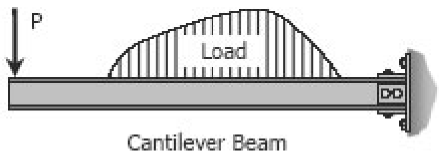
Shear & Moment in Beams

DEFINITION OF A BEAM

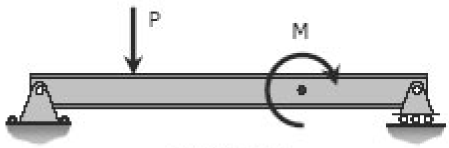
A beam is a bar subject to forces or couples that lie in a plane containing the longitudinal of the bar. According to determinacy, a beam may be determinate or indeterminate.

STATICALLY DETERMINATE BEAMS

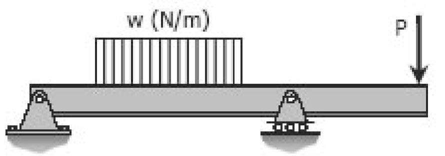
Statically determinate beams are those beams in which the reactions of the supports may be determined by the use of the equations of static equilibrium. The beams shown below are examples of statically determinate beams.



Cantilever Beam



Simple Beam

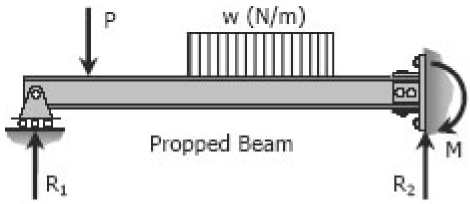


Overhanging Beam

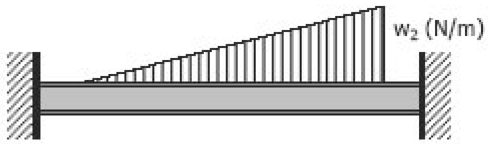
STATICALLY INDETERMINATE BEAMS

If the number of reactions exerted upon a beam exceeds the number of equations in static equilibrium, the beam is said to be statically indeterminate. In order to solve the reactions of the beam, the static equations must be supplemented by equations based upon the elastic deformations of the beam.

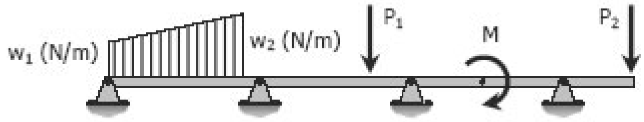
The degree of indeterminacy is taken as the difference between the umber of reactions to the number of equations in static equilibrium that can be applied. In the case of the propped beam shown, there are three reactions R_1, R_2, and M and only two equations ($\Sigma M = 0$ and sum;$F_v = 0$) can be applied, thus the beam is indeterminate to the first degree (3 – 2 = 1).



Propped Beam



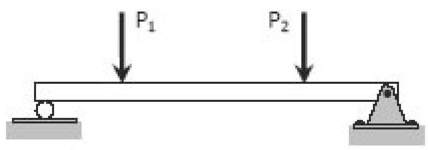
Fixed or Restrained Beam



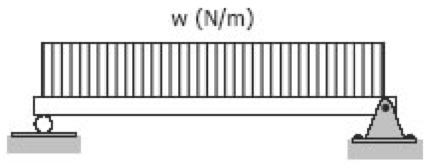
Continuous Beam

TYPES OF LOADING

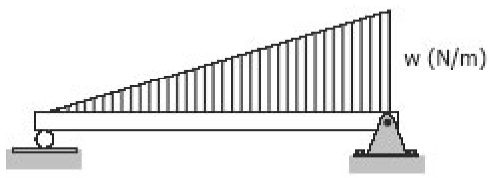
Loads applied to the beam may consist of a concentrated load (load applied at a point), uniform load, uniformly varying load, or an applied couple or moment. These loads are shown in the following figures.



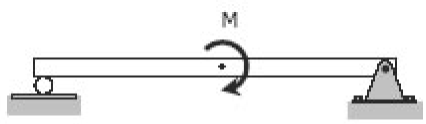
Concentrated Loads



Uniform Load

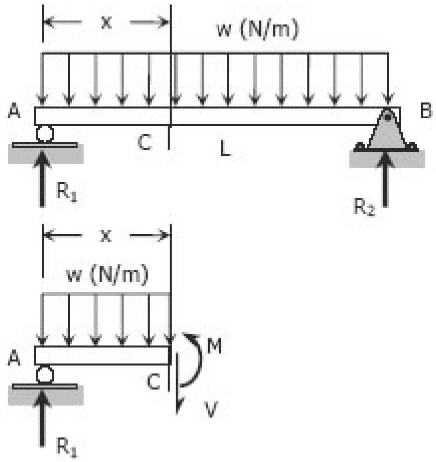


Uniformly Varying Load



Applied Couple

Shear and Moment Diagrams



Consider a simple beam shown of length L that carries a uniform load of w (N/m) throughout its length and is held in equilibrium by reactions R_1 and R_2. Assume that the beam is cut at point C a distance of x from he left support and the portion of the beam to the right of C be removed. The portion removed must then be replaced by vertical shearing force V together with a couple M to hold the left portion of the bar in equilibrium under the action of R_1 and wx. The couple M is called the resisting moment or moment and the force V is called the resisting shear or shear. The sign of V and M are taken to be positive if they have the senses indicated above.

Solved Problems in Shear and Moment Diagrams

INSTRUCTION

Write shear and moment equations for the beams in the following problems. In each problem, let x be the distance measured from left end of the beam. Also, draw shear and moment diagrams, specifying values at all change of loading positions and at points of zero shear. Neglect the mass of the beam in each problem.

Problem 403

Beam loaded as shown in Fig. P-403.

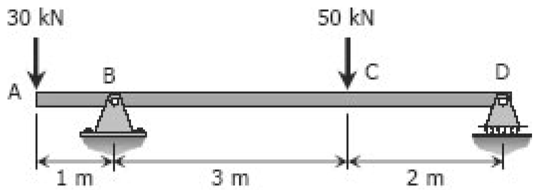


Figure P-403

Solution 403

From the load diagram:

$\sum M_B = 0$

$5R_D + 1(30) = 3(50)$

$R_D = 24\text{ kN}$

$\sum M_D = 0$

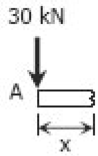
$5R_B = 2(50) + 6(30)$

$R_B = 56\text{ kN}$

Segment AB:

$V_{AB} = -30\text{ kN}$

$M_{AB} = -30x\text{ kN·m}$



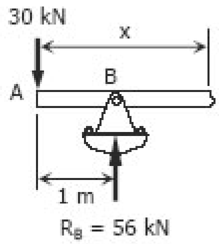
Segment BC:

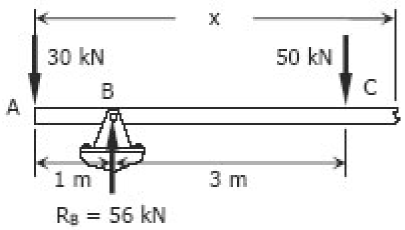
$V_{BC} = -30 + 56$

$= 26\text{ kN}$

$M_{BC} = -30x + 56(x - 1)$

$= 26x - 56\text{ kN·m}$





Segment CD:

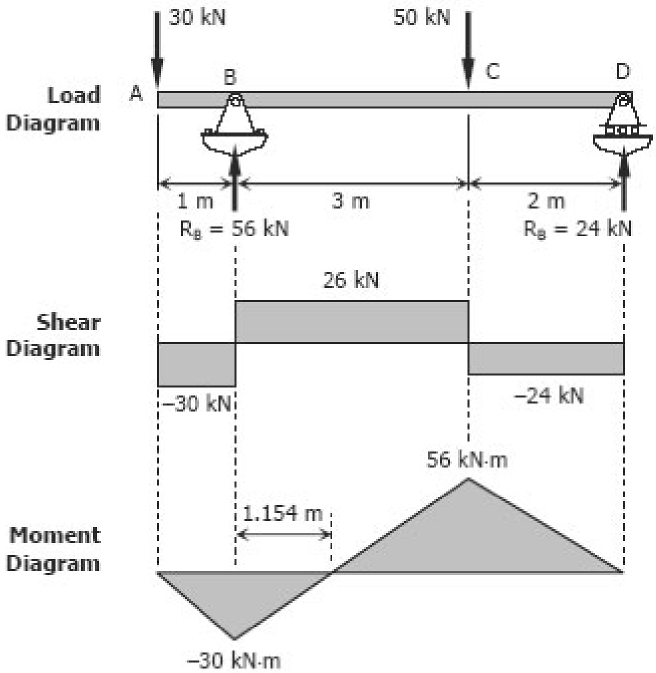
$V_{CD} = -30 + 56 - 50$

$= -24\text{ kN}$

$M_{CD} = -30x + 56(x - 1) - 50(x - 4)$

$= -30x + 56x - 56 - 50x + 200$

$= -24x + 144$



To draw the Shear Diagram:

(1) In segment AB, the shear is uniformly distributed over the segment at a magnitude of –30 kN.

(2) In segment BC, the shear is uniformly distributed at a magnitude of 26 kN.

(3) In segment CD, the shear is uniformly distributed at a magnitude of –24 kN.

To draw the Moment Diagram:

(1) The equation $M_{AB} = -30x$ is linear, at $x = 0$, $M_{AB} = 0$ and at $x = 1$ m, $M_{AB} = -30$ kN·m.

(2) $M_{BC} = 26x - 56$ is also linear. At $x = 1$ m, $M_{BC} = -30$ kN·m; at $x = 4$ m, $M_{BC} = 48$ kN·m. When $M_{BC} = 0$, $x = 2.154$ m, thus the moment is zero at 1.154 m from B.

(3) $M_{CD} = -24x + 144$ is again linear. At $x = 4$ m, $M_{CD} = 48$ kN·m; at $x = 6$ m, $M_{CD} = 0$.

Problem 404

Beam loaded as shown in Fig. P-404.

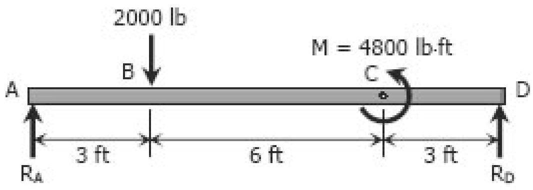


Figure P-404

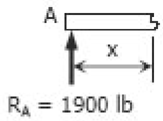
Solution 404

$$\Sigma M_A = 0$$
$$12R_D + 4800 = 3(2000)$$
$$R_D = 100 \text{ lb}$$

$$\Sigma M_D = 0$$
$$12R_A = 9(2000) + 4800$$
$$R_A = 1900 \text{ lb}$$

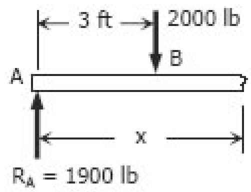
Segment AB:

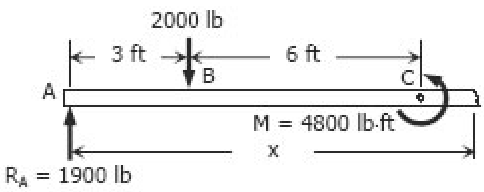
$$V_{AB} = 1900 \text{ lb}$$
$$M_{AB} = 1900x \text{ lb·ft}$$



Segment BC:

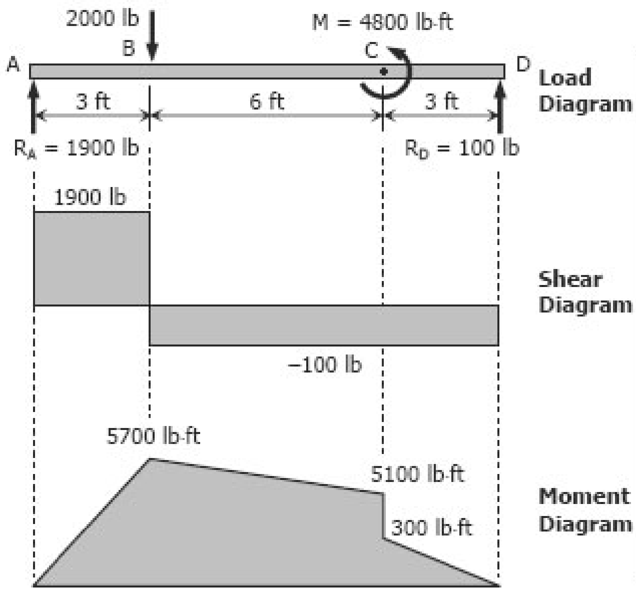
$$V_{BC} = 1900 - 2000$$
$$= -100 \text{ lb}$$
$$M_{BC} = 1900x - 2000(x - 3)$$
$$= 1900x - 2000x + 6000$$
$$= -100x + 6000$$





Segment CD:

$$V_{CD} = 1900 - 2000$$
$$= -100 \text{ lb}$$
$$M_{CD} = 1900x - 2000(x - 3) - 4800$$
$$= 1900x - 2000x + 6000 - 4800$$
$$= -100x + 1200$$



To draw the Shear Diagram:

(1) At segment AB, the shear is uniformly distributed at 1900 lb.

(2) A shear of −100 lb is uniformly distributed over segments BC and CD.

To draw the Moment Diagram:

(1) $M_{AB} = 1900x$ is linear; at $x = 0$, $M_{AB} = 0$; at $x = 3$ ft, $M_{AB} = 5700$ lb·ft.

(2) For segment BC, $M_{BC} = -100x + 6000$ is linear; at $x = 3$ ft, $M_{BC} = 5700$ lb·ft; at $x = 9$ ft, $M_{BC} = 5100$ lb·ft.

(3) $M_{CD} = -100x + 1200$ is again linear; at $x = 9$ ft, $M_{CD} = 300$ lb·ft; at $x = 12$ ft, $M_{CD} = 0$.

Problem 405

Beam loaded as shown in Fig. P-405.

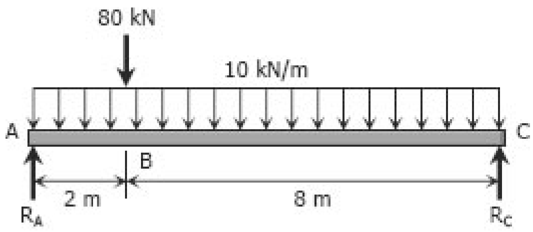
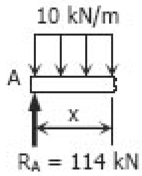


Figure P-405

Solution 405

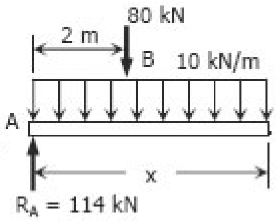
$\Sigma M_A = 0$
$10R_C = 2(80) + 5[10(10)]$
$R_C = 66 \text{ kN}$

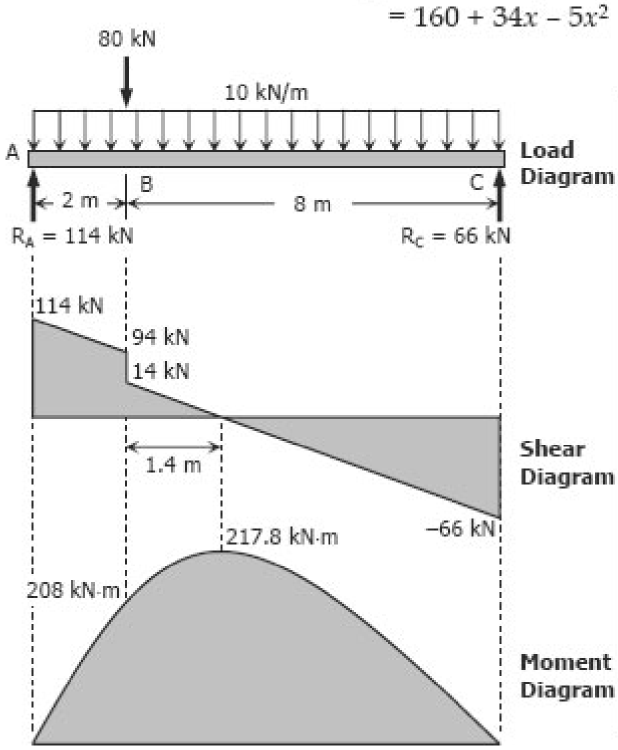
$\Sigma M_C = 0$
$10R_A = 8(80) + 5[10(10)]$
$R_A = 114 \text{ kN}$



Segment AB:
$V_{AB} = 114 - 10x$ kN
$M_{AB} = 114x - 10x(x/2)$
$= 114x - 5x^2$ kN·m

Segment BC:
$V_{BC} = 114 - 80 - 10x$
$= 34 - 10x$ kN
$M_{BC} = 114x - 80(x - 2) - 10x(x/2)$
$= 160 + 34x - 5x^2$





To draw the Shear Diagram:

(1) For segment AB, $V_{AB} = 114 - 10x$ is linear; at x = 0, V_{AB} = 14 kN; at x = 2 m, V_{AB} = 94 kN.

(2) $V_{BC} = 34 - 10x$ for segment BC is linear; at x = 2 m, V_{BC} = 14 kN; at x = 10 m, V_{BC} = −66 kN. When V_{BC} = 0, x = 3.4 m thus V_{BC} = 0 at 1.4 m from B.

To draw the Moment Diagram:

(1) $M_{AB} = 114x - 5x^2$ is a second degree curve for segment AB; at x = 0, M_{AB} = 0; at x = 2 m, M_{AB} = 208 kN·m.

(2) The moment diagram is also a second degree curve for segment BC given by $M_{BC} = 160 + 34x - 5x^2$; at x = 2 m, M_{BC} = 208 kN·m; at x = 10 m, M_{BC} = 0.

(3) Note that the maximum moment occurs at point of zero shear. Thus, at x = 3.4 m, M_{BC} = 217.8 kN·m.

Problem 406

Beam loaded as shown in Fig. P-406.

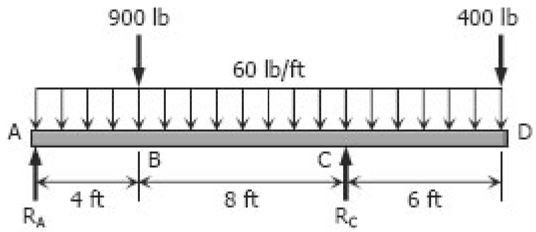


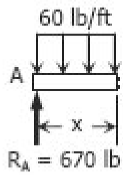
Figure P-406

Solution 406

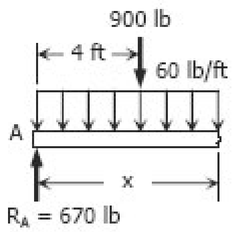
$$\sum M_A = 0$$
$$12R_C = 4(900) + 18(400) + 9[(60)(18)]$$
$$R_C = 1710 \text{ lb}$$

$$\sum M_C = 0$$
$$12R_A + 6(400) = 8(900) + 3[60(18)]$$
$$R_A = 670 \text{ lb}$$

Segment AB:

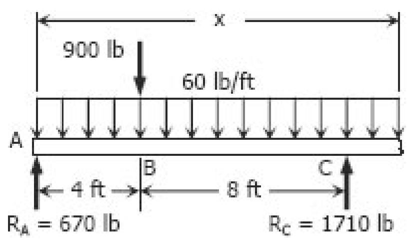


$$V_{AB} = 670 - 60x \text{ lb}$$
$$M_{AB} = 670x - 60x(x/2)$$
$$= 670x - 30x^2 \text{ lb·ft}$$



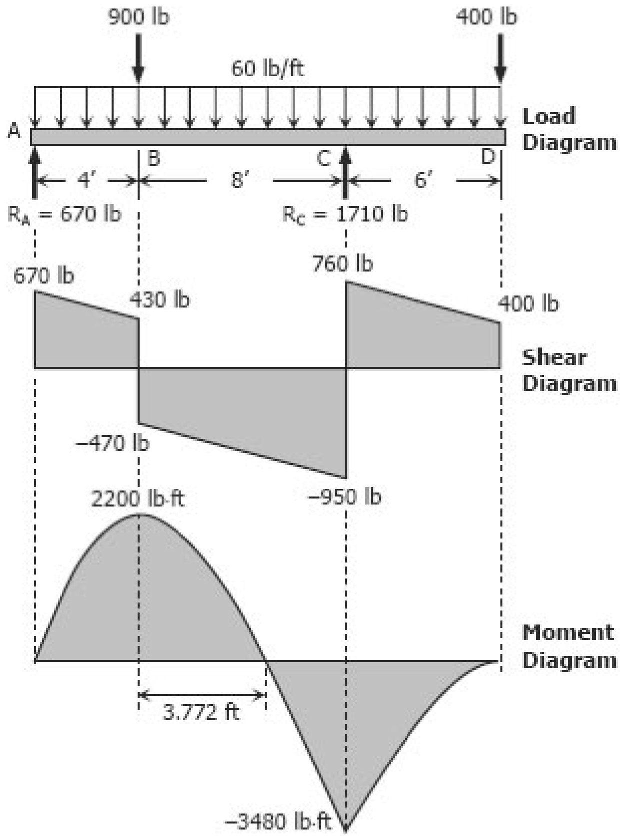
Segment BC:

$$V_{BC} = 670 - 900 - 60x$$
$$= -230 - 60x \text{ lb}$$
$$M_{BC} = 670x - 900(x - 4) - 60x(x/2)$$
$$= 3600 - 230x - 30x^2 \text{ lb·ft}$$



Segment CD:

$$V_{CD} = 670 + 1710 - 900 - 60x$$
$$= 1480 - 60x \text{ lb}$$
$$M_{CD} = 670x + 1710(x - 12)$$
$$- 900(x - 4) - 60x(x/2)$$
$$= -16920 + 1480x - 30x^2 \text{ lb·ft}$$



To draw the Shear Diagram:

(1) $V_{AB} = 670 - 60x$ for segment AB is linear; at x = 0, V_{AB} = 670 lb; at x = 4 ft, V_{AB} = 430 lb.

(2) For segment BC, $V_{BC} = -230 - 60x$ is also linear; at x= 4 ft, V_{BC} = −470 lb, at x = 12 ft, V_{BC} = −950 lb.

(3) $V_{CD} = 1480 - 60x$ for segment CD is again linear; at x = 12, V_{CD} = 760 lb; at x = 18 ft, V_{CD} = 400 lb.

To draw the Moment Diagram:

(1) $M_{AB} = 670x - 30x^2$ for segment AB is a second degree curve; at x = 0, M_{AB} = 0; at x = 4 ft, M_{AB} = 2200 lb·ft.

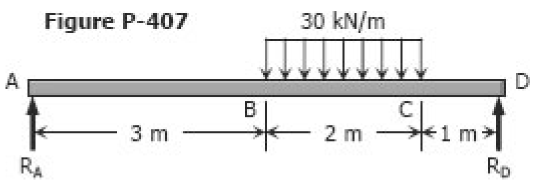
(2) For BC, $M_{BC} = 3600 - 230x - 30x^2$, is a second degree curve; at x = 4 ft, M_{BC} = 2200 lb·ft, at x = 12 ft, M_{BC} = −3480 lb·ft; When M_{BC} = 0, $3600 - 230x - 30x^2 = 0$, x = −15.439 ft and 7.772 ft. Take x = 7.772 ft, thus, the moment is zero at 3.772 ft from B.

(3) For segment CD, $M_{CD} = -16920 + 1480x - 30x^2$ is a second degree curve; at x = 12 ft, M_{CD} = −3480 lb·ft; at x = 18 ft, M_{CD} = 0.

Problem 407

Beam loaded as shown in Fig. P-407.

Figure P-407



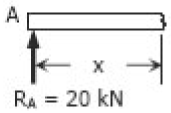
Solution 407

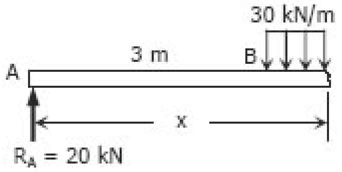
$$\Sigma M_A = 0$$
$$6R_D = 4[2(30)]$$
$$R_D = 40 \text{ kN}$$

$$\Sigma M_D = 0$$
$$6R_A = 2[2(30)]$$
$$R_A = 20 \text{ kN}$$

Segment AB:

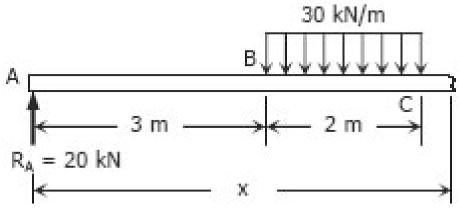
$$V_{AB} = 20 \text{ kN}$$
$$M_{AB} = 20x \text{ kN}\cdot\text{m}$$





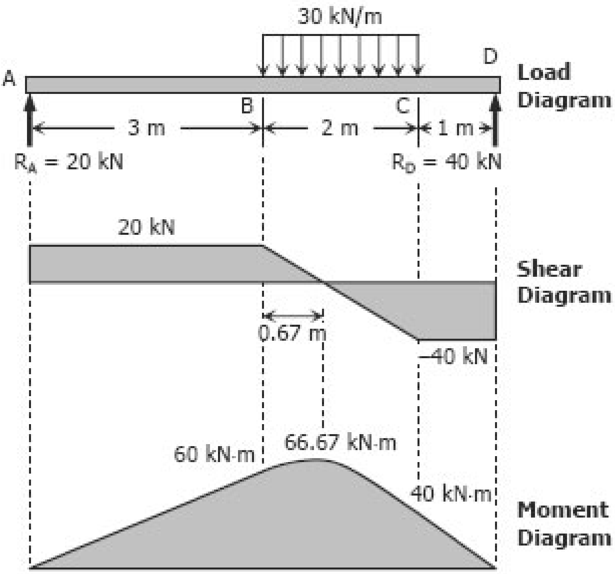
Segment BC:

$$V_{BC} = 20 - 30(x - 3)$$
$$= 110 - 30x \text{ kN}$$
$$M_{BC} = 20x - 30(x - 3)(x - 3)/2$$
$$= 20x - 15(x - 3)^2$$



Segment CD:

$$V_{CD} = 20 - 30(2)$$
$$= -40 \text{ kN}$$
$$M_{CD} = 20x - 30(2)(x - 4)$$
$$= 20x - 60(x - 4)$$



To draw the Shear Diagram:

(1) For segment AB, the shear is uniformly distributed at 20 kN.
(2) V_{BC} = 110 – 30x for segment BC; at x = 3 m, V_{BC} = 20 kN; at x = 5 m, V_{BC} = –40 kN. For V_{BC} = 0, x = 3.67 m or 0.67 m from B.
(3) The shear for segment CD is uniformly distributed at –40 kN.

To draw the Moment Diagram:

(1) For AB, M_{AB} = 20x; at x = 0, M_{AB} = 0; at x = 3 m, M_{AB} = 60 kN·m.
(2) $M_{BC} = 20x - 15(x - 3)^2$ for segment BC is second degree curve; at x = 3 m, M_{BC} = 60 kN·m; at x = 5 m, M_{BC} = 40 kN·m. Note that maximum moment occurred at zero shear; at x = 3.67 m, M_{BC} = 66.67 kN·m.
(3) M_{CD} = 20x – 60(x – 4) for segment BC is linear; at x = 5 m, M_{CD} = 40 kN·m; at x = 6 m, M_{CD} = 0.

Problem 408

Beam loaded as shown in Fig. P-408.

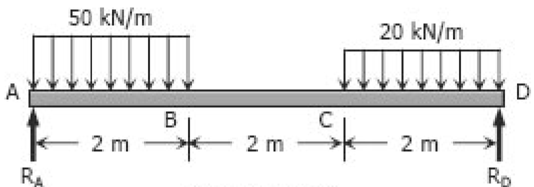
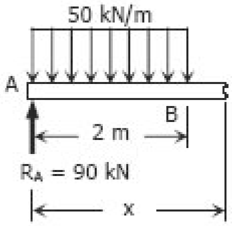


Figure P-408

Solution 408

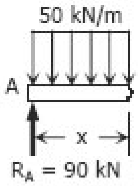
$\Sigma M_A = 0$	$\Sigma M_D = 0$
$6R_D = 1[2(50)] + 5[2(20)]$	$6R_A = 5[2(50)] + 1[2(20)]$
$R_D = 50 \text{ kN}$	$R_A = 90 \text{ kN}$



Segment AB:

$$V_{AB} = 90 - 50x \text{ kN}$$

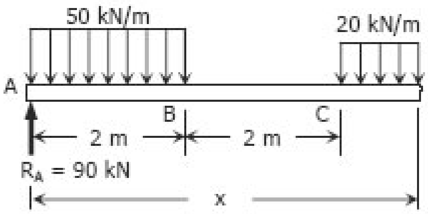
$$M_{AB} = 90x - 50x(x/2)$$
$$= 90x - 25x^2$$



Segment BC:

$$V_{BC} = 90 - 50(2)$$
$$= -10 \text{ kN}$$

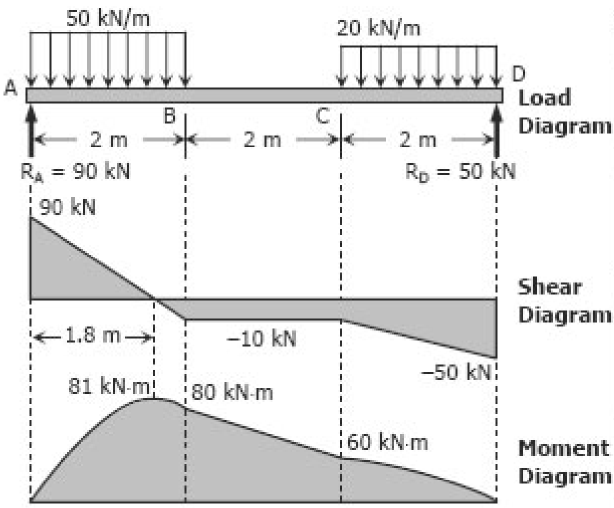
$$M_{BC} = 90x - 2(50)(x - 1)$$
$$= -10x + 100 \text{ kN·m}$$



Segment CD:

$$V_{CD} = 90 - 2(50) - 20(x - 4)$$
$$= -20x + 70 \text{ kN}$$

$$M_{CD} = 90x - 2(50)(x - 1)$$
$$- 20(x - 4)(x - 4)/2$$
$$= 90x - 100(x - 1) - 10(x - 4)^2$$
$$= -10x^2 + 70x - 60 \text{ kN·m}$$



To draw the Shear Diagram:

(1) $V_{AB} = 90 - 50x$ is linear; at $x = 0$, $V_{BC} = 90$ kN; at $x = 2$ m, $V_{BC} = -10$ kN. When $V_{AB} = 0$, $x = 1.8$ m.

(2) $V_{BC} = -10$ kN along segment BC.

(3) $V_{CD} = -20x + 70$ is linear; at $x = 4$ m, $V_{CD} = -10$ kN; at $x = 6$ m, $V_{CD} = -50$ kN.

To draw the Moment Diagram:

(1) $M_{AB} = 90x - 25x^2$ is second degree; at $x = 0$, $M_{AB} = 0$; at $x = 1.8$ m, $M_{AB} = 81$ kN·m; at $x = 2$ m, $M_{AB} = 80$ kN·m.

(2) $M_{BC} = -10x + 100$ is linear; at $x = 2$ m, $M_{BC} = 80$ kN·m; at $x = 4$ m, $M_{BC} = 60$ kN·m.

(3) $M_{CD} = -10x^2 + 70x - 60$; at $x = 4$ m, $M_{CD} = 60$ kN·m; at $x = 6$ m, $M_{CD} = 0$.

Problem 409

Cantilever beam loaded as shown in Fig. P-409.

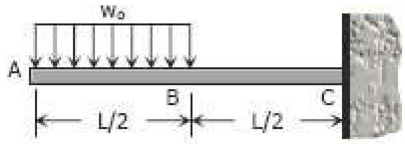


Figure P-409

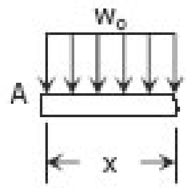
Solution 409

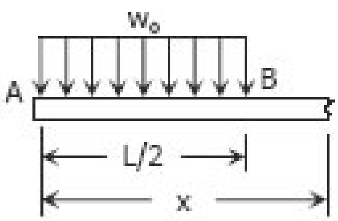
Segment AB:

$V_{AB} = -w_o x$

$M_{AB} = -w_o x(x/2)$

$= -\frac{1}{2} w_o x^2$





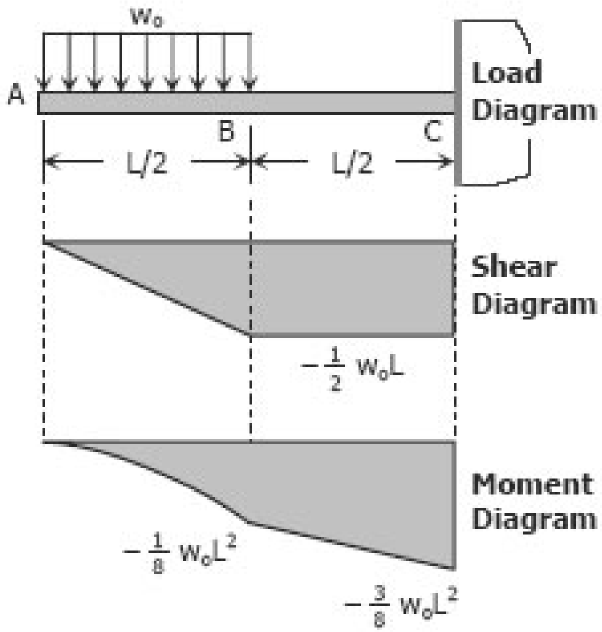
Segment BC:

$V_{BC} = -w_o(L/2)$

$= -\frac{1}{2} w_o L$

$M_{BC} = -w_o(L/2)(x - L/4)$

$= -\frac{1}{2} w_o L x + \frac{1}{8} w_o L^2$



To draw the Shear Diagram:

(1) $V_{AB} = -w_o x$ for segment AB is linear; at x = 0, V_{AB} = 0; at x = L/2, $V_{AB} = -\frac{1}{2} w_o L$.

(2) At BC, the shear is uniformly distributed by $-\frac{1}{2} w_o L$.

To draw the Moment Diagram:

(1) $M_{AB} = -\frac{1}{2} w_o x^2$ is a second degree curve; at x = 0, M_{AB} = 0; at x = L/2, $M_{AB} = -\frac{1}{8} w_o L^2$.

(2) $M_{BC} = -\frac{1}{2} w_o L x + \frac{1}{8} w_o L^2$ is a second degree; at x = L/2, $M_{BC} = -\frac{1}{8} w_o L^2$; at x = L, $M_{BC} = -\frac{3}{8} w_o L^2$.