.71$ MPa

$$\sum F_V = 0$$

$$P_{st} + P_{co} = 300$$

$$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 300$$

$$85.71A_{st} + 6[\tfrac{1}{4}\pi(200)^2 - A_{st}] = 300(1000)$$

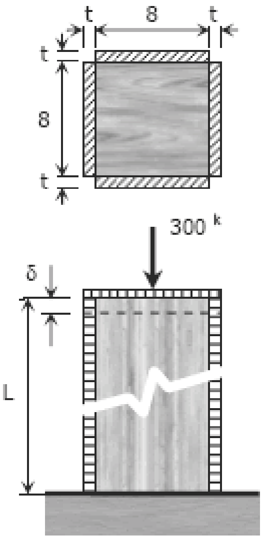
$$79.71A_{st} + 60\,000\pi = 300\,000$$

$$A_{st} = \mathbf{1398.9\ mm^2}$$

Problem 235

A timber column, 8 in. × 8 in. in cross section, is reinforced on each side by a steel plate 8 in. wide and t in. thick. Determine the thickness t so that the column will support an axial load of 300 kips without exceeding a maximum timber stress of 1200 psi or a maximum steel stress of 20 ksi. The moduli of elasticity are 1.5×10^6 psi for timber, and 29×10^6 psi for steel.

Solution 235



$$\delta_{st} = \delta_t$$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{timber}$$

$$\frac{\sigma_{st} L}{29 \times 10^6} = \frac{\sigma_{timber} L}{1.5 \times 10^6}$$

$$1.5\sigma_{st} = 29\sigma_{timber}$$

When $\sigma_{timber} = 1200$ psi

$$1.5\sigma_{st} = 29(1200)$$

$$\sigma_{st} = 23\,200 \text{ psi} = 23.2 \text{ ksi} > 20 \text{ ksi (not ok!)}$$

When $\sigma_{st} = 20$ ksi

$$1.5(20 \times 1000) = 29\sigma_{timber}$$

$$\sigma_{timber} = 1034.48 \text{ psi} < 1200 \text{ psi (ok!)}$$

Use $\sigma_{st} = 20$ ksi and $\sigma_{timber} = 1.03$ ksi

$$\sum F_V = 0$$

$$F_{steel} + F_{timber} = 300$$

$$\sigma_{st} A_{st} + \sigma_{timber} A_{timber} = 300$$

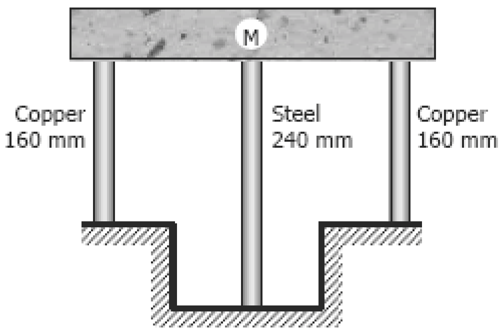
$$20[4(8t)] + 1.03(8^2) = 300$$

$$\mathbf{t = 0.365 \text{ in}}$$

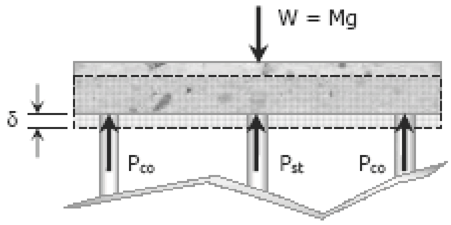
Problem 236

A rigid block of mass M is supported by three symmetrically spaced rods as shown in fig P-236. Each copper rod has an area of 900 mm^2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm^2; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass M which can be supported.

Figure P-236 and P-237



Solution 236



$$\delta_{co} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{co}(160)}{120000} = \frac{\sigma_{st}(240)}{200000}$$
$$10\sigma_{co} = 9\sigma_{st}$$

When $\sigma_{st} = 140$ MPa

$$\sigma_{co} = \tfrac{9}{10}(140)$$
$$\sigma_{co} = 126 \text{ MPa} > 70 \text{ MPa (not ok!)}$$

When $\sigma_{co} = 70$ MPa

$$\sigma_{st} = \tfrac{10}{9}(70)$$
$$\sigma_{st} = 77.78 \text{ MPa} < 140 \text{ MPa (ok!)}$$

Use $\sigma_{co} = 70$ MPa and $\sigma_{st} = 77.78$ MPa

$$\sum Fv = 0$$
$$2P_{co} + P_{st} = W$$
$$2(\sigma_{co}A_{co}) + \sigma_{st}A_{st} = Mg$$
$$2[70(900)] + 77.78(1200) = M(9.81)$$
$$M = \mathbf{22\,358.4\ kg}$$

Problem 237

In Prob. 236, how should the lengths of the two identical copper rods be changed so that each material will be stressed to its allowable limit?

Solution 237

Use $\sigma_{co} = 70$ MPa and $\sigma_{st} = 140$ MPa

$$\delta_{co} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{70L_{co}}{120000} = \frac{140(240)}{200000}$$
$$L_{co} = \mathbf{288\ mm}$$

Problem 238

The lower ends of the three bars in Fig. P-238 are at the same level before the uniform rigid block weighing 40 kips is attached. Each steel bar has a length of 3 ft, and area of 1.0 in.^{2}, and E = 29 × 10^6 psi. For the bronze bar, the area is 1.5 in.^{2} and E = 12 × 10^6 psi. Determine (a) the length of the bronze bar so that the load on each steel bar is twice the load on the bronze bar, and (b) the length of the bronze that will make the steel stress twice the bronze stress.

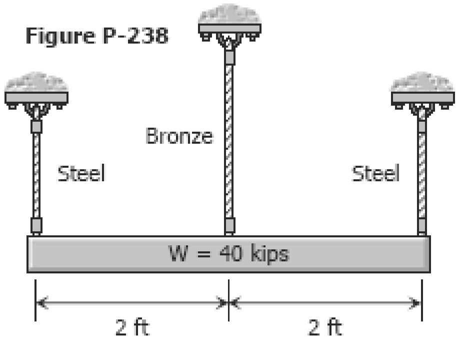
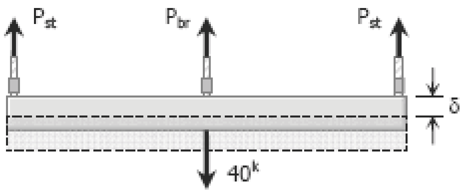


Figure P-238

Solution 238



(a) Condition: $P_{st} = 2P_{br}$

$\sum F_V = 0$

$2P_{st} + P_{br} = 40$

$2(2P_{br}) + P_{br} = 40$

$P_{br} = 8 \text{ kips}$

$P_{st} = 2(8) = 16 \text{ kips}$

$\delta_{br} = \delta_{st}$

$$\left(\frac{PL}{AE}\right)_{br} = \left(\frac{PL}{AE}\right)_{st}$$

$$\frac{8000L_{br}}{1.5(12\times10^6)} = \frac{16\,000(3\times12)}{1.0(29\times10^6)}$$

$L_{br} = 44.69 \text{ in}$

$L_{br} = \mathbf{3.72 \text{ ft}}$

(b) Condition: $\sigma_{st} = 2\sigma_{br}$

$\sum F_V = 0$

$2P_{st} + P_{br} = 40$

$2(\sigma_{st}A_{st}) + \sigma_{br}A_{br} = 40$

$2[(2\sigma_{br})A_{st}] + \sigma_{br}A_{br} = 40$

$4\sigma_{br}(1.0) + \sigma_{br}(1.5) = 40$

$\sigma_{br} = 7.27 \text{ ksi}$

$\sigma_{st} = 2(7.27) = 14.54 \text{ ksi}$

$\delta_{br} = \delta_{st}$

$$\left(\frac{\sigma L}{E}\right)_{br} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{7.27(1000)L_{br}}{12\times10^6} = \frac{14.54(1000)(3\times12)}{29\times10^6}$$

$L_{br} = 29.79 \text{ in}$

$L_{br} = \mathbf{2.48 \text{ ft}}$

Problem 239

The rigid platform in Fig. P-239 has negligible mass and rests on two steel bars, each 250.00 mm long. The center bar is aluminum and 249.90 mm long. Compute the stress in the aluminum bar after the center load P = 400 kN has been applied. For each steel bar, the area is 1200 mm^2 and E = 200 GPa. For the aluminum bar, the area is 2400 mm^2 and E = 70 GPa.

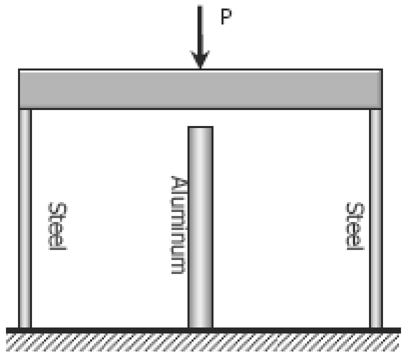
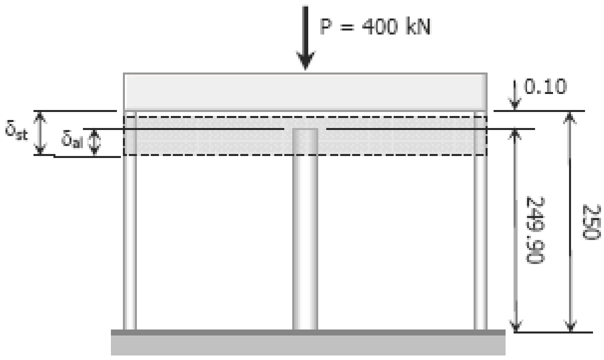


Figure P-239

Solution 239



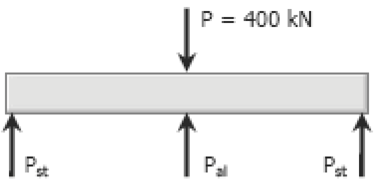
$$\delta_{st} = \delta_{al} + 0.10$$

$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al} + 0.10$$

$$\frac{\sigma_{st}(250)}{200\,000} = \frac{\sigma_{al}(249.90)}{70\,000} + 0.10$$

$$0.00125\sigma_{st} = 0.00357\sigma_{al} + 0.10$$

$$\sigma_{st} = 2.856\sigma_{al} + 80$$



$$\sum F_V = 0$$

$$2P_{st} + P_{al} = 400\,000$$

$$2\sigma_{st}\,A_{st} + \sigma_{al}\,A_{al} = 400\,000$$

$$2(2.856\sigma_{al} + 80)1200 + \sigma_{al}\,(2400) = 400\,000$$

$$9254.4\sigma_{al} + 192\,000 = 400\,000$$

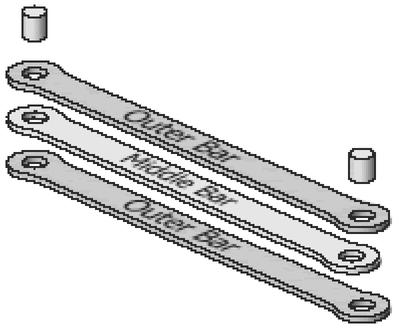
$$\sigma_{al} = \mathbf{22.48\ MPa}$$

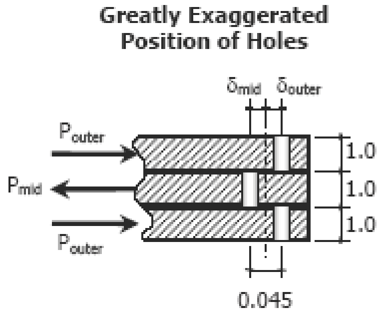
Problem 240

Three steel eye-bars, each 4 in. by 1 in. in section, are to be assembled by driving rigid 7/8-in.-diameter drift pins through holes drilled in the ends of the bars. The center-line spacing between the holes is 30 ft in the two outer bars, but 0.045 in. shorter in the middle bar. Find the shearing stress developed in the drip pins. Neglect local deformation at the holes.

Solution 240

Middle bar is 0.045 inch shorter between holes than outer bars.





$$\sum F_H = 0$$
$$P_{mid} = 2P_{outer}$$

$$\delta_{outer} + \delta_{mid} = 0.045$$
$$\left(\frac{PL}{AE}\right)_{outer} + \left(\frac{PL}{AE}\right)_{mid} = 0.045$$
$$\frac{P_{outer}(30\times 12)}{[1.0(4.0)]E} + \frac{P_{mid}(30\times 12 - 0.045)}{[1.0(4.0)]E} = 0.045$$
$$360P_{outer} + 359.955P_{mid} = 0.18E$$
$$360P_{outer} + 359.955(2P_{outer}) = 0.18E$$
(For steel: $E = 29 \times 106$ psi)

$$1079.91P_{outer} = 0.18(29 \times 10^6)$$
$$P_{outer} = 4833.74 \text{ lb}$$

$$P_{mid} = 2(4833.74)$$
$$P_{mid} = 9667.48 \text{ lb}$$

Use shear force $V = P_{mid}$

Shearing stress of drip pins (double shear):

$$\tau = \frac{V}{A} = \frac{9667.48}{2[\frac{1}{4}\pi(\frac{7}{8})^2]}$$
$$\tau = \mathbf{8038.54\ psi}$$

Problem 241

As shown in Fig. P-241, three steel wires, each 0.05 in.^{2} in area, are used to lift a load W = 1500 lb. Their unstressed lengths are 74.98 ft, 74.99 ft, and 75.00 ft. (a) What stress exists in the longest wire? (b) Determine the stress in the shortest wire if W = 500 lb.

Solution 241

Let L_1 = 74.98 ft; L_2 = 74.99 ft; and L_3 = 75.00 ft

(a) Bring L_1 and L_2 into L_3 = 75 ft length:

(For steel: E = 29 × 106 psi)

$$\delta = \frac{PL}{AE}$$

For L_1:

$$(75 - 74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$

P_1 = 386.77 lb

For L_2

$$(75 - 74.99)(12) = \frac{P_2(74.99 \times 12)}{0.05(29 \times 10^6)}$$

P_2 = 193.36 lb

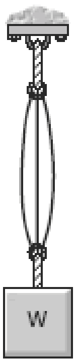


Figure P-241

Let $P = P_3$ (Load carried by L_3)

$P + P_2$ (Total load carried by L_2)

$P + P_1$ (Total load carried by L_1)

$\sum F_V = 0$

$(P + P_1) + (P + P_2) + P = W$

$3P + 386.77 + 193.36 = 1500$

$P = 306.62 \text{ lb} = P_3$

$$\sigma_3 = \frac{P_3}{A} = \frac{306.62}{0.05}$$

σ_3 = **6132.47 psi**

(b) From the above solution:

$P_1 + P_2$ = 580.13 lb > 500 lb (L_3 carries no load)

Bring L_1 into L_2 = 74.99 ft

$$\left[\delta = \frac{PL}{AE}\right] \quad (74.99 - 74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$

P_1 = 193.38 lb

Let $P = P_2$ (Load carried by L_2)

$P + P_1$ (Total load carried by L_1)

$\sum F_V = 0$

$(P + P_1) + P = 500$

$2P + 193.38 = 500$

P = 153.31 lb

$P + P_1 = 153.31 + 193.38$

$P + P_1$ = 346.69 lb

$$\sigma = \frac{P + P_1}{A} = \frac{346.69}{0.05}$$

σ = **6933.8 psi**