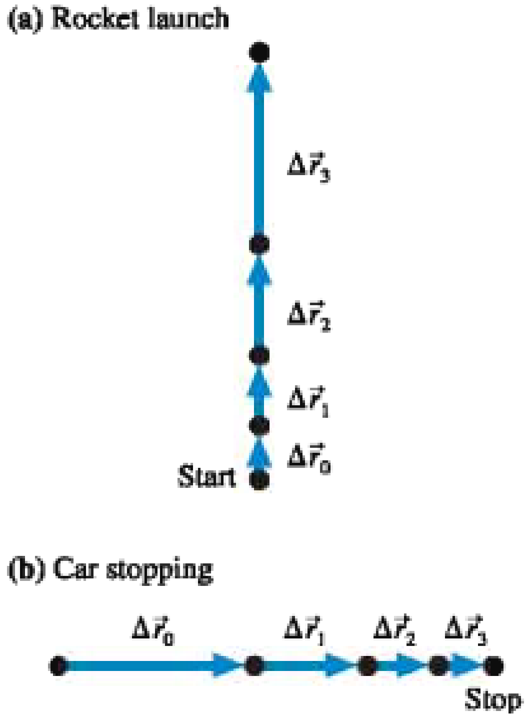
LECTURE 3

Application to Motion Diagrams

The first step in analyzing a motion diagram is to determine all of the displacement vectors. As Figure 1.9 shows, the displacement vectors are simply the arrows connecting each dot to the next. Label each arrow with a *vector* symbol $\Delta\vec{r}_n$, starting with $n = 0$. FIGURE 1.10 shows the motion diagrams of Figure 1.4 redrawn to include the displacement vectors. You do not need to show the position vectors.

- An object is speeding up if its displacement vectors are increasing in length.
- An object is slowing down if its displacement vectors are decreasing in length.

FIGURE 1.10 Motion diagrams with the displacement vectors.



1.4 Velocity

$$\text{average speed} = \frac{\text{distance traveled}}{\text{time interval spent traveling}}$$

If you drive 15 miles (mi) in 30 minutes $\left(\frac{1}{2}\text{ hour}\right)$, your average speed is

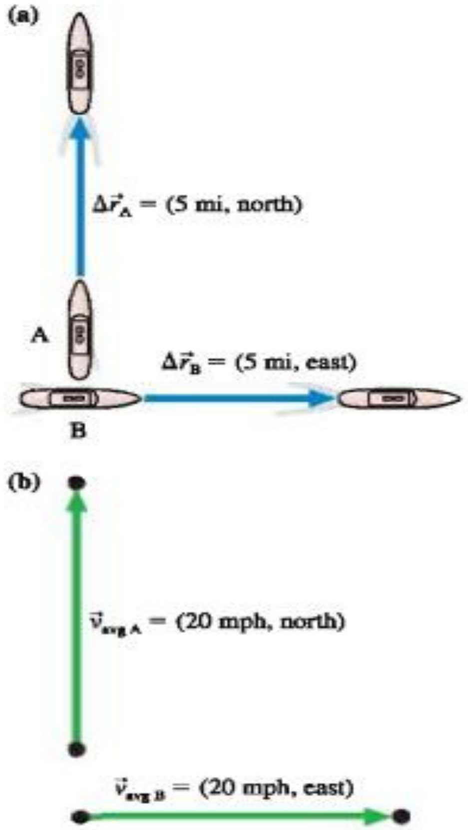
$$\text{average speed} = \frac{15\text{ mi}}{\frac{1}{2}\text{ hour}} = 30\text{ mph}$$

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t}$$

As an example, FIGURE 1.12a shows two ships that start from the same position and move 5 miles in 15 minutes. Both ships have a speed of 20 mph, but their velocities are different. Because their displacements during Δt are $\Delta\vec{r}_{\text{A}} = (5\text{ mi, north})$ and $\Delta\vec{r}_{\text{B}} = (5\text{ mi, east})$, we can write their velocities as

$$\begin{aligned} \vec{v}_{\text{avg A}} &= (20\text{ mph, north}) \\ \vec{v}_{\text{avg B}} &= (20\text{ mph, east}) \end{aligned} \tag{1.6}$$

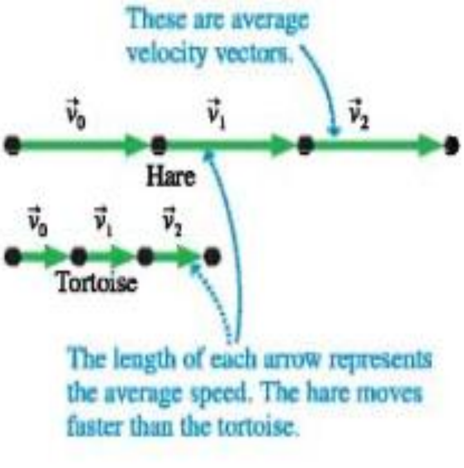
FIGURE 1.12 The displacement vectors and velocities of ships A and B.



Motion Diagrams with Velocity Vectors

The velocity vector, as we've defined it, points in the same direction as the displacement $\Delta\vec{r}$, and the length of $\vec{v}$ is directly proportional to the length of $\Delta\vec{r}$. Consequently, the vectors connecting each dot of a motion diagram to the next, which we previously labeled as displacement vectors, could equally well be identified as velocity vectors.

FIGURE 1.13 Motion diagram of the tortoise racing the hare.



1.5 Linear Acceleration

How can we measure the change of velocity in a meaningful way? When we wanted to measure changes in position, the ratio $\Delta\vec{r}/\Delta t$ was useful. This ratio is the *rate of change of position.* By analogy, consider an object whose velocity changes from $\vec{v}_1$ to $\vec{v}_2$ during the time interval Δt. Just as $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ is the change of position, the quantity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ is the change of velocity. The ratio $\Delta\vec{v}/\Delta t$ is then the *rate of change of velocity.* But what does it measure?

its symbol is $\vec{a}_{avg}$. **The average acceleration of an object during the time interval Δt, in which the object's velocity changes by $\Delta\vec{v}$, is the vector**

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \tag{1.7}$$

Finding the Acceleration Vectors on a Motion Diagram

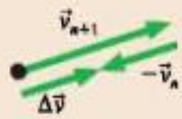
TACTICS BOX 1.3 Finding the acceleration vector MP

To find the acceleration as the velocity changes from $\vec{v}_n$ to $\vec{v}_{n+1}$:

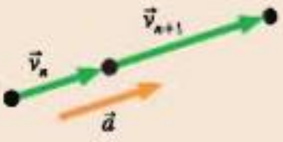
❶ Draw the velocity vector $\vec{v}_{n+1}$.

❷ Draw $-\vec{v}_n$ at the tip of $\vec{v}_{n+1}$.

❸ Draw $\Delta\vec{v} = \vec{v}_{n+1} - \vec{v}_n$
$= \vec{v}_{n+1} + (-\vec{v}_n)$
This is the direction of $\vec{a}$.



❹ Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_n$ and $\vec{v}_{n+1}$.



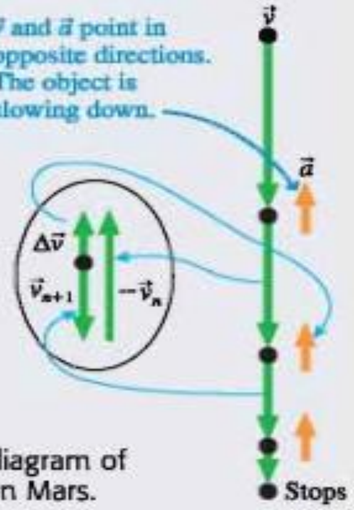


FIGURE 1.16 Motion diagram of a spaceship landing on Mars.