Series :-

$\Rightarrow a_1 + a_2 + a_3 \cdots = \sum_{n=1}^{\infty} a_n = S_n$

$\Rightarrow S_n = S_{n-1} + a_n$

Two question on series

Find the sum if exist

↓

2 method

- Geometric series
  - $\sum (r)^n$
- partial sum (telescoping test)
  - $\sum$ اقتران - اقتران

الى هنا مادة السكند

Test for convergent or divergent

↓

" 7 Test "

- divergent test
- p - test
- integral test
- compersion test
- limit compersion
- ratio test
- root test

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* 1 * Divergent Test :- (D.T)

$$\Rightarrow \lim_{n\to\infty} a_n \begin{cases} = 0 \rightarrow \text{test faild} \rightarrow \text{test ع دور غيره} \\ \neq 0 \text{ or D.N.E} \rightarrow \text{"divergent" by D.T} \end{cases}$$

Note :

If a_n is Convergent series then $\lim_{n\to\infty} a_n = 0$

Ex :- Test for Convergence or divergence ?

$\boxed{1}$ $\displaystyle\sum_{n=1}^{\infty} \frac{1+n}{5+3n}$

$$\hookrightarrow \lim_{n\to\infty} \frac{1+n}{5+3n} = \frac{\infty}{\infty} !! \xrightarrow{C.R} = \boxed{\frac{1}{3}} \neq 0$$

"div by D.T"

$\boxed{2}$ $\displaystyle\sum_{n=5}^{\infty} \frac{1}{2+3^{-n}}$

$$\hookrightarrow \lim_{n\to\infty} \frac{1}{2+3^{-n}} = \frac{1}{2+3^{-\infty}} = \frac{1}{2+0} = \boxed{\frac{1}{2}} \neq 0$$

"div by D.T"

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$\boxed{3}$ $\sum_{n=3}^{\infty} \left(1+\frac{-2}{n}\right)^n$

$\hookrightarrow \lim_{n\to\infty} \left(1+\frac{-2}{n}\right)^n = e^{-2} \neq 0$

"div by D.T"

$\boxed{4}$ $\sum_{n=1}^{\infty} n^2 \cdot \sin\left(\frac{1}{n}\right)$

$\hookrightarrow \lim_{n\to\infty} n^2 \cdot \sin\left(\frac{1}{n}\right) \xrightarrow{①} \lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n^2}} = \frac{0}{0}$!! use L.R

$\downarrow$ $\infty \neq 0$

$\downarrow ②$

Sandwesh Theorem

$\hookrightarrow \infty \neq 0$ "div by D.T"

Ex :- Given $\sum_{n=1}^{\infty} a_n = 2$, find $\lim_{n\to\infty} (4a_n)^2 + 5$?

$\downarrow$

So Convergent

a_n Convergent $\Rightarrow \lim_{n\to\infty} a_n = 0$

<u>Now</u> $\lim_{n\to\infty} (4a_n)^2 + 5$

$= 0 + 5 = \boxed{5}$

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* 2 * p-test : (p-series)

$$\sum_{n=1}^{\infty} \frac{1}{n^{(p)}} \begin{cases} p > 1 \rightarrow \text{Convergent by p-test} \\ p \leq 1 \rightarrow \text{divergent by p-test} \end{cases}$$

Ex : Test for Convergence or divergence ?

$\boxed{1}$ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$ $\quad \frac{1}{2} \leq 1 \rightarrow$ div by p-test

$\boxed{2}$ $\sum_{n=1}^{\infty} 3 . n^{-5} = \sum_{n=1}^{\infty} \frac{3}{n^{5}}$, $5 > 1 \rightarrow$ Conv by p-test

$\boxed{3}$ $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}} = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{2}{3}}}$, $\frac{2}{3} \leq 1 \rightarrow$ div by p-test

$\boxed{4}$ $\sum_{n=1}^{\infty} \frac{5 - 2\sqrt{n}}{n^3} = \sum_{n=1}^{\infty} \frac{5}{n^3} - \frac{2\sqrt{n}}{n^3}$ $\quad \sqrt{n} \rightarrow n^{\frac{1}{2}}$

$= \sum_{n=1}^{\infty} \frac{5}{n^3} - \frac{2}{n^{5/2}}$ = Conv − Conv

= Conv by p-test

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* 3 * Integral Test :-

« شروط استخدامه »

① Continuous
متصلة

② positive
موجبة

③ decreasing
متناقصة

$$\Rightarrow \sum_{n=1}^{\infty} a_n = \int_1^{\infty} a_n \, dn = \lim_{a \to \infty} \int_1^{a} a_n \, dn$$

* ويكون سلوك وجواب ال Series نفس سلوك التكامل *

Conv → Conv , div → div

* طيب كيف بدي اعرف انها متناقصة (decreasing) ؟

بسيطة ←
- درجة البسط > درجة المقام
- المشتقة سالبة > 0
- الحدود تتناقص : $a_1 > a_2$

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Ex :- $\sum_{n=1}^{\infty} n \cdot e^{-n}$ D.T ∝ P.T ∝

* Continuous ✓
* positive ✓
* decreasing ✓

} → Use Integral Test

$$\sum_{n=1}^{\infty} n \cdot e^{-n} = \lim_{a \to \infty} \int_1^a n \cdot e^{-n} \, dn$$ "by parts"

$u = n$ $\quad dv = e^{-n} \, dn$

$du = 1 \, dn$ $\quad v = \frac{e^{-n}}{-1}$

$\int -$

$$= n * \frac{-1}{e^n} + \int e^{-n} \, dn$$

$$= \frac{-n}{e^n} + \frac{e^{-n}}{-1} \Big]_1^a$$

$$= \left(\frac{-a}{e^a} - \frac{1}{e^a} \right) - \left(\frac{-1}{e} - \frac{1}{e} \right)$$

$$\lim_{a \to \infty} \frac{-a}{e^a} - \frac{1}{e^a} + \frac{1}{e} + \frac{1}{e}$$

$$= \frac{N}{\infty} - \frac{1}{\infty} + \frac{1}{e} + \frac{1}{e} = \boxed{\frac{2}{e}}$$ So Conv by I. Test

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Ex : $\sum_{n=3}^{\infty} \frac{1}{n.\ln(n)}$ $\sum \frac{عدد}{n.(\ln(n))^{p}}$

شكل مشهور على ال Integral test

* Continuous ✓
* positive ✓
* decreasing ✓

$$\Rightarrow \sum_{n=3}^{\infty} \frac{1}{n.\ln(n)} = \lim_{a\to\infty} \int_{3}^{a} \frac{1}{n.\ln(n)}\,dn$$

" يحل بالتعويض "

$y = \ln(n)$

$dy = \frac{1}{n}\,dn$

$dn = n\,dy$

$$\Rightarrow \int \frac{1}{\cancel{n}.(y)} . \cancel{n}\,dy$$

$$\int \frac{1}{y}\,dy = \ln|y|$$

$$= \ln|\ln(n)| \Big]_{3}^{a}$$

$$= \ln(\ln(a)) - \ln(\ln(3))$$

$$\lim_{a\to\infty} \ln(\ln(a)) - \ln(\ln(3))$$

$$= \ln(\ln(\infty)) - \ln(\ln(3))$$

$$= \infty - \ln(\ln(3)) = \infty$$ div by I. Test

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