LECTURE 5

Vectors

3.1 Coordinate Systems

3.2 Vector and Scalar Quantities

3.3 Basic Vector Arithmetic

3.4 Components of a Vector and Unit Vectors

---3.1 Coordinate Systems

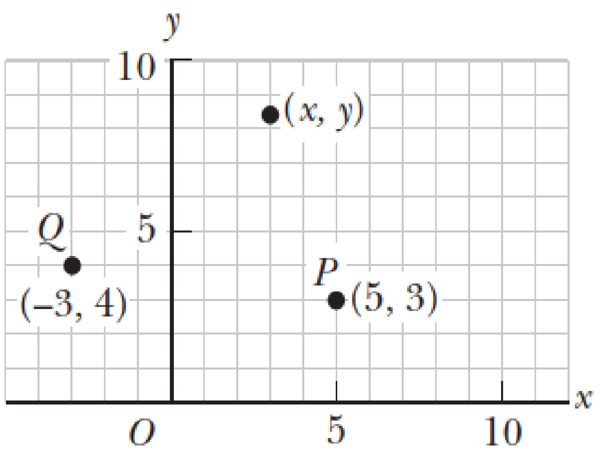


Figure 3.1 Designation of points in a Cartesian coordinate system. Every point is labeled with coordinates (x, y).

$$x = r\cos\theta \tag{3.1}$$

$$y = r\sin\theta \tag{3.2}$$

◀ Cartesian coordinates in terms of polar coordinates

Conversely, if we know the Cartesian coordinates, the definitions of trigonometry tell us that the polar coordinates are given by

$$\tan\theta = \frac{y}{x} \tag{3.3}$$

$$r = \sqrt{x^2 + y^2} \tag{3.4}$$

◀ Polar coordinates in terms of Cartesian coordinates

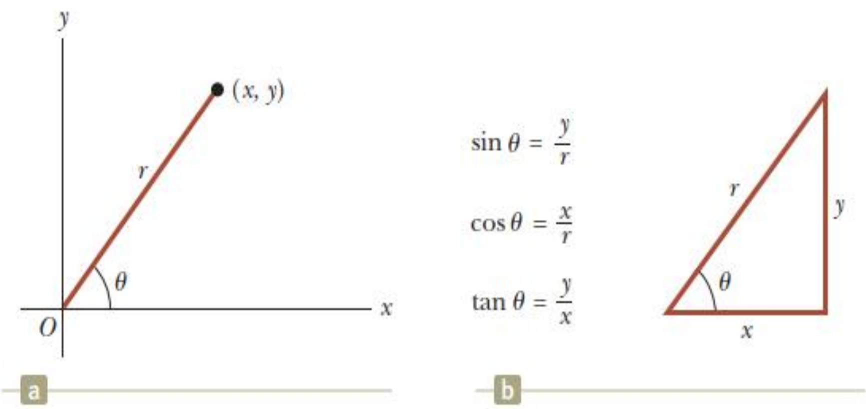


Figure 3.2 (a) The plane polar coordinates of a point are represented by the distance r and the angle θ, where θ is measured counterclockwise from the positive x axis. (b) The right triangle used to relate (x, y) to (r, θ).

Example

The Cartesian coordinates of a point in the xy plane are (x, y) = (-3.50, -2.50) m as shown in Figure 3.3. Find the polar coordinates of this point.

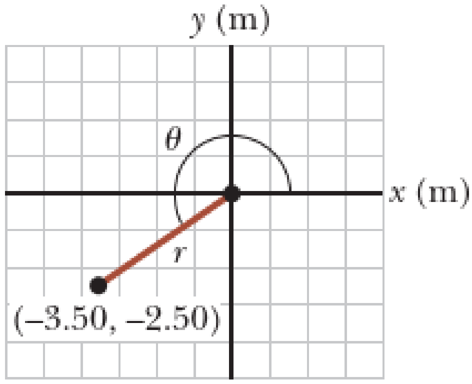


Figure 3.3 (Example 3.1) Finding polar coordinates when Cartesian coordinates are given.

SOLUTION

Conceptualize The drawing in Figure 3.3 helps us conceptualize the problem. We wish to find r and u. Based on the figure and the data given in the problem statement, we expect r to be a few meters and u to be between 180^{o} and 270^{o} .

Use Equation 3.4 to find r:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50\text{ m})^2 + (-2.50\text{ m})^2} = 4.30\text{ m}$$

Use Equation 3.3 to find θ:

$$\tan\theta = \frac{y}{x} = \frac{-2.50\text{ m}}{-3.50\text{ m}} = 0.714$$

$$\theta = 216^\circ$$

3.2 Vector and Scalar Quantities

<u>A scalar quantity</u> is completely specified by a single value with an appropriate unit and has no direction. Temperature is therefore an example of a scalar quantity. Some examples of scalar quantities are temperature, volume, mass, speed, time, and time intervals.

<u>A vector quantity</u> is completely specified by a number with an appropriate unit (the magnitude of the vector) plus a direction. Some examples of vector quantities are displacement, velocity, and acceleration

3.3 Basic Vector Arithmetic

For many purposes, two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ may be defined to be *equal* if they have the same magnitude and if they point in the same direction. That is, $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ only if $A = B$ and if $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ point in the same direction along parallel lines. For example, all the vectors in Figure 3.5 are equal even though they have different starting points. This property allows us to move a vector to a position parallel to itself in a diagram without affecting the vector.

The rules for **vector addition** are conveniently described by a graphical method. To add vector $\vec{\mathbf{B}}$ to vector $\vec{\mathbf{A}}$, first draw vector $\vec{\mathbf{A}}$ on graph paper, with its magnitude represented by a convenient length scale, and then draw vector $\vec{\mathbf{B}}$ to the same scale, with its tail starting from the tip of $\vec{\mathbf{A}}$, as shown in Figure 3.6. The **resultant vector** $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the vector drawn from the tail of $\vec{\mathbf{A}}$ to the tip of $\vec{\mathbf{B}}$.

A geometric construction can also be used to add more than two vectors as shown in Figure 3.7 for the case of three vectors. The resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$ is the vector that completes the polygon. In other words, $\vec{\mathbf{R}}$ is the vector drawn from the tail of the first vector to the tip of the last vector. This technique for adding vectors is often called the "head to tail method."

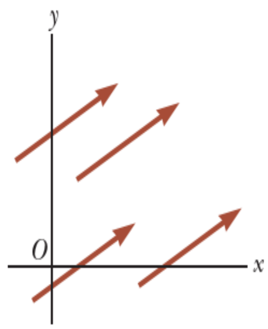


Figure 3.5 These four vectors are equal because they have equal lengths and point in the same direction.

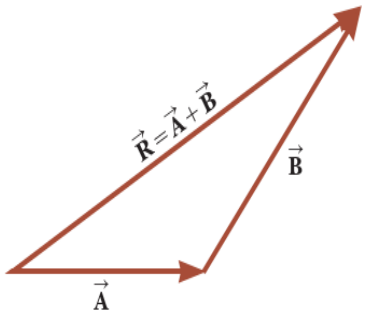


Figure 3.6 When vector $\vec{B}$ is added to vector $\vec{A}$ the resultant $\vec{R}$ is the vector that runs from the tail of $\vec{A}$ to the tip of $\vec{B}$.

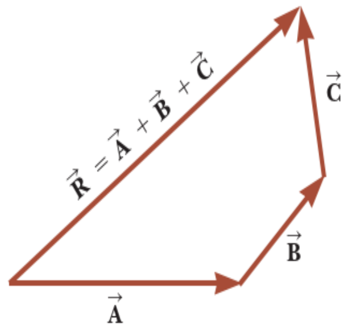
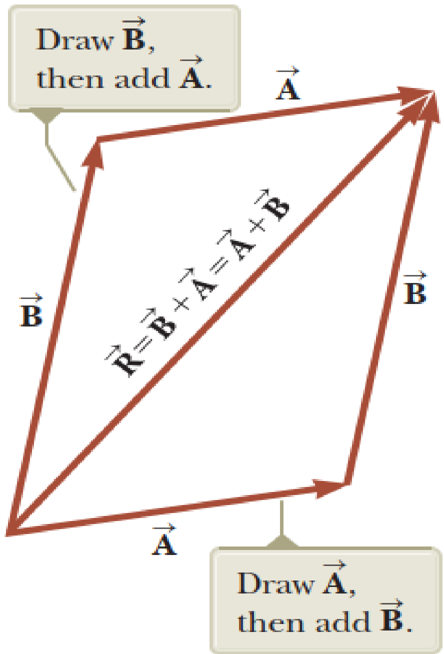


Figure 3.7 Geometric construction for summing three vectors. The resultant vector $\vec{R}$ is by definition the one that completes the polygon.

Vector Addition

a) Commutative law of addition:

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



b) Associative law of addition:

$$\vec{\mathbf{A}} + (\vec{\mathbf{B}} + \vec{\mathbf{C}}) = (\vec{\mathbf{A}} + \vec{\mathbf{B}}) + \vec{\mathbf{C}}$$

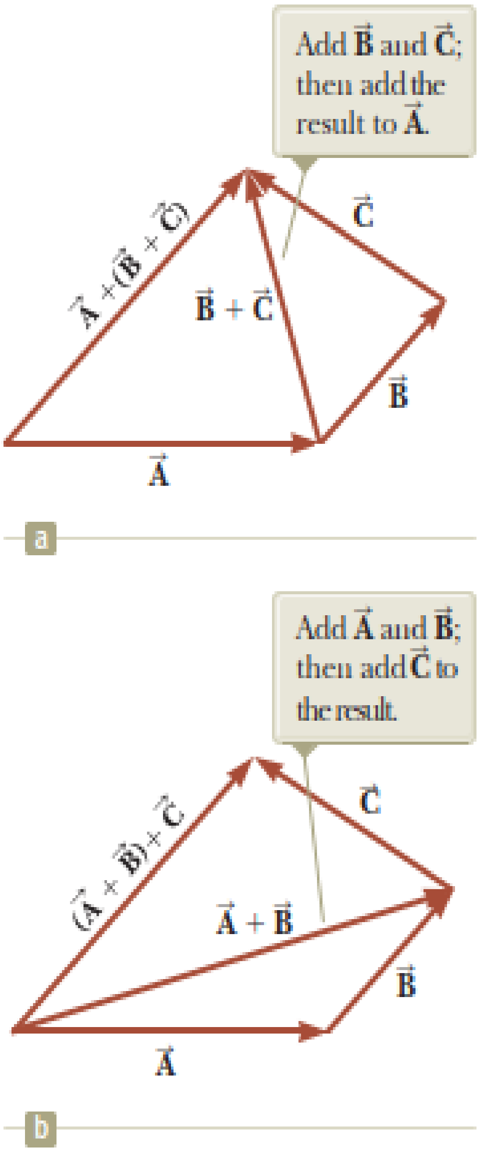


Figure 3.9 Geometric constructions for verifying the associative law of addition. (a) Vectors $\vec{B}$ and $\vec{C}$ are added first and added to $\vec{A}$. (b) Vectors $\vec{A}$ and $\vec{B}$ are added first, and then $\vec{C}$ is added.

c) Scalar multiplication

Scalar multiplication of vectors is straightforward. If vector $\vec{\mathbf{A}}$ is multiplied by a positive scalar quantity m, the product $m\,\vec{\mathbf{A}}$ is a vector that has the same direction as $\vec{\mathbf{A}}$ and magnitude mA. If vector $\vec{\mathbf{A}}$ is multiplied by a negative scalar quantity $-m$,

the product $-m\vec{\mathbf{A}}$ is directed opposite $\vec{\mathbf{A}}$. For example, the vector $5\vec{\mathbf{A}}$ is five times as long as $\vec{\mathbf{A}}$ and points in the same direction as $\vec{\mathbf{A}}$; the vector $-\frac{1}{3}\vec{\mathbf{A}}$ is one-third the length of $\vec{\mathbf{A}}$ and points in the direction opposite $\vec{\mathbf{A}}$.

Vector subtraction:

We define the operation

$\vec{\mathbf{A}} - \vec{\mathbf{B}}$ as vector $-\vec{\mathbf{B}}$ added to vector $\vec{\mathbf{A}}$:

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$$

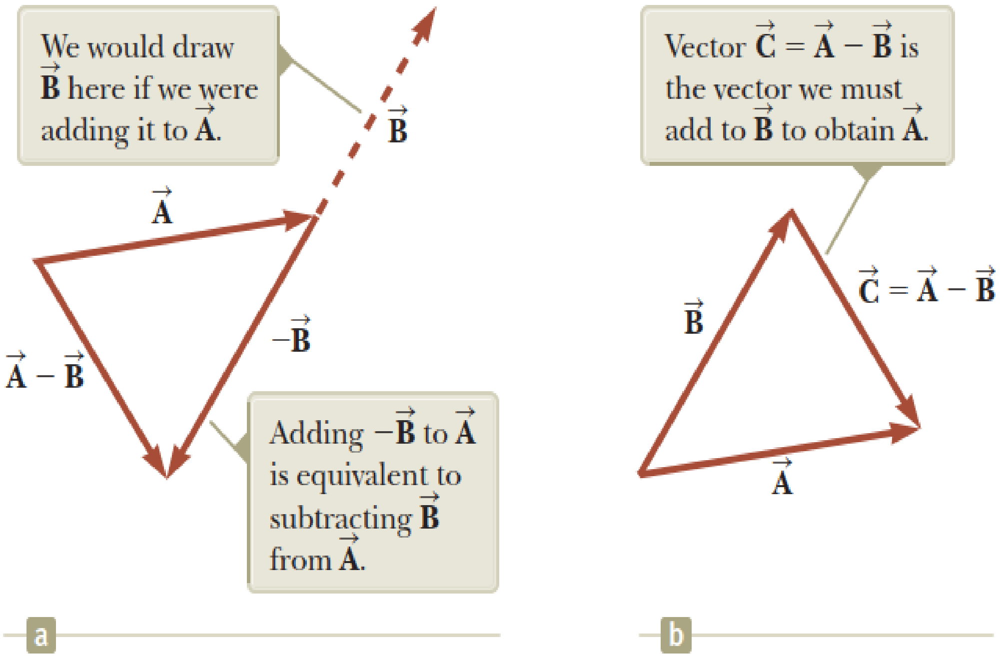


Figure 3.10 (a) Subtracting vector $\vec{\mathbf{B}}$ from vector $\vec{\mathbf{A}}$. The vector $-\vec{\mathbf{B}}$ is equal in magnitude to vector $\vec{\mathbf{B}}$ and points in the opposite direction. (b) A second way of looking at vector subtraction.