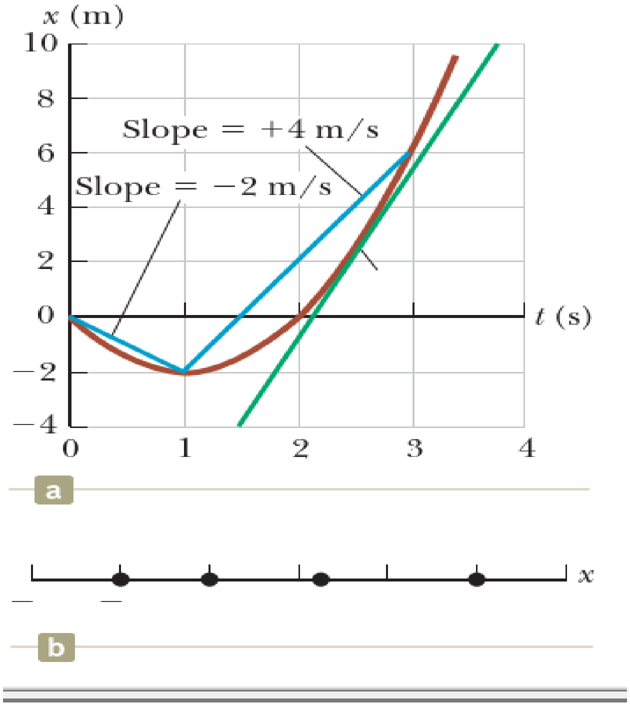
Average and Instantaneous Velocity

Example

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[4] The position–time graph for this motion is shown in Figure 2.4a. Because the position of the particle is given by a mathematical function, the motion of the particle is known at all times, *unlike* that of the car in Figure 2.1, where data is only provided at six instants of time. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

SOLUTION



In the first time interval, set $t_i = t_A = 0$ and $t_f = t_B = 1$ s. Substitute these values into $x = -4t + 2t^2$ and use Equation 2.1 to find the displacement:

$$\Delta x_{A \to B} = x_f - x_i = x_B - x_A$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2] = -2\text{ m}$$

For the second time interval ($t = 1$ s to $t = 3$ s), set $t_i = t_B = 1$ s and $t_f = t_D = 3$ s:

$$\Delta x_{B \to D} = x_f - x_i = x_D - x_B$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2] = +8\text{ m}$$

These displacements can also be read directly from the position–time graph.

(B) Calculate the average velocity during these two time intervals.

SOLUTION

In the first time interval, use Equation 2.2 with $\Delta t = t_f - t_i = t_B - t_A = 1$ s:

$$v_{x,\text{avg}\,(A \to B)} = \frac{\Delta x_{A \to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = -2\text{ m/s}$$

In the second time interval, $\Delta t = 2$ s:

$$v_{x,\text{avg}\,(B \to D)} = \frac{\Delta x_{B \to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = +4\text{ m/s}$$

These values are the same as the slopes of the blue lines joining these points in Figure 2.4a.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

SOLUTION

Calculate the slope of the green line at $t = 2.5$ s (point C) in Figure 2.4a by reading position and time values for the ends of the green line from the graph:

$$v_x = \frac{10\text{ m} - (-4\text{ m})}{3.8\text{ s} - 1.5\text{ s}} = +6\text{ m/s}$$

Notice that this instantaneous velocity is on the same order of magnitude as our previous results, that is, a few meters per second. Is that what you would have expected?

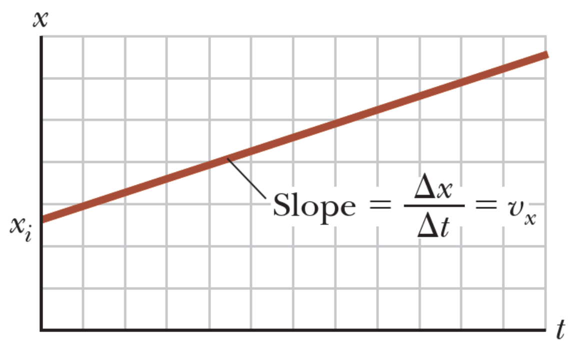
Particle Under Constant Velocity

Imagine a moving object that can be modeled as a particle. If it moves at a constant speed through a displacement Δx in a straight line in a time interval Δt, its constant velocity is

$$v_x = \frac{\Delta x}{\Delta t} \qquad \textbf{(2.6)}$$

The position of the particle as a function of time is given by

$$x_f = x_i + v_x t \qquad \textbf{(2.7)}$$



$$v_x = \frac{\Delta x}{\Delta t} \qquad (2.6)$$

Remembering that $\Delta x = x_f - x_i$, we see that $v_x = (x_f - x_i)/\Delta t$, or

$$x_f = x_i + v_x \Delta t$$

This equation tells us that the position of the particle is given by the sum of its original position x_i at time $t = 0$ plus the displacement $v_x \Delta t$ that occurs during the time interval Δt. In practice, we usually choose the time at the beginning of the interval to be $t_i = 0$ and the time at the end of the interval to be $t_f = t$, so our equation becomes

$$\boxed{x_f = x_i + v_x t \quad \text{(for constant } v_x\text{)}} \qquad (2.7)$$

Graphical Relationships Between x, v_x, and a_x

$$v_x = dx/dt,$$

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

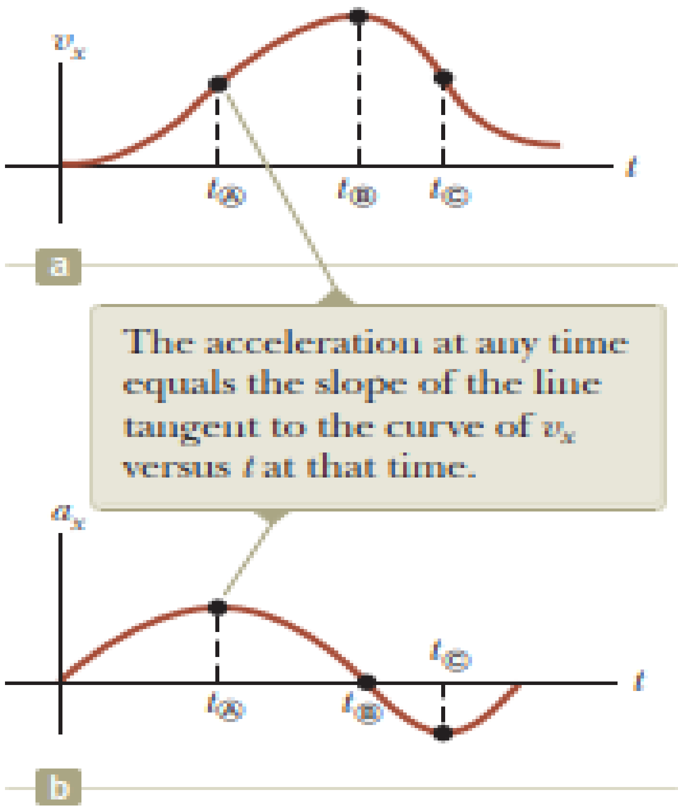
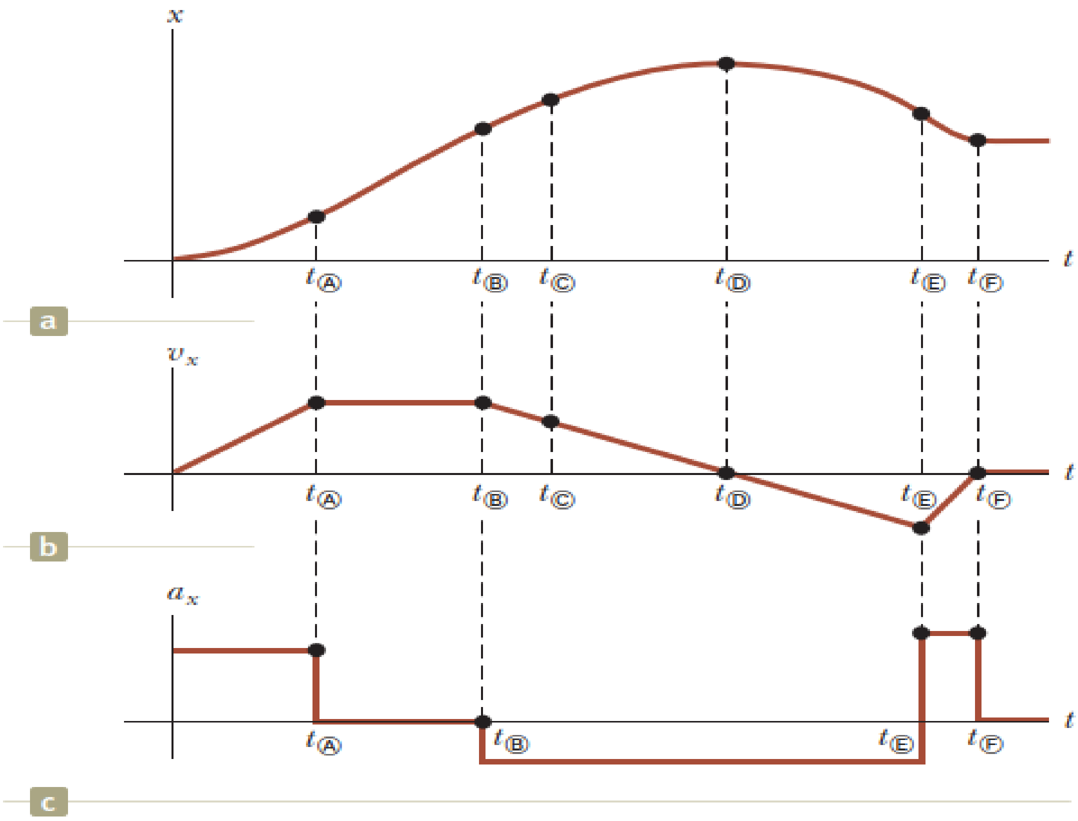
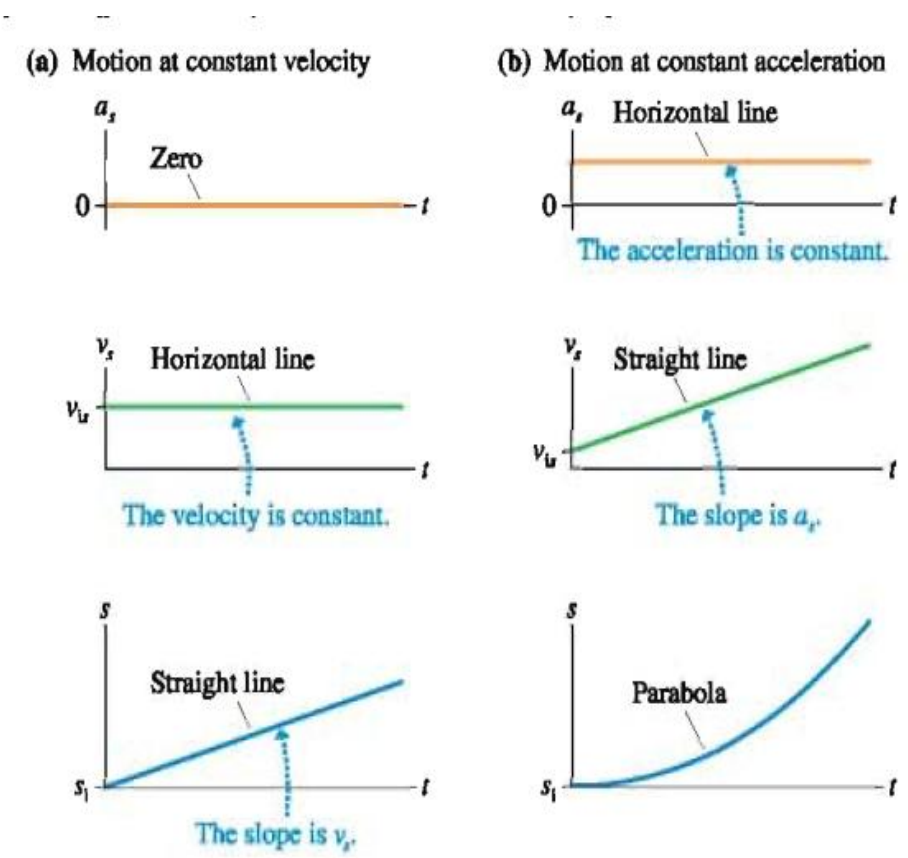


Figure 2.7 (a) The velocity–time graph for a particle moving along the x axis. (b) The instantaneous acceleration can be obtained from the velocity–time graph.


x
t
$t_Ⓐ$
$t_Ⓑ$
$t_Ⓒ$
$t_Ⓓ$
$t_Ⓔ$
$t_Ⓕ$
a
v_x
b
a_x
c

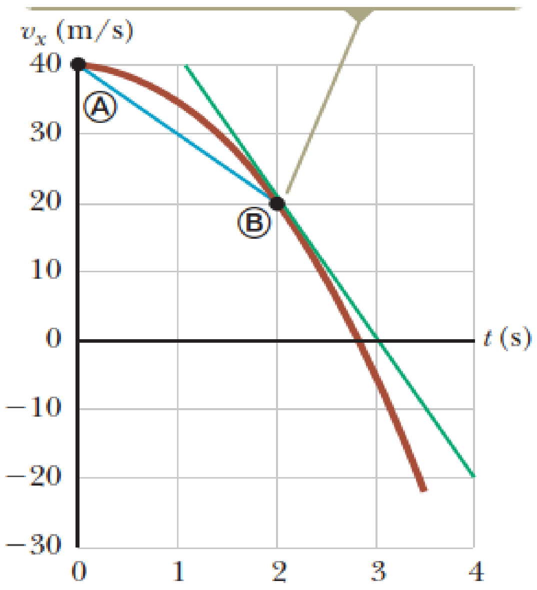


Example

The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

S O L U T I O N



$$v_{x\text{Ⓐ}} = 40 - 5t_{\text{Ⓐ}}^{\,2} = 40 - 5(0)^2 = +40 \text{ m/s}$$

$$v_{x\text{Ⓑ}} = 40 - 5t_{\text{Ⓑ}}^{\,2} = 40 - 5(2.0)^2 = +20 \text{ m/s}$$

$$a_{x,\text{avg}} = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{x\text{Ⓑ}} - v_{x\text{Ⓐ}}}{t_{\text{Ⓑ}} - t_{\text{Ⓐ}}} = \frac{20 \text{ m/s} - 40 \text{ m/s}}{2.0 \text{ s} - 0 \text{ s}}$$

$$= \boxed{-10 \text{ m/s}^2}$$

(B) Determine the acceleration at $t = 2.0$ s.

SOLUTION

Knowing that the initial velocity at any time t is $v_{xi} = 40 - 5t^2$, find the velocity at any later time $t + \Delta t$:

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Find the change in velocity over the time interval Δt:

$$\Delta v_x = v_{xf} - v_{xi} = -10t\,\Delta t - 5(\Delta t)^2$$

To find the acceleration at any time t, divide this expression by Δt and take the limit of the result as Δt approaches zero:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\,\Delta t) = -10t$$

Substitute $t = 2.0$ s:

$$a_x = (-10)(2.0)\ \text{m/s}^2 = \boxed{-20\ \text{m/s}^2}$$

Because the velocity of the particle is positive and the acceleration is negative at this instant, the particle is slowing down.