Thermal Stress

Temperature changes cause the body to expand or contract. The amount δ_T, is given by

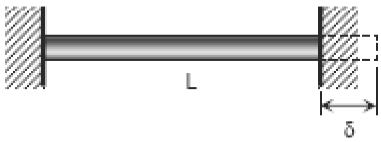
$$\delta_T = \alpha L(T_f - T_i) = \alpha L\,\Delta T$$

where α is the coefficient of thermal expansion in m/m°C, L is the length in meter, and T_i and T_f are the initial and final temperatures, respectively in °C.

For steel, $\alpha = 11.25 \times 10^{-6}$ / °C.

If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress.

For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as:



deformation due to temperature changes;

$$\delta_T = \alpha L\,\Delta T$$

deformation due to equivalent axial stress;

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$

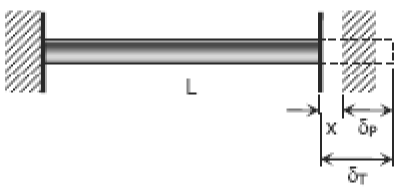
$$\delta_T = \delta_P$$

$$\alpha L\,\Delta T = \frac{\sigma L}{E}$$

$$\sigma = E\,\alpha\,\Delta T$$

where σ is the thermal stress in MPa and E is the modulus of elasticity of the rod in MPa.

If the wall yields a distance of x as shown, the following calculations will be made:



$$\delta_T = x + \delta_P$$

$$\alpha L\,\Delta T = x \;\frac{\sigma L}{E}$$

where σ represents the thermal stress.

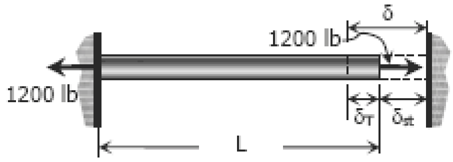
Take note that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension.

Solved Problems in Thermal Stress

Problem 261

A steel rod with a cross-sectional area of 0.25 in^2 is stretched between two fixed points. The tensile load at 70°F is 1200 lb. What will be the stress at 0°F? At what temperature will the stress be zero? Assume $\alpha = 6.5 \times 10^{-6}$ in / (in·°F) and $E = 29 \times 10^{6}$ psi.

Solution 261



For the stress at 0°C:

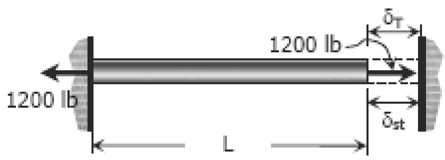
$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$\sigma = (6.5 \times 10^{-6})(29 \times 10^{6})(70) + \frac{1200}{0.25}$$

$$\sigma = 17\,995 \text{ psi} = \mathbf{18\ ksi}$$



For the temperature that causes zero stress:

$$\delta_T = \delta_{st}$$

$$\alpha \cancel{L}(\Delta T) = \frac{P\cancel{L}}{AE}$$

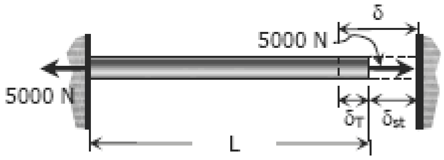
$$(6.5 \times 10^{-6})(T - 70) = \frac{1200}{0.25(29 \times 10^{6})}$$

$$T = \mathbf{95.46°C}$$

Problem 262

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution 262



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

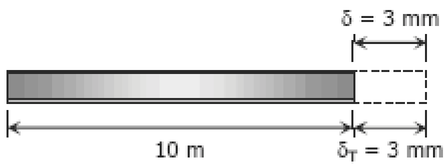
$$A = \frac{5000}{36.4} = 137.36 \text{ mm}^2$$

$$\tfrac{1}{4}\pi\, d^2 = 137.36; \quad \boldsymbol{d = 13.22 \text{ mm}}$$

Problem 263

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution 263



Temperature at which $\delta_T = 3$ mm:

$$\delta_T = \alpha L(\Delta T)$$

$$\delta_T = \alpha L(T_f - T_i)$$

$$3 = (11.7 \times 10^{-6})(10\,000)(T_f - 15)$$

$$\boldsymbol{T_f = 40.64°C}$$

Required stress:

$$\delta = \delta_T$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T)$$

$$\sigma = \alpha E(T_f - T_i)$$

$$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$$

$$\boldsymbol{\sigma = 60 \text{ MPa}}$$

Problem 264

A steel rod 3 feet long with a cross-sectional area of 0.25 in.^{2} is stretched between two fixed points. The tensile force is 1200 lb at 40°F. Using E = 29 × 10^6 psi and α = 6.5 × 10^{-6} in./(in.·°F), calculate (a) the temperature at which the stress in the bar will be 10 ksi; and (b) the temperature at which the stress will be
zero.

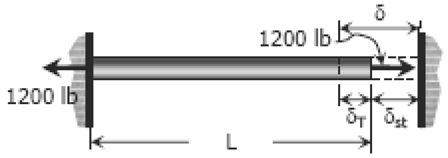
Solution 264

(a) Without temperature change:

$\sigma = P/A = 1200/0.25 = 4800$ psi

$\sigma = 4.8$ ksi < 10 ksi

A drop of temperature is needed to increase the stress to 10 ksi. See accompanying figure.



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

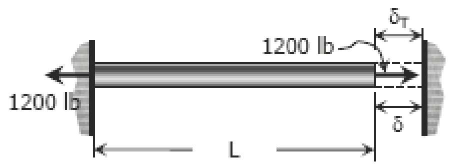
$$10\,000 = (6.5 \times 10^{-6})(29 \times 10^{6})(\Delta T) + \frac{1200}{0.25}$$

$$\Delta T = 27.59°\text{F}$$

Required temperature:
(temperature must drop from 40°F)

$T = 40 - 27.59 = \mathbf{12.41°F}$

(b) From the figure below:



$$\delta = \delta_T$$

$$\frac{P\cancel{L}}{AE} = \alpha \cancel{L}(\Delta T)$$

$$P = \alpha AE(T_f - T_i)$$

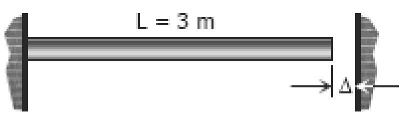
$$1200 = (6.5 \times 10^{-6})(0.25)\ (29 \times 10^{6})(T_f - 40)$$

$T_f = \mathbf{65.46°F}$

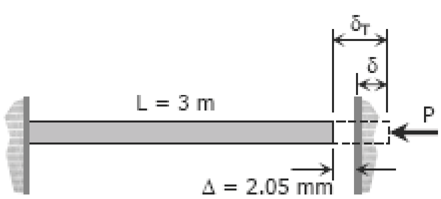
Problem 265

A bronze bar 3 m long with a cross sectional area of 320 mm^2 is placed between two rigid walls as shown in Fig. P-265. At a temperature of -20°C, the gap Δ = 25 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use $\alpha = 18.0 \times 10^{-6}$ m/(m·°C) and E = 80 GPa.

Figure P-265



Solution 265



$$\delta_T = \delta + \Delta$$

$$\alpha L(\Delta T) = \frac{\sigma L}{E} + 2.5$$

$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80000} + 2.5$$

$$\Delta T = 70.6°\text{C}$$

$$T = 70.6 - 20$$

$T = \mathbf{50.6°C}$