University Of Anbar

College Of Engineering

Electrical Engineering Department

MATHEMATICS-1

1st class students

Dr. Hamid Raddam Hussein

References

1. Stewart, J., Clegg, D. K., & Watson, S. (2020). Calculus: early transcendental. Cengage Learning.
2. Thomas, G. B., Haas, J., Heil, C., & Weir, M. (2018). Thomas' Calculus. Pearson Education Limited.
3. Stroud, K. A., & Booth, D. J. (2020). Engineering mathematics. Bloomsbury Publishing.

CHAPTER 1

Functions 1 الدوال

1.1 Sets of Numbers and Inequalities.

- Sets on Numbers.
- Intervals.
- Properties of Inequalities.
- Linear Inequalities.
- Properties of Absolute Value.
- Solve Inequalities; Quadratic, Numerator and the Denominator.

1.2 Functions: Basic Definitions and Examples.

- Definition of Function.
- The Vertical Line test of a Function.
- Some Types of Functions.
- Domain and Range of a Function.
- Representation of Functions.

1.3 Properties of Functions and Their Combination.

- Symmetry.
- Even and Odd Functions.
- Increasing and Decreasing Functions.
- Basic Operations on Functions.
- Composition of Functions.

1.4 Inverse Functions

- One-to-One Functions.
- Inverse Functions.

1.5 Trigonometric Functions

- Degree/Radians Conversion Factors.
- Trigonometric Functions.
- Trigonometric Functions using the Unit Circle.
- Values of Sine and Cosine.
- Trigonometric Identities.
- Graphs of the Trigonometric Functions.

1.6 Inverse Trigonometric Functions

- Inverse of Sine and Cosine Functions.
- Inverse of Tangent and Cotangent Functions.
- Inverse of Secant and Cosecant Functions.

CHAPTER 1

Functions 1 الدوال

1.1 Sets of Numbers and Inequalities

مجموعات الأعداد و المتباينات

SETS OF NUMBERS مجموعات الأعداد

مجموعة الأعداد الطبيعية - natural numbers set

N={ 1 , 2 , 3 , ... }

مجموعة الأعداد الكلية - Whole numbers set

W={ 0 , 1 , 2 , 3 , ... }

مجموعة الأعداد الصحيحة - integers numbers set

Z={ ... ,-3 ,-2 ,-1 ,0 ,1, 2 , 3 , ... }

$N \subset W \subset Z$ رمز مجموعة جزئية $\subset$

مجموعة الأعداد النسبية - rational numbers set

$Q = \{ \frac{p}{q} \mid p,q \in Z ,\ q \neq 0 \}$

$\frac{2}{3}, \frac{5}{7}, \frac{9}{8}, \frac{-6}{7}$

$5 \in Q$? $\frac{5}{1} = 5$ T

$\frac{2}{5} = 0.4$

$\frac{1}{3} = 0.333333... = 0.\overline{3}$

$\frac{2}{7} = 0.285714285714285714285714 \ldots$

تذكير

$\sqrt{9} = 3$ لأن $3 \times 3 = 9$, $\sqrt{9} = \pm 3$? ***F***

$\sqrt{16} = 4$

$\sqrt{25} = 5$

$\sqrt{36} = 6$

$\sqrt{49} = 7$

$\sqrt{64} = 8$

$\sqrt{81} = 9$

مجموعة الأعداد غير النسبية - irrational numbers set

يرمز لها بالرمز - *I*

$\sqrt{2} = 1.414212356237...$ $\sqrt{6} =$ ٢,٤٤٩٤٨٩٧٤٢٧٨٣١٧٨٥٥٥

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$

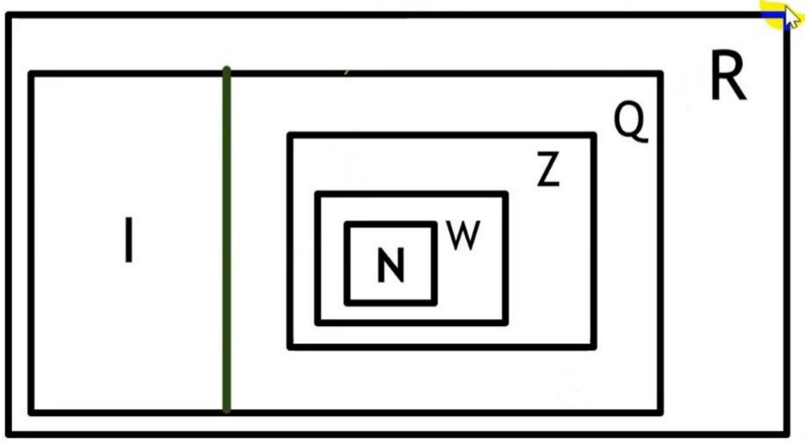
$\Pi = 3.14159...$

$\sqrt{-2}$ ليس نسبية أو غير نسبية

مجموعة الأعداد الحقيقية - real numbers set

$R = Q \cup I$

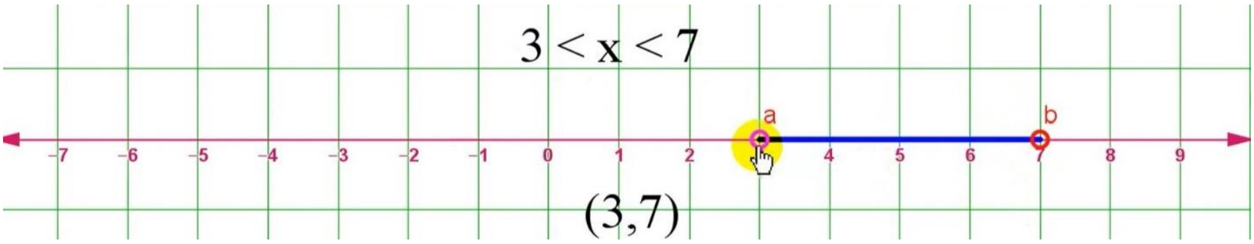
$N \subset W \subset Z \subset Q \subset R$



Intervals - الفترات

open interval الفترة المفتوحة

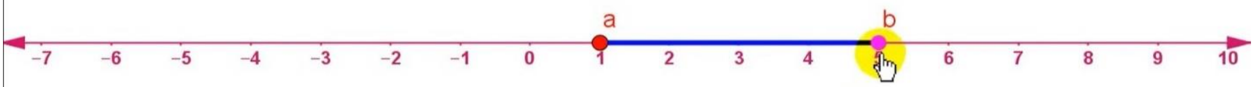
1. The open interval $(a,b)=\{x \mid a<x<b\}$



Closed interval الفترة المغلقة

2. The closed interval $[a,b]=\{x \mid a\le x\le b\}$

$1 \le x \le 5$



[1,5]

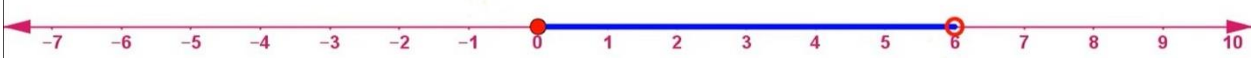
$3 \le x \le 8$

[3,8]

half-open interval نصف فترة مفتوحة

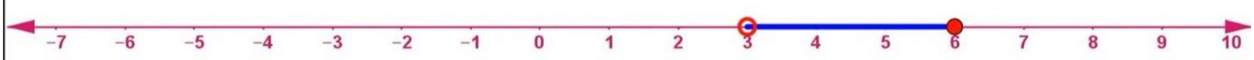
3. The half-open interval $(a,b]=\{x \mid a<x\le b\}$ or $[a,b)=\{x \mid a\le x<b\}$

$0 \le x < 6$



[0,6)

$3 < x \leq 6$



$(3,6]$

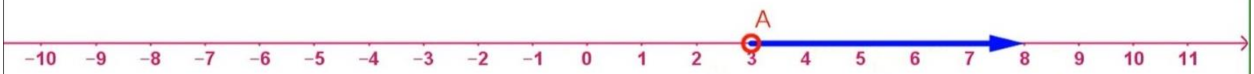
infinite-open interval فترة مفتوحة لانهائية

4. The infinite-open interval $(a, \infty) = \{x \mid x > a\}$ or $(-\infty, b) = \{x \mid x < b\}$

$x < 3$

$(-\infty, 3)$

$3 < x$

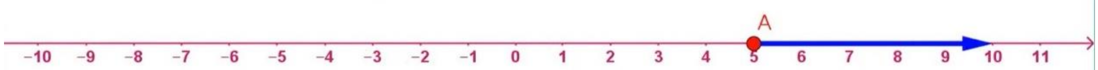


$(3, \infty)$

infinite-closed interval فترة مغلقة لانهائية

5. The infinite-closed interval $[a, \infty) = \{x \mid x \geq a\}$ or $(-\infty, b] = \{x \mid x \leq b\}$

$5 \leq x$



$[5, \infty)$

EXAMPLE: Solve the following inequalitie حل المتباينات التالية

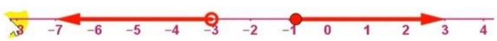
$2 < x < 5$ solution: $(2, 5)$

$7 \geq x > 4$ solution: $(4, 7]$

$x < -3$ solution: $(-\infty, -3)$

$x \geq -1$ solution: $[-1, \infty)$

$x < -3$ or $x \geq -1$ solution: $(-\infty, -3) \cup [-1, \infty)$



Properties of Inequalities خصائص المتباينات

If a, b, c in $\mathbb{R}$, then

1. If $a < b$ then $a + c < b + c$ and $a - c < b - c$

$2 < 5$, $2+4 < 5+4$, $6 < 9$

$2 < 5$, $2-4 < 5-4$, $-2 < 1$

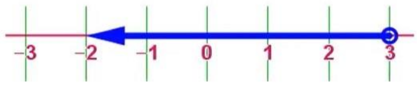
Solve the following inequalities حل المتباينات التالية

$x+2 < 5$

$x < 5-2$

$x < 3$

solution: $(-\infty , 3)$



2. If $a < b$ and $b < c$ then $a < c$

$3 < 7$ and $7 < 9$ then $3 < 9$

3. If $a < b$ and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

$2 < 5$, $2 \cdot 3 < 5 \cdot 3$, $6 < 15$

$6 < 9$, $\frac{6}{3} < \frac{9}{3}$, $2 < 3$

4. If $a < b$ and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

$4 < 8$, $4(-2) > 8(-2)$, $-8 > -16$

$4 < 8$, $\frac{4}{-2} > \frac{8}{-2}$, $-2 > -4$

LINEAR INEQUALITIES المتباينات الخطية

EXAMPLE 1.1.1: Solve the following inequalities: حل المتباينات التالية

$$6 - 2x \geq 3x - 4$$

$$6 + 4 \geq 3x + 2x$$

$$10 \geq 5x$$

$$2 \geq x$$

Solution : $(-\infty, 2]$

EXAMPLE 1.1.1: Solve the following inequalities: حل المتباينات التالية

$$4(x - 3) > -2x - 16$$

$$4x - 12 > -2x - 16$$

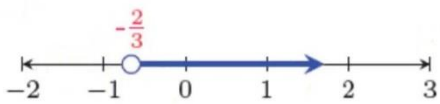
$$4x + 2x > 12 - 16$$

$$6x > -4$$

$$x > \frac{-4}{6}$$

$$x > \frac{-2}{3}$$

Solution : $(-\frac{2}{3}, \infty)$



EXAMPLE 1.1.1: Solve the following inequalities: حل المتباينات التالية

$$-3 < 5 - 2x \le 7$$

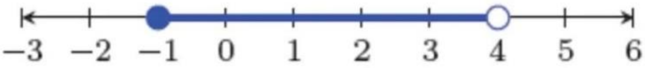
$$-3 - 5 < -2x \le 7 - 5$$

$$-8 < -2x \le 2$$

$$\frac{-8}{-2} > x \ge \frac{2}{-2}$$

$$4 > x \ge -1$$

Solution : $[-1, 4)$



RELATED PROBLEM 1 أسئلة لها صلة بالسابق

Solve the following inequalities: حل المتباينات التالية

a. $4 - x < 3x + 2$

b. $3(2x - 1) > 4x - 11$

c. $1 \le 5 + 2x < 4$

Answers

a. $\left(\frac{1}{2}, \infty\right)$

b. $(-4, \infty)$

c. $[-2, -\frac{1}{2})$

ABSOLUTE VALUES القيمة المطلقة

$$|\ 5\ | = 5$$

$$|-5| = -(-5)$$

$$|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a & \text{if } a < 0 \end{cases}$$

EXAMPLE 1.1.2: Rewrite each expression without absolute value:

أعد كتابة كل تعبير بدون قيمة مطلقة

$$|\sqrt{2}-1| = |\sqrt{2}-\sqrt{1}| = \sqrt{2}-1$$

$$|3-\Pi| = -(3-\Pi) = \Pi - 3$$

$$|3x-1| = \begin{cases} 3x-1, & \text{if } 3x-1 \geq 0 \\ 1-3x, & \text{if } 3x-1 < 0 \end{cases} = \begin{cases} 3x-1, & \text{if } x \geq \frac{1}{3} \\ 1-3x, & \text{if } x < \frac{1}{3} \end{cases}$$

تذكير

$\sqrt{2} > 1$? $\sqrt{2} > \sqrt{1}$ T

$\sqrt{5} > 3$? $\sqrt{5} > \sqrt{9}$ F

EXAMPLE 1.1.2: Rewrite each expression without absolute value:

أعد كتابة كل تعبير بدون قيمة مطلقة

$$|\sqrt{2}-1| = |\sqrt{2}-\sqrt{1}| = \sqrt{2}-1$$

$$|3-\Pi| = -(3-\Pi) = \Pi - 3$$

$$|3x-1| = \begin{cases} 3x-1, & \text{if } 3x-1 \geq 0 \\ 1-3x, & \text{if } 3x-1 < 0 \end{cases} = \begin{cases} 3x-1, & \text{if } x \geq \frac{1}{3} \\ 1-3x, & \text{if } x < \frac{1}{3} \end{cases}$$

Properties of Absolute Value خصائص القيمة المطلقة

For all real numbers *a* and *b*

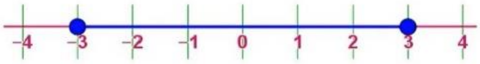
1. $|a| \geq 0$.
2. $|ab| = |a||b|$
3. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \quad b \neq 0$
4. $|a+b| \leq |a| + |b|$ (Triangle Inequality) متباينة المثلث
5. $\sqrt{a^2} = |a|$, $\sqrt{(-3)^2} = |-3| = 3$

For all real numbers *a* and *b*

6. If $a > 0$, then $|x| < a$ if and only if $-a < x < a$.

EXAMPLE $|x| \leq 3$ $-3 \leq x \leq 3$

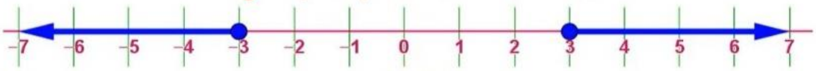
Solution : $[-3, 3]$



7. If $a > 0$, then $|x| > a$ if and only if $x > a$ or $x < -a$

EXAMPLE $|x| \geq 3$ $x \geq 3$ or $x \leq -3$

Solution : $[3, \infty) \cup (-\infty, -3]$

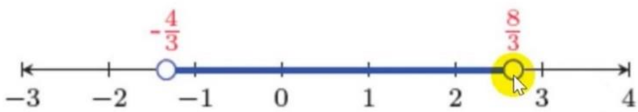


8. $|f(x)| < |g(x)|$ if and only if $f^2(x) \leq g^2(x)$.

EXAMPLE 1.1.3: Solve the following inequalities:

$$|3x - 2| < 6$$
$$-6 < 3x - 2 < 6$$
$$-4 < 3x < 8$$
$$\frac{-4}{3} < x < \frac{8}{3}$$

Solution : $\left(\frac{-4}{3}, \frac{8}{3}\right)$



EXAMPLE 1.1.3: Solve the following inequalities:

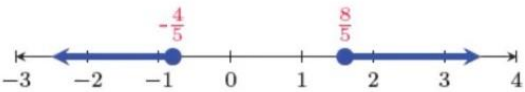
$$| 2 - 5x | \geq 6$$

$2 - 5x \geq 6$ or $-6 \geq 2 - 5x$

$-5x \geq 4$ or $-8 \geq -5x$

$x \leq \frac{-4}{5}$ or $\frac{8}{5} \leq x$

Solution : $(-\infty, \frac{-4}{5}] \cup [\frac{8}{5}, \infty)$



EXAMPLE 1.1.3: Solve the following inequalities:

$|4x+3| \leq -2$ Solution : ϕ

RELATED PROBLEM 2 أسئلة لها صلة بالسابق

Solve the following inequalities:

a. $|2x+3| < 2$ b. $|4-2x| \geq 8$ c. $|5x-8| \geq -1$

Answers

a. $\left(-\frac{5}{2}, -\frac{1}{2}\right)$ b. $(-\infty, -2] \cup [6, \infty)$ c. $\mathbb{R}$

Solve inequalities - حل المتباينات
Quadratic - التربيعية
numerator and the denominator - البسط و المقام

تذكير

تحليل مقدار من الدرجة الثانية

$$(x-2)(x-3) = x^2 - 3x - 2x + 6 = x^2 - 5x + 6$$

تحقق من صحة المساواة

$$x^2 + 4x - 12 = (x+3)(x-4) \qquad F$$

$$x^2 + 4x - 12 = (x+6)(x-2)$$

تذكير

مفكوك المربع الكامل

$(a+b)^2 = a^2 + 2ab + b^2$

$(x-3)^2 = x^2 - 2.3x + 3^2 = x^2 - 6x + 9$

تذكير

الفرق بين مربعين

$a^2 - b^2 = (a-b)(a+b)$

$x^2 - 4 = (x-2)(x+2)$

EXAMPLE 1.1.4: Solve the following inequalities:

$$x^2 - 5x < -6$$

$$x^2 - 5x + 6 < 0$$

$$(x-2)(x-3) < 0$$

	$(-\infty, 2)$	$(2, 3)$	$(3, \infty)$
$(x-2)$	−	+	+
$(x-3)$	−	−	+
$(x-2)(x-3)$	+	−	+

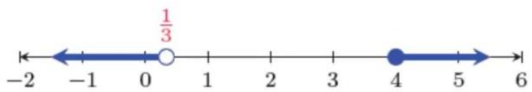
Solution : $(2, 3)$

EXAMPLE 1.1.4: Solve the following inequalities:

$$\frac{4-x}{3x-1} \leq 0$$

	$(-\infty, \frac{1}{3})$	$(\frac{1}{3}, 4)$	$(4, \infty)$
$4-x$	+	+	−
$3x-1$	−	+	+
$\frac{4-x}{3x-1}$	−	+	−

Solution : $(-\infty, \frac{1}{3}) \cup [4, \infty)$



EXAMPLE 1.1.4: Solve the following inequalities:

$$\frac{2}{1-3x} - 1 \geq 0$$

$$\frac{2}{1-3x} - \frac{1-3x}{1-3x} \geq 0$$

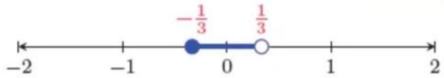
$$\frac{2-(1-3x)}{1-3x} \geq 0$$

$$\frac{2-1+3x}{1-3x} \geq 0$$

$$\frac{1+3x}{1-3x} \geq 0$$

	$(-\infty, -\frac{1}{3})$	$(-\frac{1}{3}, \frac{1}{3})$	$(\frac{1}{3}, \infty)$
$1+3x$	−	+	+
$1-3x$	+	+	−
$\frac{1+3x}{1-3x}$	−	+	−

Solution : $[-\frac{1}{3}, \frac{1}{3})$



$|x+3| \leq |2x-1|$

8. $|f(x)| < |g(x)|$ if and only if $f^2(x) \leq g^2(x)$

$$|x+3| \leq |2x-1|$$

$$(x+3)^2 \leq (2x-1)^2$$

$$(x+3)^2 - (2x-1)^2 \leq 0$$

$$x^2+6x+9-(4x^2-4x+1) \leq 0$$

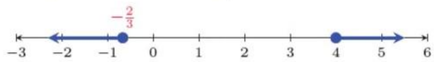
$$x^2+6x+9-4x^2+4x-1 \leq 0$$

$$-3x^2+10x+8 \leq 0$$

$$(-x+4)(3x+2) \leq 0$$

	$(-\infty, -\frac{2}{3})$	$(-\frac{2}{3}, 4)$	$(4, \infty)$
$-x+4$	+	+	−
$3x+2$	−	+	+
$(-x+4)(3x+2)$	−	+	−

Solution : $(-\infty, -\frac{2}{3}] \cup [4, \infty)$



$$\frac{3}{x-9} \geq \frac{2}{x+2}$$

$$\frac{3}{x-9} - \frac{2}{x+2} \geq 0$$

$$\frac{3(x+2)-2(x-9)}{(x-9)(x+2)} \geq 0$$

$$\frac{3x+6-2x+18}{(x-9)(x+2)} \geq 0$$

$$\frac{x+24}{(x-9)(x+2)} \geq 0$$

	$(-\infty\ -24)$	$(-24, -2)$	$(-2, 9)$	$(9, \infty)$
$x+24$	−	+	+	+
$x-9$	−	−	−	+
$x+2$	−	−	+	+
$\frac{x+24}{(x-9)(x+2)}$	−	+	−	+

السبورة
-24 -2 9

Solution : $[-24, -2) \cup (9, \infty)$

EXAMPLE 1.1.4: Solve the following inequalities:

f. $\dfrac{1}{x^2+1} \geq 0$ Solution : R

RELATED PROBLEM 3

Solve the following inequalities:

a. $x^2 < 2x + 3$

b. $\dfrac{2x-3}{x-2} \geq 0$

c. $|3x+4| < |x-2|$

d. $\dfrac{2}{x-3} \geq \dfrac{1}{x+1}$

e. $x^2 + 4 \leq 0$

Answers

a. $(-1, 3)$

b. $(-\infty, \frac{3}{2}] \cup (2, \infty)$

c. $\left(-3, -\frac{1}{2}\right)$

d. $[-5, -1) \cup (3, \infty)$

e. ϕ

EXERCISES 1.1

In Exercises 1 – 8, solve the inequalities

1. $3x - 4 < 8$
2. $1 - 2x < -4$
3. $3x - 4 < -4$
4. $4 - 5x < 2x - 7$
5. $3(x-4) - 2 \geq 2(x-7)$.
6. $-3 < 2x - 4 \leq 7$
7. $-2 < \dfrac{2x-7}{3} \leq 4$
8. $3(2x-4) - 2(x-7) \geq 7 + 3(x-5)$.

In Exercises 9 – 15, solve the inequalities

9. $|2x+4| \leq 3$
10. $\left|\dfrac{2x+5}{3}\right| \leq 4$
11. $|3x-2| > 5$
12. $|2x+4| + 4 \leq 3$
13. $-2|5x+2| + 4 \leq 3$
14. $|x| \leq |x-5|$
15. $|3x-2| > |2x-5|$.

In Exercises 16 – 23, solve the inequalities

16. $x(x-4) < 0$
17. $x^2 - 3x < 4$
18. $x^2 < x$
19. $x^2 - 4x + 4 \geq 0$
20. $\dfrac{2}{x-3} \leq 0$
21. $\dfrac{2x-4}{x+3} \leq 0$
22. $\dfrac{1}{x+3} \leq 4$

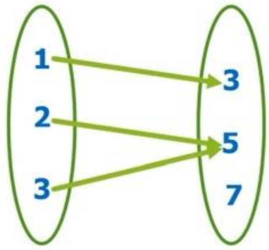
SECTION 1.2 FUNCTIONS: BASIC DEFINITIONS AND EXAMPLES

Definition of Function - تعريف الدالة

D_f domain المجال

C_f codomain المجال المقابل

R_f range المدى



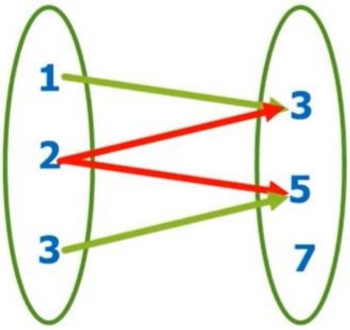
domain المجال A

B المجال المقابل codomain

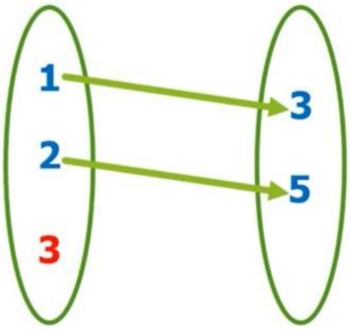
f: A ⟶ B

range المدى ={ 3 , 5 }

Not a Function ليست الدالة



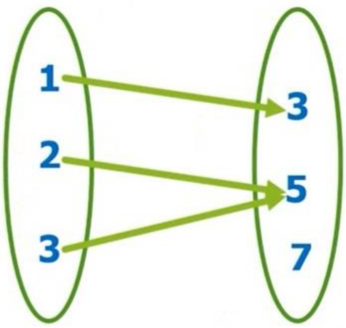
A B



A B

كتابة الدالة كمجموعة من الأزواج المرتبة

f={ (1,3) , (2,5) , (3,5) }



A B

f: A ⟶ B

Function - الدالة

EXAMPLE 1.2.1 Determine which of the following sets is a function. If it is a function, what is its domain and range?

حدد المجموعة التي تمثل دالة ، إذا كانت المجموعة تمثل دالة حدد المجال و المدى

a. $f = \{(1,2),(3,4),(-1,5),(2,0),(0,0)\}$

Solution المجال $D_f = \{1,3,-1,2,0\}$ المدى $R_f = \{2,4,5,0\}$.

b. $g = \{(5,-3),(1,4),(-5,2),(1,0),(0,0)\}$

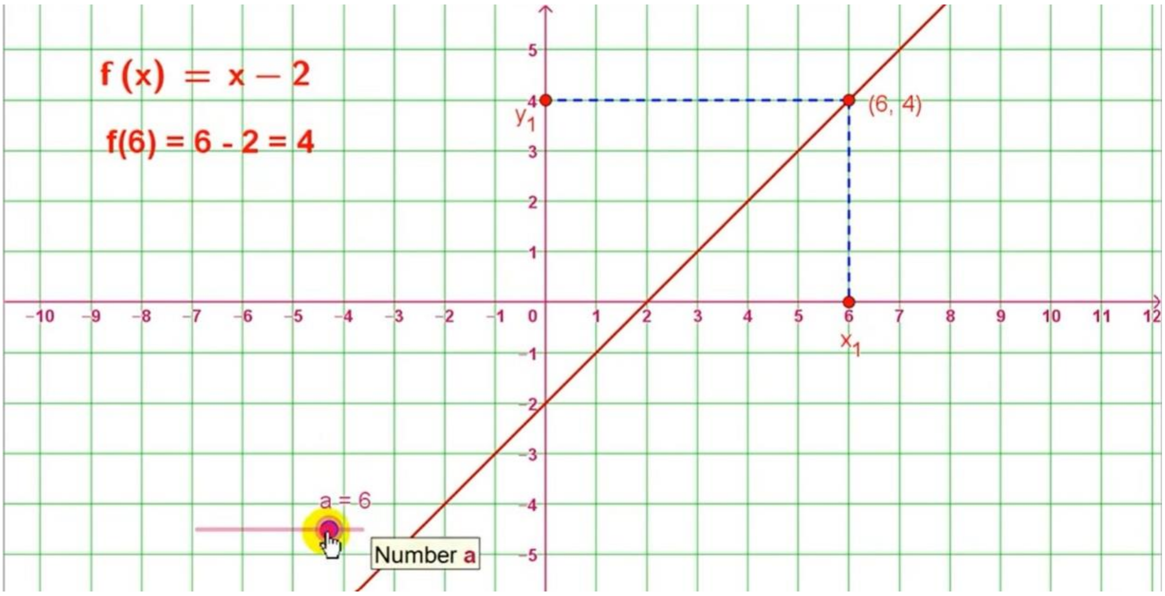
Solution g is not a function $(1,4)$ and $(1,0)$ have the same x − coordinate.

لهما الاحداثي x نفسة

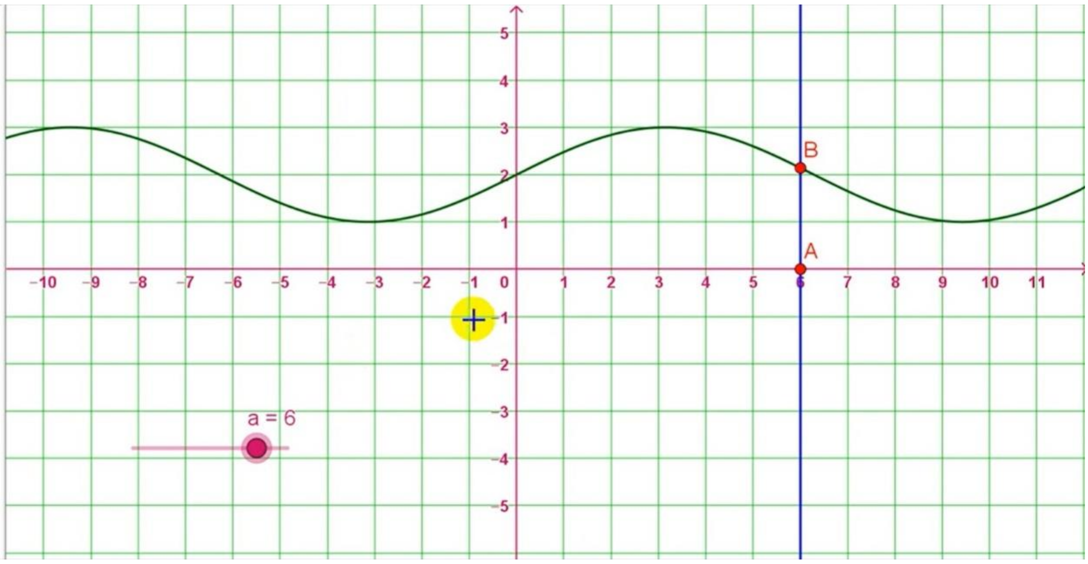
$(1,4)\ ,\ (1,0) \in g$

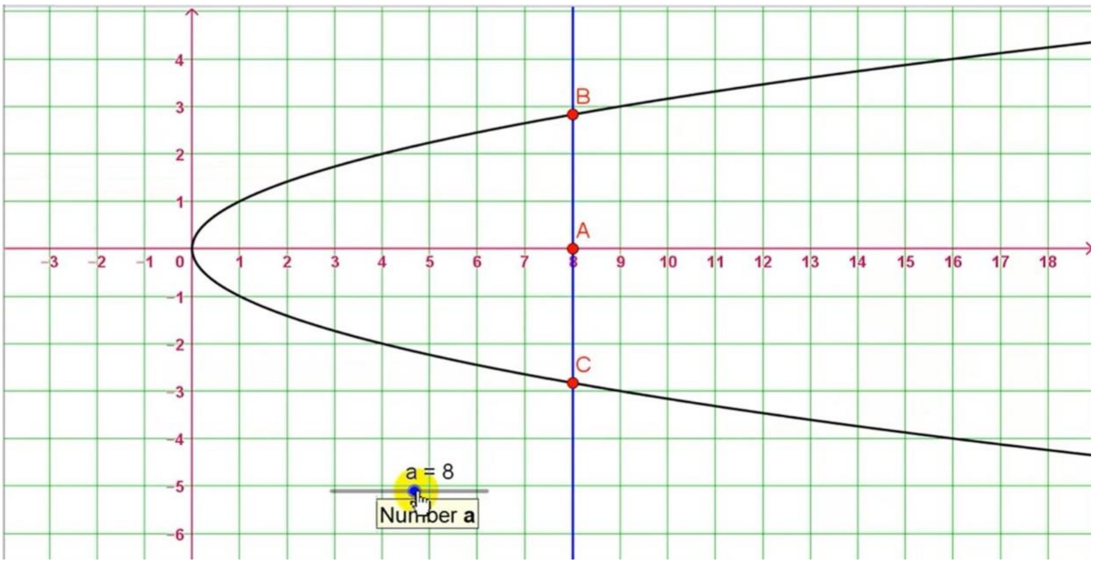
y= x -2

f(x)= x -2



THE VERTICAL LINE TEST OF A FUNCTION اختبار الخط العمودي لدالة





SOME TYPES OF FUNCTIONS بعض أنواع الدوال

POLYNOMIALS كثيرات الحدود

f(x)= 3x⁵ + 4x⁴- 2x³+ 3x²+ 8x- 4 كثيرة حدود من الدرجة (***degree***) الخامسة

3 , 4 , -2 , 3 , 8 , -4 (coefficients of the polynomial) معاملات كثيرة الحدود

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

n (*degree*) من الدرجة (polynomial) كثيرة حدود

$a_0, a_1, a_2, \cdots, a_n$ **(coefficients of the polynomial)** معاملات كثيرة الحدود

$a_n \neq 0$, $n \in N$

$p(x) = ax^2 + bx + c$ **quadratic functions** **دالة تربيعية**

$p(x) = ax^3 + bx^2 + cx + d$ ***cubic functions*** **دالة تكعيبية**

example

$$f(x) = \sqrt{2}x^3 + x^5 - \frac{5}{2}x^2 + 1 \qquad \text{degree } 5$$

$$g(x) = 5x^{-2} - x^3 + x^7$$

is not a polynomial because $x^{-2} = \frac{1}{x^2}$

RATIONAL FUNCTIONS **الدوال النسبية**

$$f(x) = \frac{p(x)}{q(x)} \qquad q(x) \neq 0$$

example

$$f(x) = \frac{-3x^3 + 1}{x^2 - x} \qquad \text{rational function}$$

$$f(x) = \frac{x^2 + 1}{\sqrt{x} - x} \qquad \text{not a rational function}$$

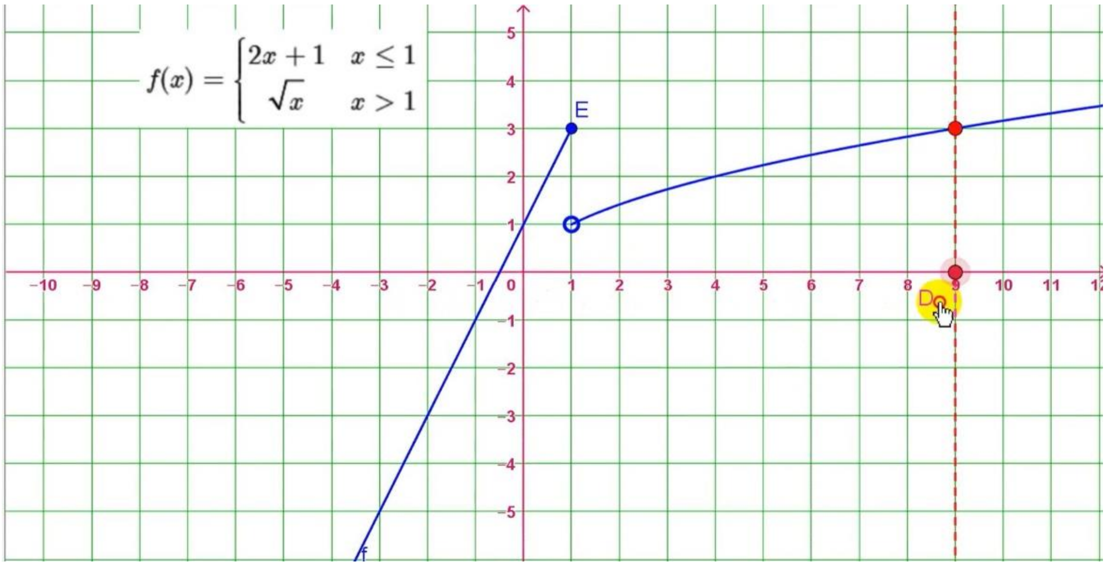
RADICAL FUNCTIONS **الدوال الجذرية**

$f(x) = \sqrt{x^3 - 4}$ $\qquad x^3 - 4 \geq 0$ $\qquad \sqrt{-9} = ?$

$g(x) = \sqrt[3]{x^2 + 2}$ $\qquad \sqrt[3]{27} = 3$ $\qquad \sqrt[3]{-27} = -3$

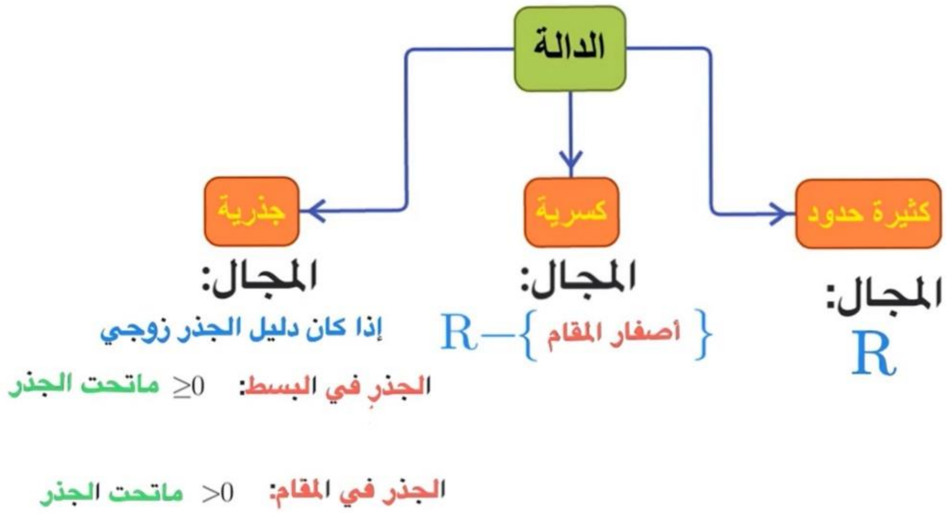
PIECEWISE FUNCTIONS **دالة متعددة التعريف**

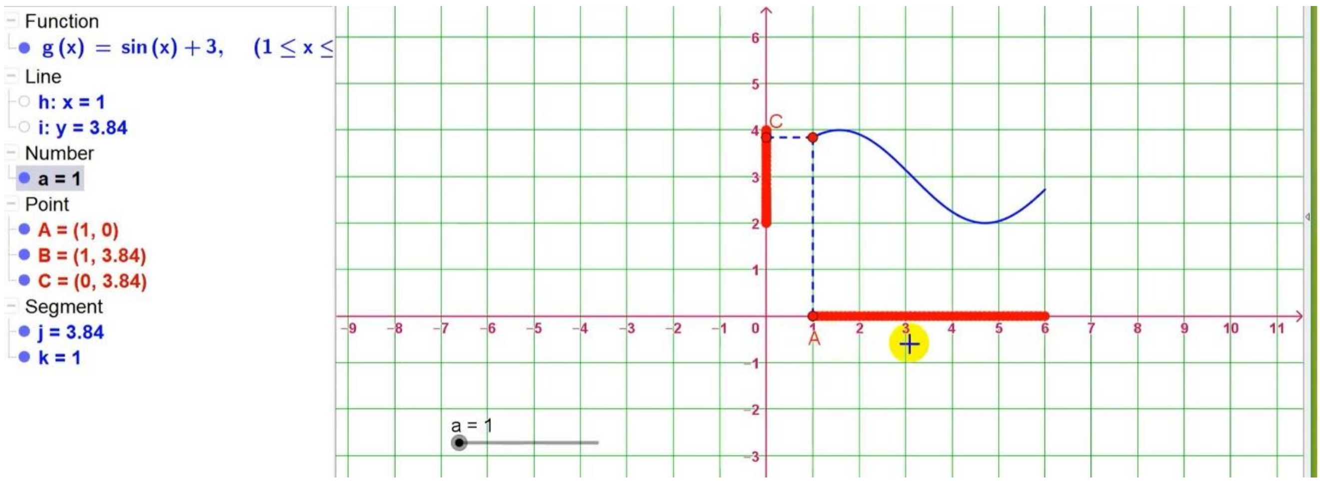
$$f(x) = \begin{cases} 2x + 1 & x \leq 1 \\ \sqrt{x} & x > 1 \end{cases}$$



DOMAIN AND RANGE OF A FUNCTION - المجال و المدى في الدوال

تحديد مجال الدالة جبريا





EXAMPLE 1.2.3 Find the **domain** of each of the following functions

أوجد المجال لجميع الدوال التالية

a. $f(x) = x^2 - 2x$ Solution $D_f = R$

b. $f(x) = \dfrac{3x-4}{x-5}$

Solution $x-5=0$ $x=5$

$D_f = (-\infty, 5) \cup (5, \infty)$

$D_f = R - \{5\}$

EXAMPLE 1.2.3 Find the **domain** of each of the following functions

أوجد المجال لجميع الدوال التالية

c. $f(x) = \dfrac{2x-3}{x^2-4x-5}$

Solution $x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$(x-5) = 0$ or $(x+1) = 0$

$x = 5$ $x = -1$

$D_f = (-\infty, -1) \cup (-1, 5) \cup (5, \infty)$

$D_f = R - \{5, -1\}$

EXAMPLE 1.2.3 Find the **domain** of each of the following functions

أوجد المجال لجميع الدوال التالية

d. $f(x) = \sqrt{5x-2}$

Solution $5x - 2 \geq 0$

$5x \geq 2$

$x \geq \frac{2}{5}$

$D_f = [\frac{2}{5}, \infty)$

e. $f(x) = \sqrt[3]{x^2 - 4x}$

Solution $D_f = R$

RELATED PROBLEM 2 Find the domain of each of the following functions.

أسئلة لها صلة بالسابق

a. $f(x) = x^3 + x^2 - 3$

b. $f(x) = \dfrac{x-2}{x+4}$

c. $f(x) = \dfrac{4x-5}{x^2 - x - 30}$

d. $f(x) = \sqrt{4-3x}$

e. $f(x) = \sqrt[5]{x-8}$

Answer

a. $D_f = \mathbb{R}$

b. $D_f = \{x \in \mathbb{R} : x \neq -4\}$ $D_f =$ R – { -4 }

c. $D_f = \{x \in \mathbb{R} : x \neq -5, x \neq 6\}$ $D_f =$ R – { -5 , 6}

d. $D_f = \{x \in \mathbb{R} : x \leq \frac{4}{3}\}$ $D_f = (-\infty, \frac{4}{8}]$

e. $D_f = \mathbb{R}$

Representation of functions - تمثيل الدوال

DEFINITION 1.2.4

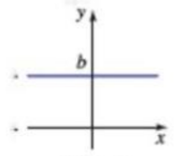
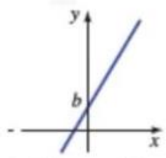
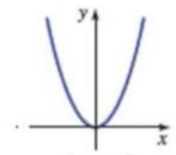
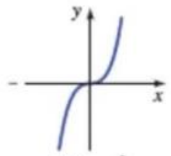
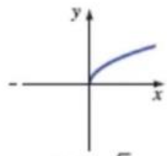
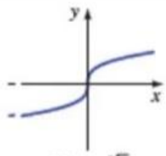
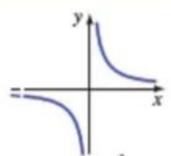
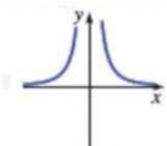
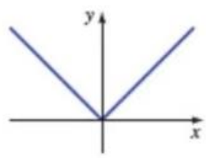
The two functions f and g are equal, if f and g have the same domain and $f(x) = g(x)$ for each x in the common domain. تعريف تساوي دالتين

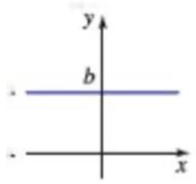
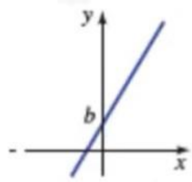
EXAMPLE 1.2.4

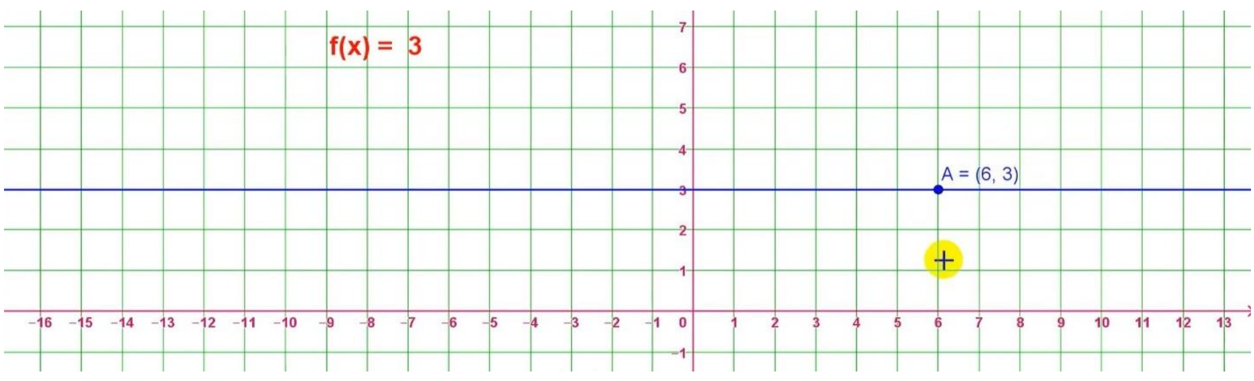
$f(x) = \sqrt{x} + 1$ $D_f = [\,0, \infty)$

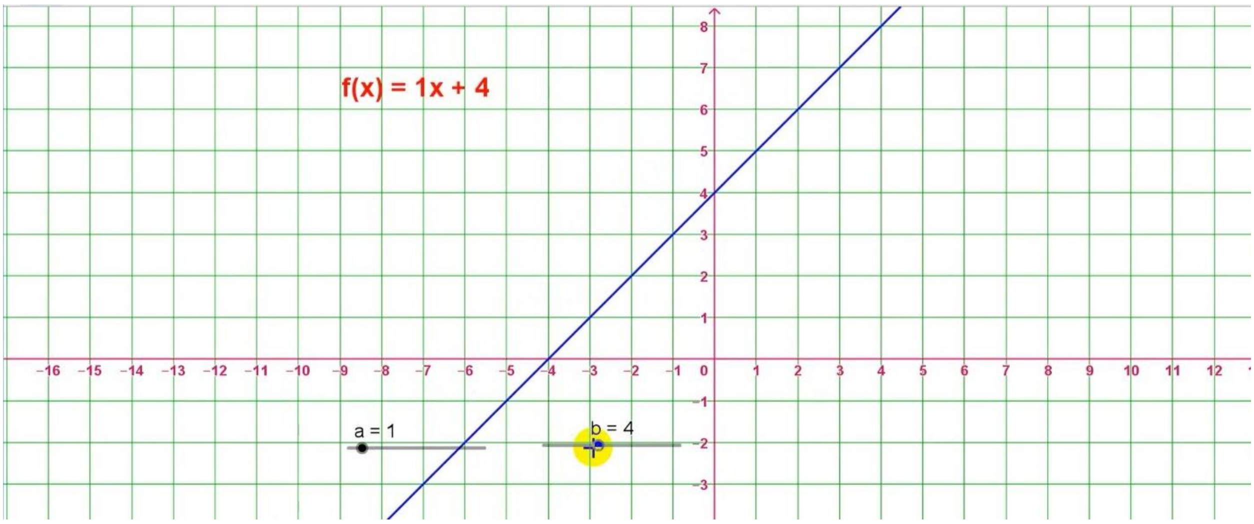
$g(x) = \frac{1}{2}\left(2\sqrt{x} + 2\right)$ $D_f = [\,0, \infty)$

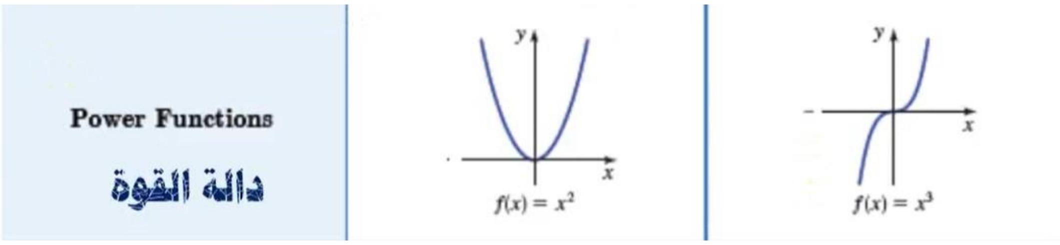
f(x) = g(x)

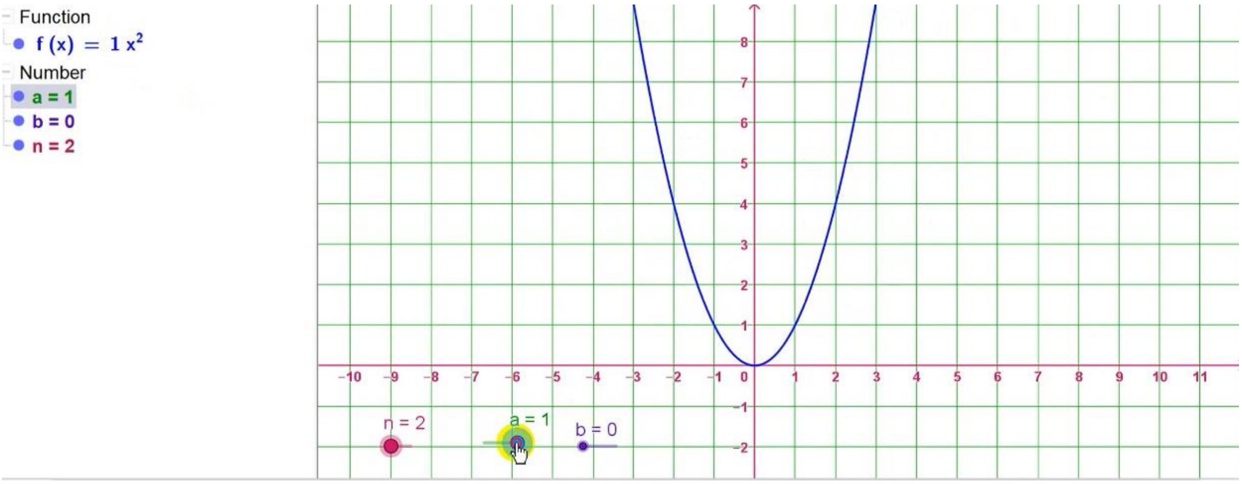
Linear Functions دالة خطية	 $f(x) = b$	 $f(x) = mx + b$
Power Functions دالة القوة	 $f(x) = x^2$	 $f(x) = x^3$
Root Functions دالة الجذرية	 $f(x) = \sqrt{x}$	 $f(x) = \sqrt[3]{x}$
Reciprocal Functions دالة المعكوس	 $f(x) = \frac{1}{x}$	 $f(x) = \frac{1}{x^2}$
Absolute Value Function دالة القيمة المطلقة		

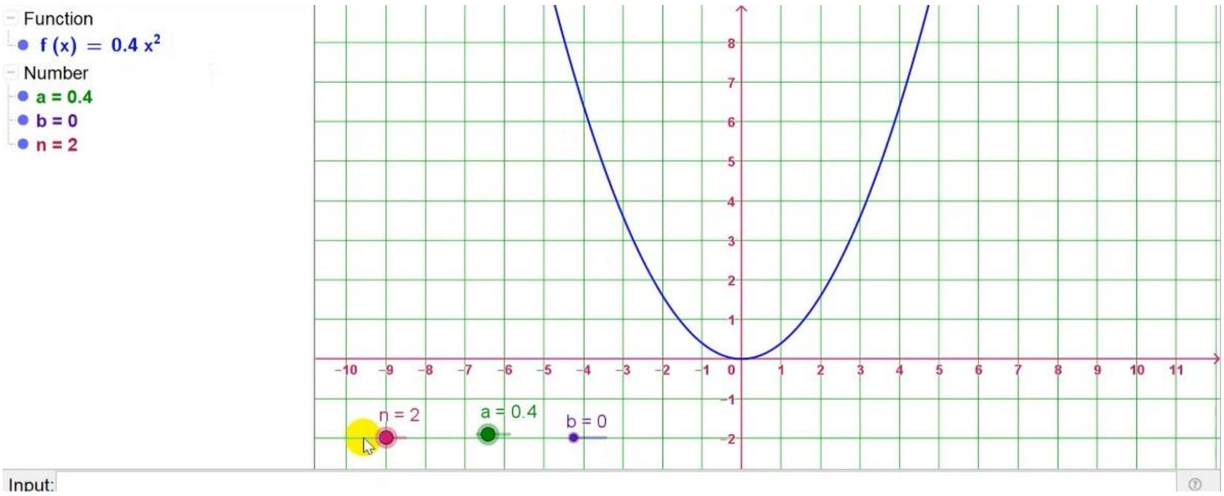
Linear Functions دالة خطية	 $f(x) = b$	 $f(x) = mx + b$

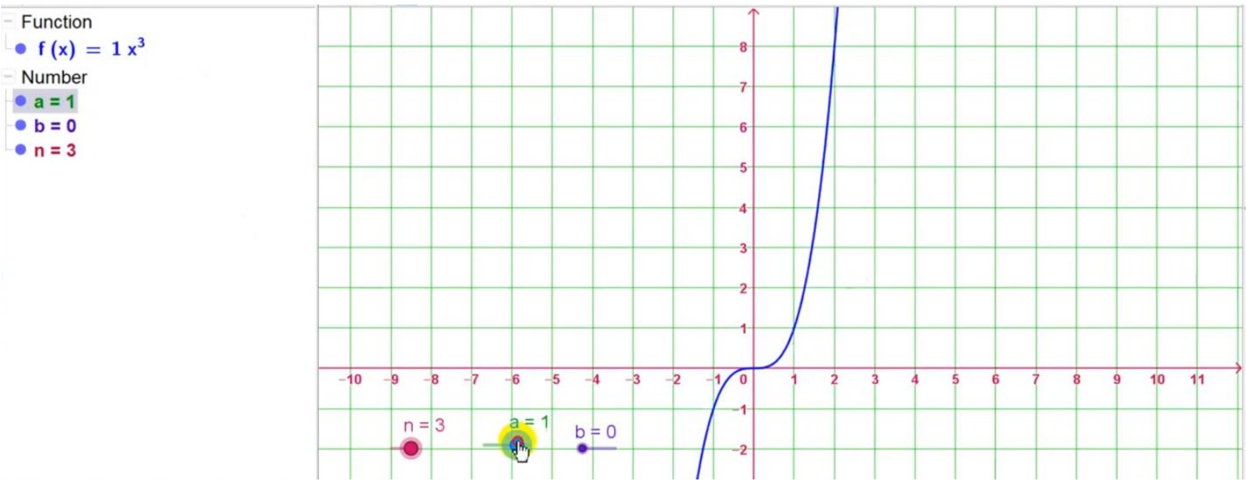


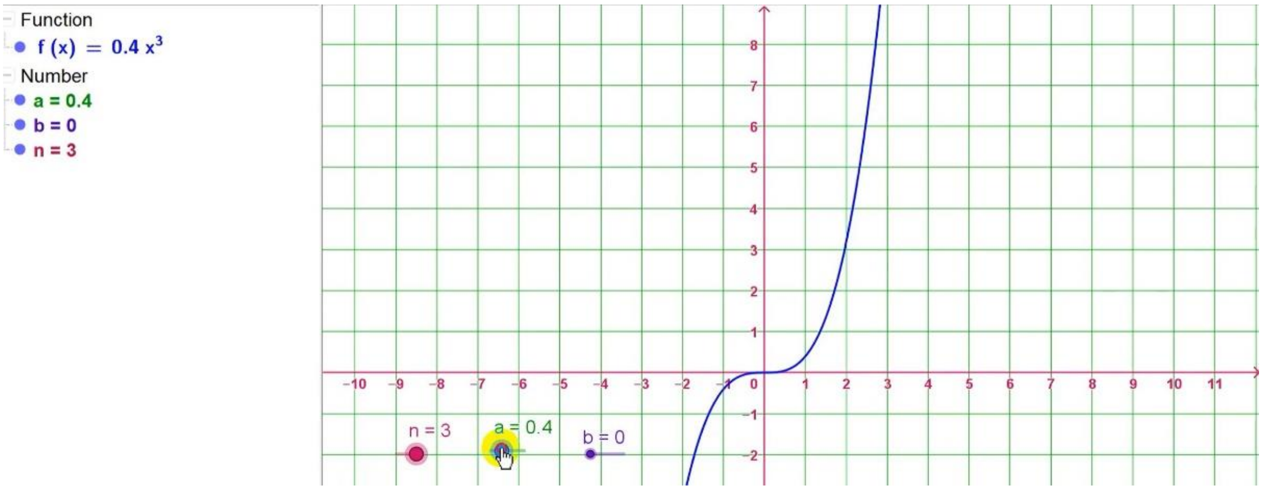

f(x) = 1x + 4
a = 1
b = 4

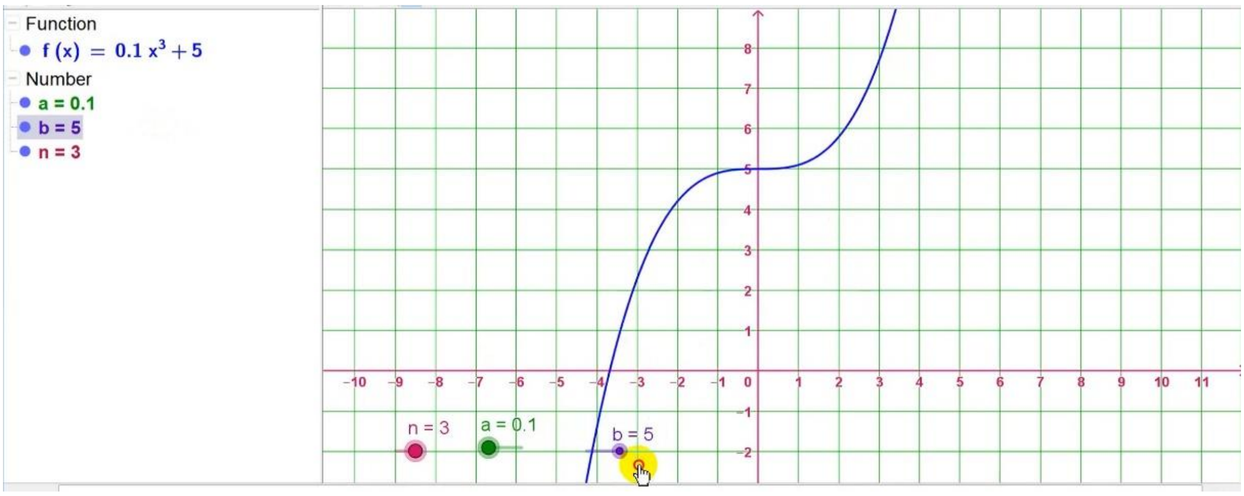

Power Functions
دالة القوة
f(x) = x²
f(x) = x³


Function
f (x) = 1 x²
Number
a = 1
b = 0
n = 2


Function
f (x) = 0.4 x²
Number
a = 0.4
b = 0
n = 2
Input:

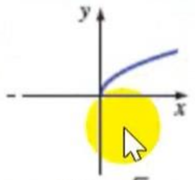

Function
f (x) = 1 x³
Number
a = 1
b = 0
n = 3
n = 3
a = 1
b = 0

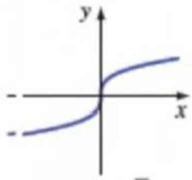

Function
f (x) = 0.4 x³
Number
a = 0.4
b = 0
n = 3
n = 3
a = 0.4
b = 0

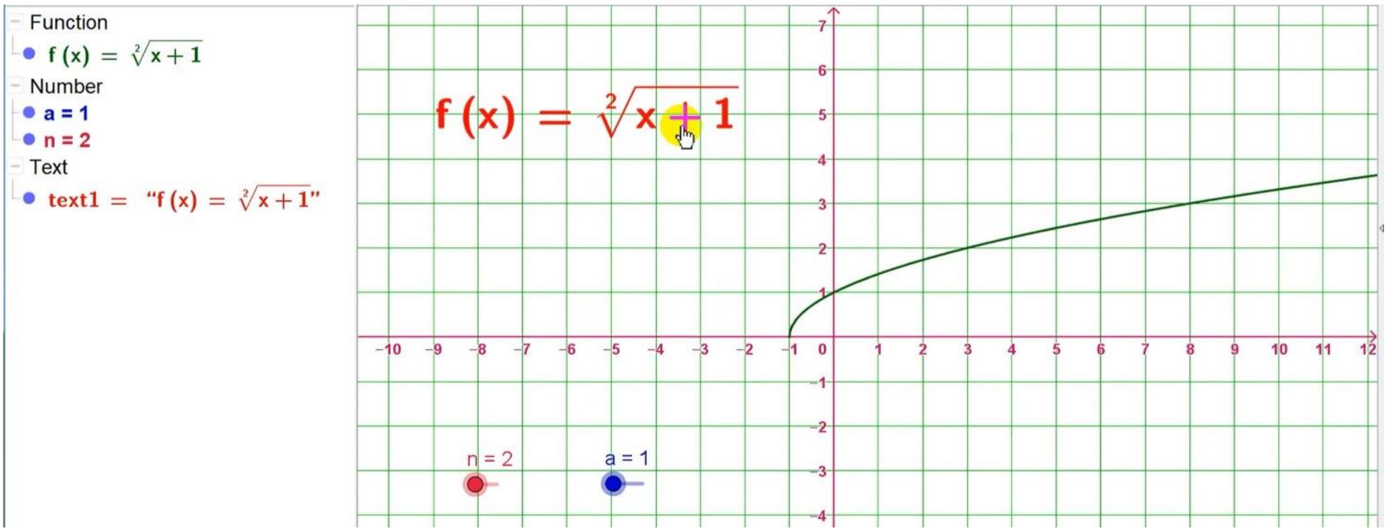

Function
f (x) = 0.1 x³ + 5
Number
a = 0.1
b = 5
n = 3
n = 3
a = 0.1
b = 5

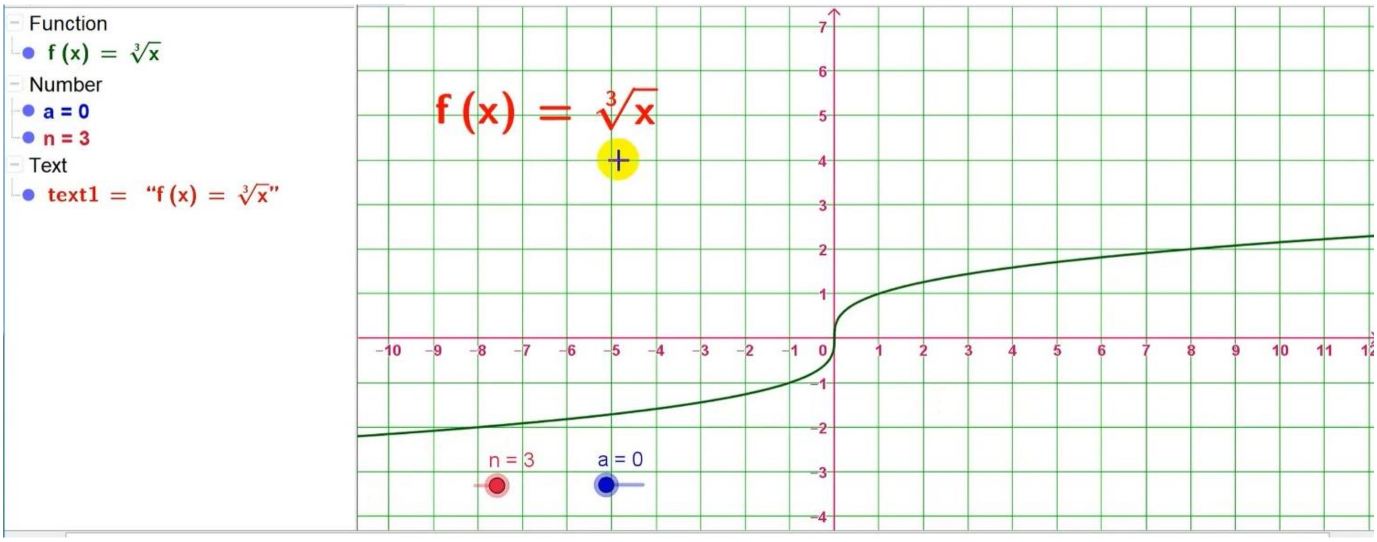
Root Functions

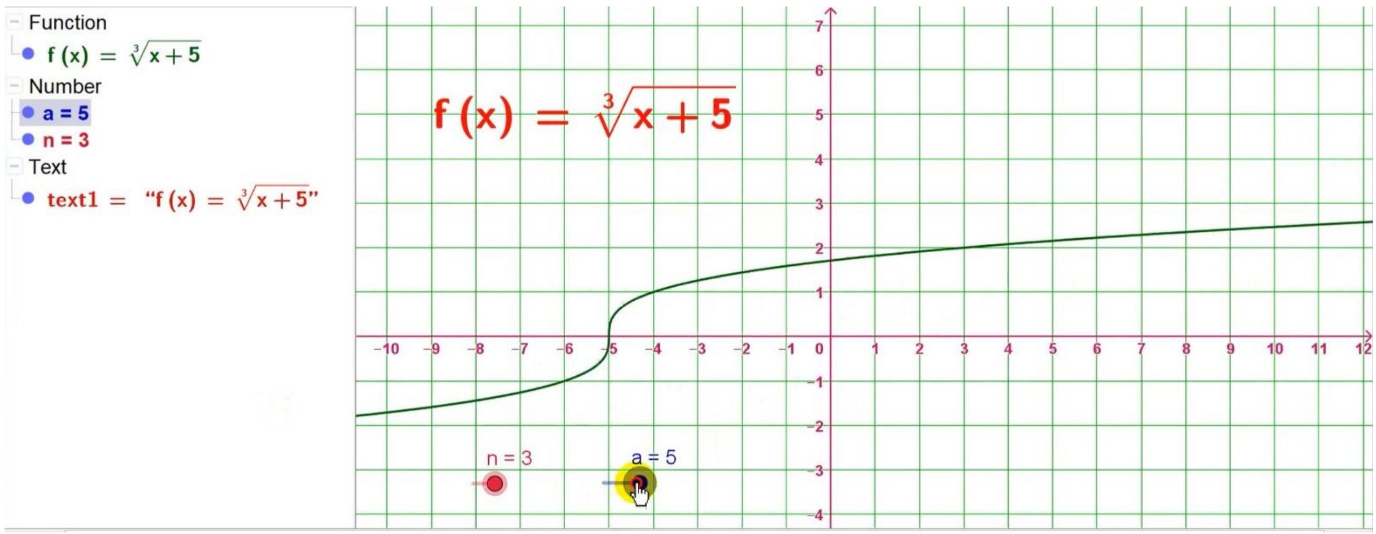
دالة الجذرية





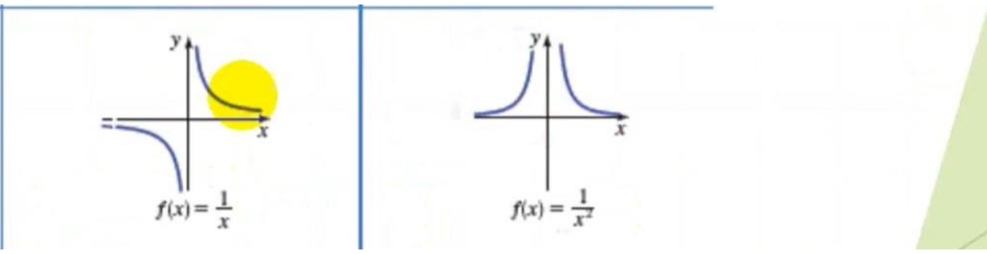


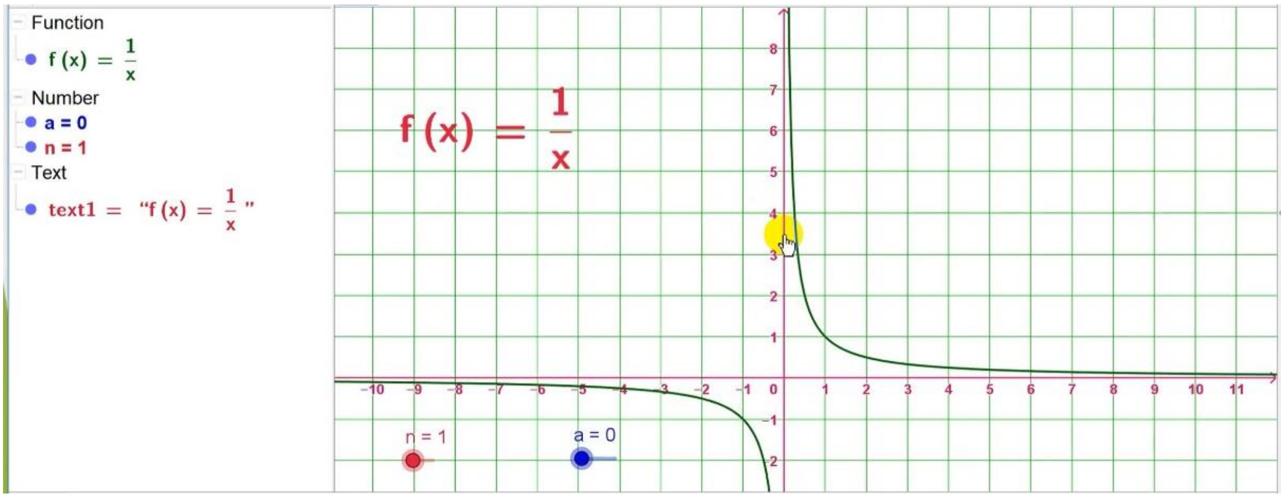


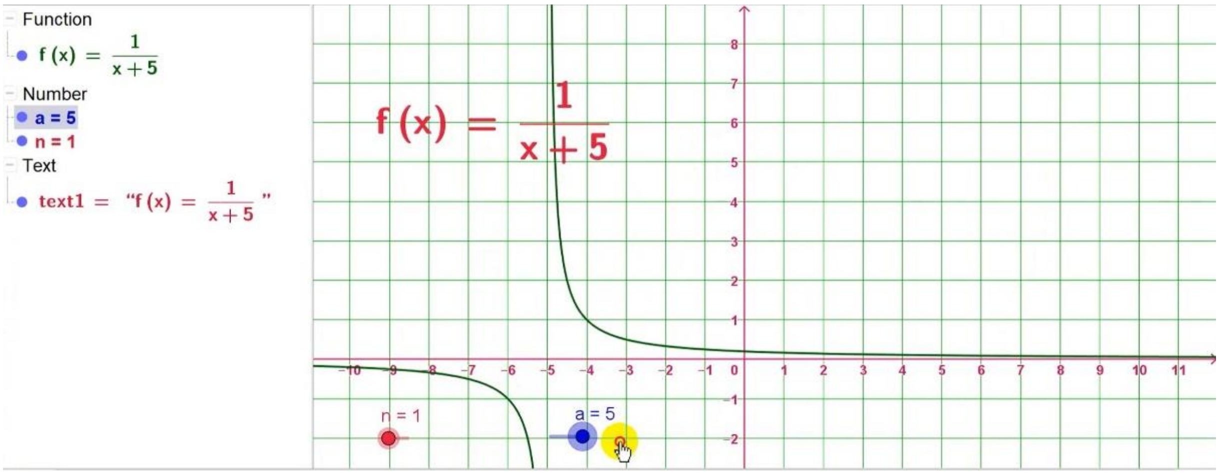


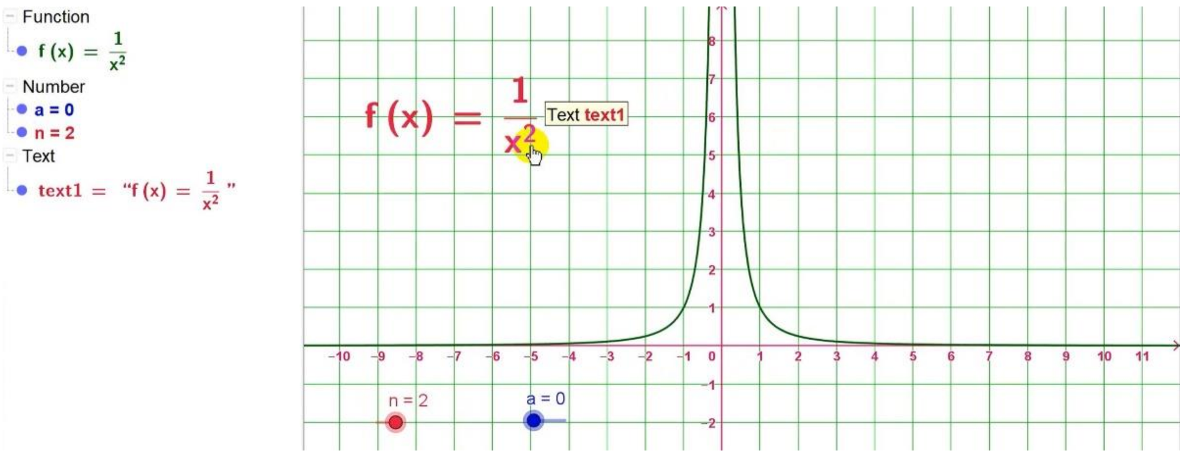
Reciprocal Functions

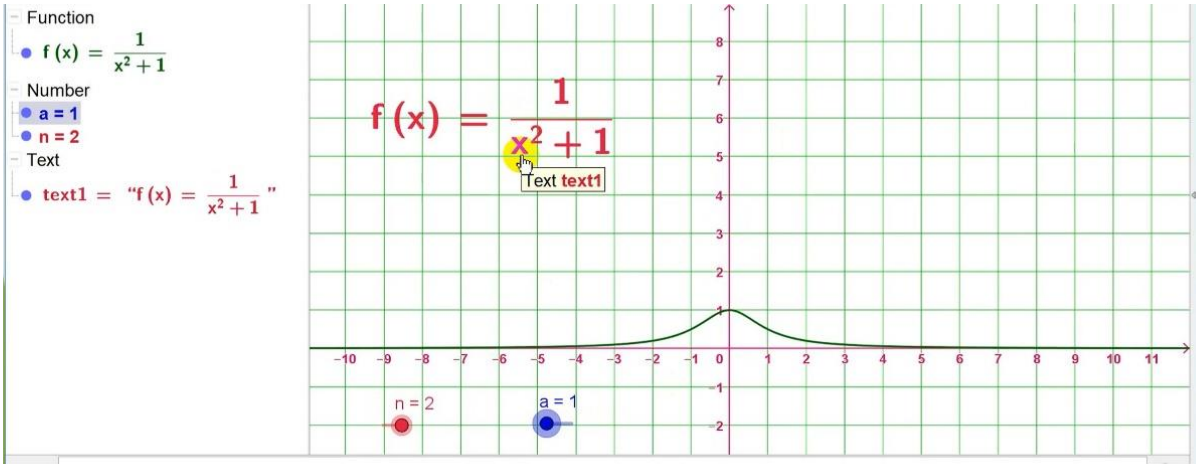
دالة المعكوس

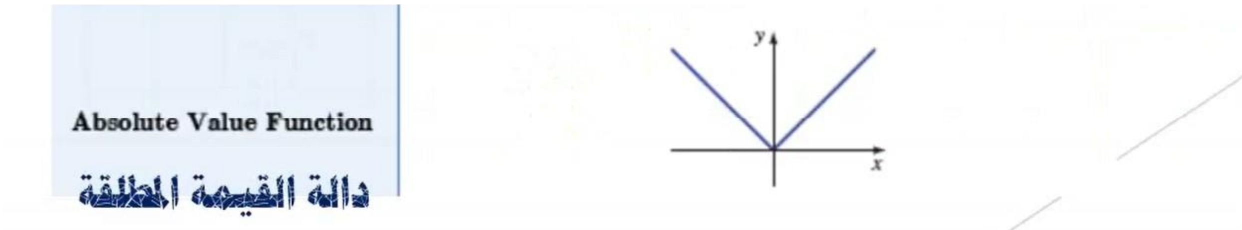


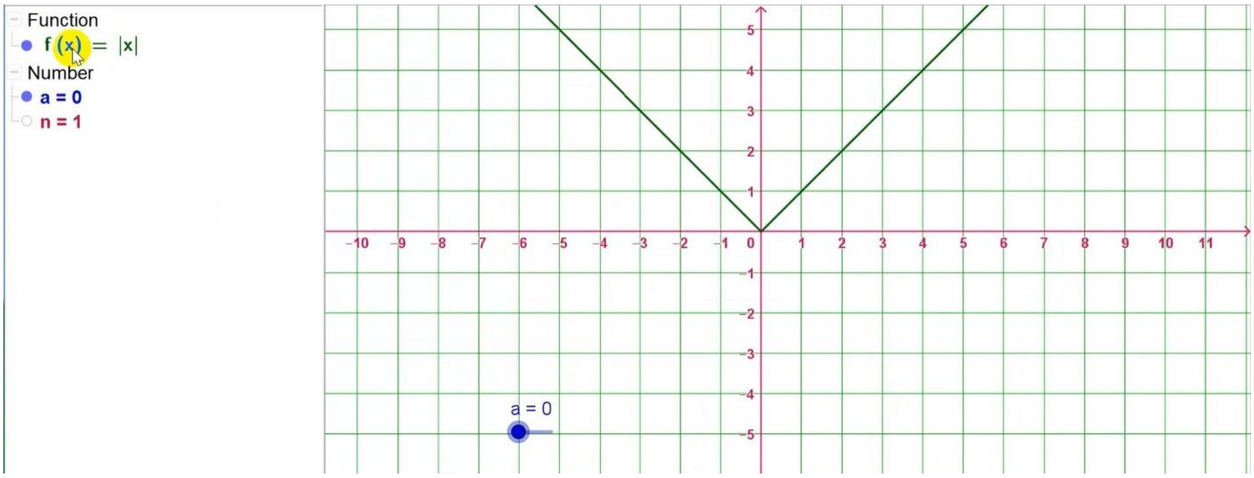


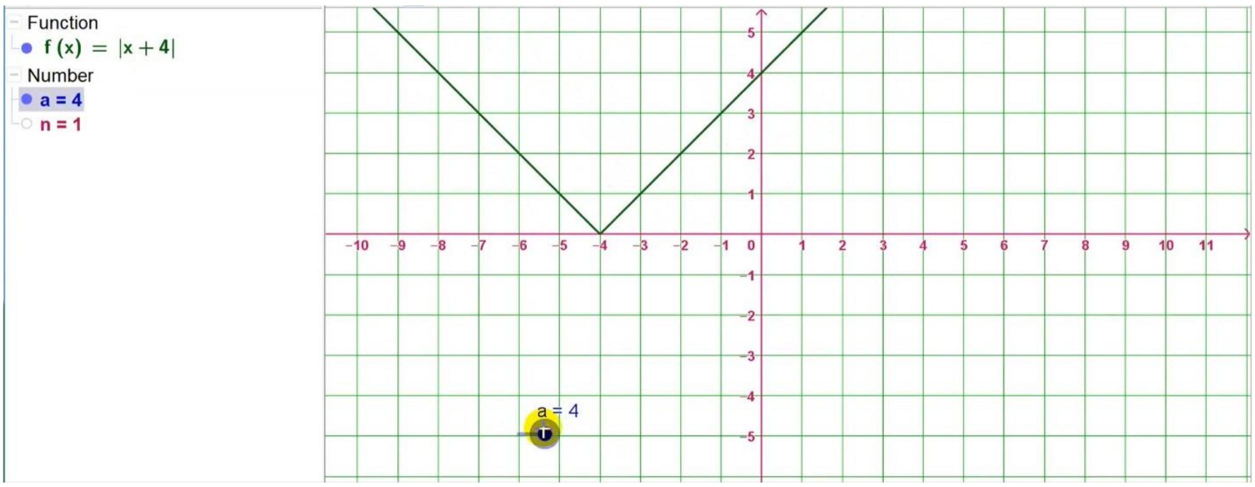





Function
f(x) = 1/(x² + 1)
Number
a = 1
n = 2
Text
text1 = "f(x) = 1/(x² + 1)"
f(x) = 1/(x² + 1)
Text text1
n = 2
a = 1


Absolute Value Function
دالة القيمة المطلقة
y
x


Function
f(x) = |x|
Number
a = 0
n = 1
a = 0


Function
f(x) = |x + 4|
Number
a = 4
n = 1
a = 4

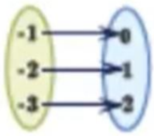
EXERCISES 1.2

In Exercises 1 – 2, determine which of the following sets is a function. If it is a function, what is its domain and range?

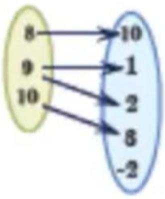
1. $f=\{(2,3),(3,3),(-2,3),(1,3),(0,3)\}$
2. $g=\{(5,1),(2,2),(-1.5,2),(5,3),(1,7)\}$.

In Exercises 3 – 4, determine which of the following diagrams represent a function. Explain your reason for any that do not define a function.

3.

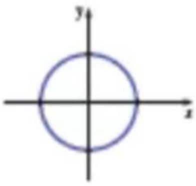


4.

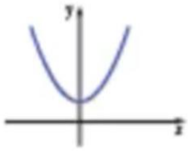


In Exercises 5 – 6 use the vertical line to test to identify if the graphs are functions or not.

5.



6.



In Exercises 7 – 8 find the numerical value of the function at the given value of x.

7. $f(x)=2x^3-3;\ x=0,-1$
8. $g(x)=\dfrac{3x^2-4x-1}{2x^2+5x-3};\ x=-1$

In Exercises 9 – 19, find the domain of each function

9. $f(x)=x^3-4x+1$
10. $f(x)=\sqrt[3]{2x^2-3x+1}$
11. $f(x)=\dfrac{x^2-2x}{x-4}$
12. $f(x)=\sqrt{3x-9}$
13. $f(x)=\dfrac{1}{\sqrt{x-5}}$
14. $f(x)=\sqrt{\dfrac{2x+1}{x+2}}$
15. $f(x)=\dfrac{\sqrt{x}+4x}{x^3-x}$
16. $g(w)=\dfrac{w-1}{w^2-w-6}$

Find the domain of $f(x)=\sqrt{x^2-x-6}$

$$x^2-x-6\ge 0$$

$$(x-3)(x+2)\ge 0$$

	$(-\infty,-2)$	$(-2,3)$	$(3,\infty)$
$x-3$	-	-	+
$x+2$	−	+	+
$(x-3)(x+2)$	+	-	+

Solution : $(-\infty,-2]\cup[3,\infty)$

Solve $1 - 2|2x - 3| \geq -6$

$-2\,|2x-3| \geq -7$

$|2x-3| \leq \frac{7}{2}$

$-\frac{7}{2} \leq 2x-3 \leq \frac{7}{2}$

$-\frac{7}{2} + 3 \leq 2x \leq \frac{7}{2} + 3$

$\frac{-7+6}{2} \leq 2x \leq \frac{7+6}{2}$

$\frac{-1}{2} \leq 2x \leq \frac{13}{2}$

$\frac{-1}{4} \leq x \leq \frac{13}{4}$

Solution : $[\frac{-1}{4}, \frac{13}{4}]$

Solve the following inequality, and write your answer in interval notation

$-5 < 2x - 3 \leq 7$

$-3 + 3 < 2x \leq 7 + 3$

$0 < 2x \leq 10$

$0 < x \leq 5$

Solution : $(0, 5]$

Solve the following inequality and write your answer in interval notation

$\sqrt{(x-2)^2} \leq 3$

$|x - 2| \leq 3$

$-3 \leq x - 2 \leq 3$

$-1 \leq x \leq 5$

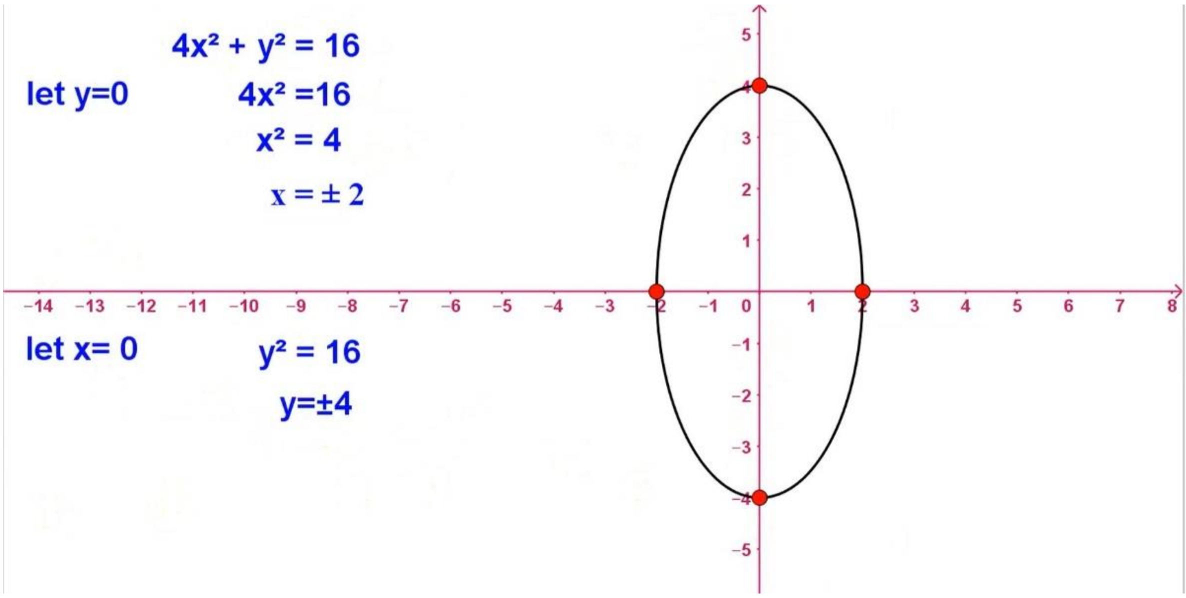
Solution : $[-1, 5]$

Section 1.3
PROPERTIES OF FUNCTIONS, AND THEIR COMBINATION
خصائص الدوال و تركيباتها

INTERCEPTS المقطع

EXAMPLE 1.3.1 Find the x and y intercepts of the graph of the equation
أوجد مقطع x , y للمنحنى الذي معادلته

$$4x^2 + y^2 = 16$$



RELATED PROBLEM 1 Find the x and y intercepts of the graph of the equation
سؤال مشابه

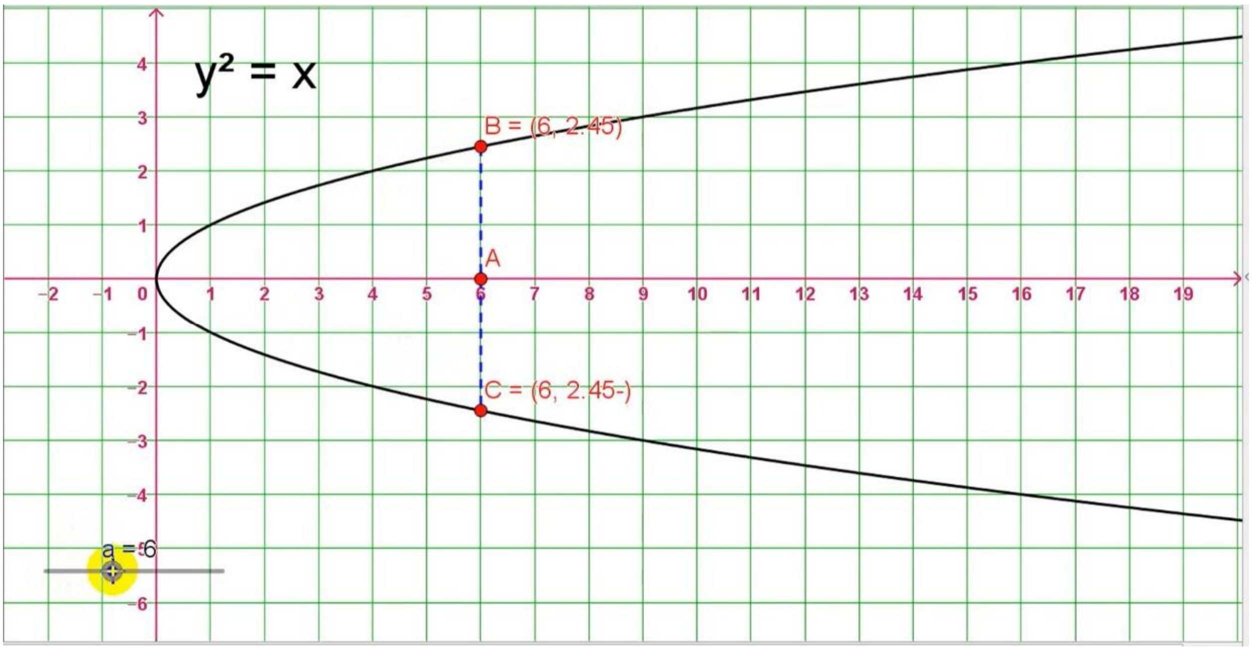
$$9x^2 - 2y^2 = 16$$

SYMMETRY التناظر

DEFINITION 1.3.1 (Symmetry with Respect to x -axis) (التناظر حول محور x)

A graph is said to be symmetric with respect to x – axis provided that whenever (x, y) is on the graph, then $(x, -y)$ is also on the graph.

يقال المنحنى A متناظر حول المحور x إذا كان (x,y) عنصر في A فأن (x,-y) عنصر في A



EXAMPLE 1.3.2 Show that the graph of the equation $x = y^2$ is symmetric with respect to x – axis.

تحقق المنحنى الذي معادلته $x = y^2$ متناظر حول المحور x

Solution **If (*x*,*y*) is on the graph,**

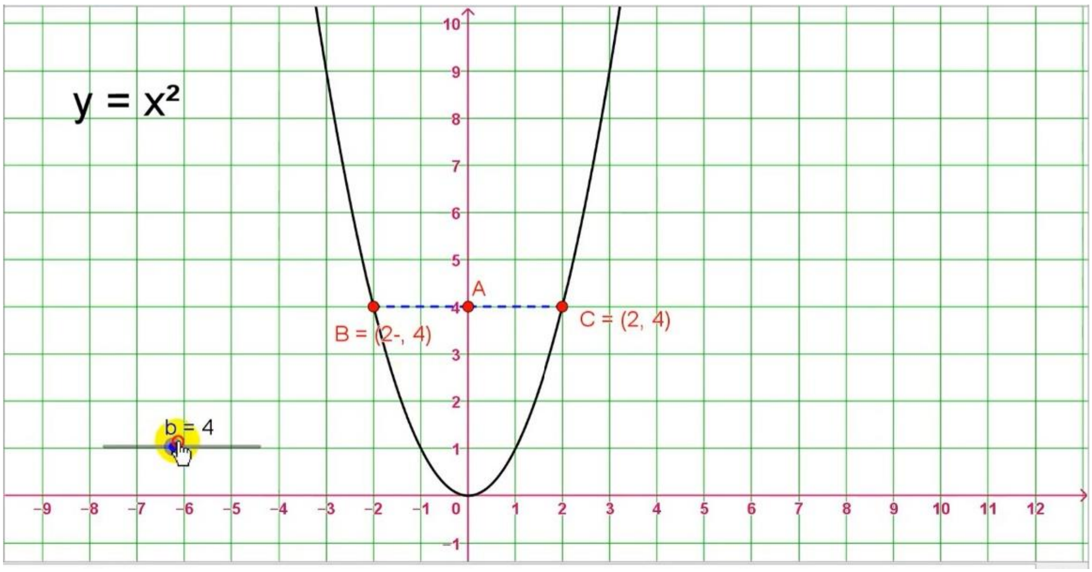
$x = y^2$ than $x = (-y)^2 = y^2$

then (*x*,-*y*) is also on the graph.

DEFINITION 1.3.2 (Symmetry with Respect to y -axis) (التناظر حول محور y)

A graph is said to be symmetric with respect to y – axis provided that whenever (x, y) is on the graph, then $(-x, y)$ is also on the graph.

يقال المنحنى A متناظر حول المحور y إذا كان (x,y) عنصر في A فأن (-x, y) عنصر في A



EXAMPLE 1.3.3 Show that the graph of the equation $y = x^2$ is symmetric with respect to y $-$ axis.

تحقق المنحنى الذي معادلته y = x² متناظر حول المحور y

Solution **If (*x*,*y*) is on the graph,**

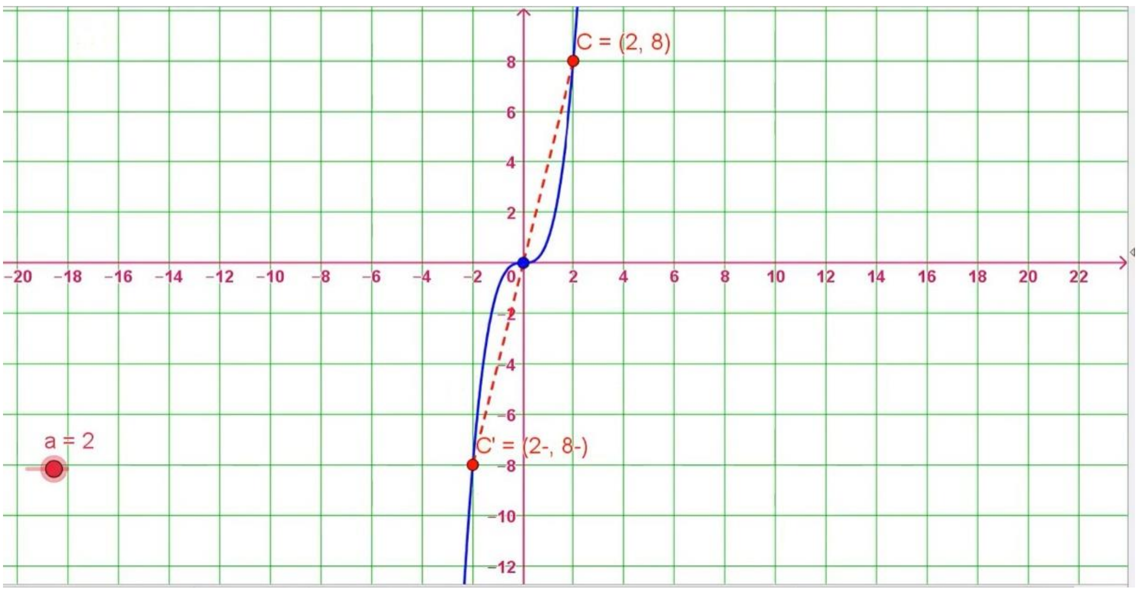
y =x² than y =(-x)²= x²

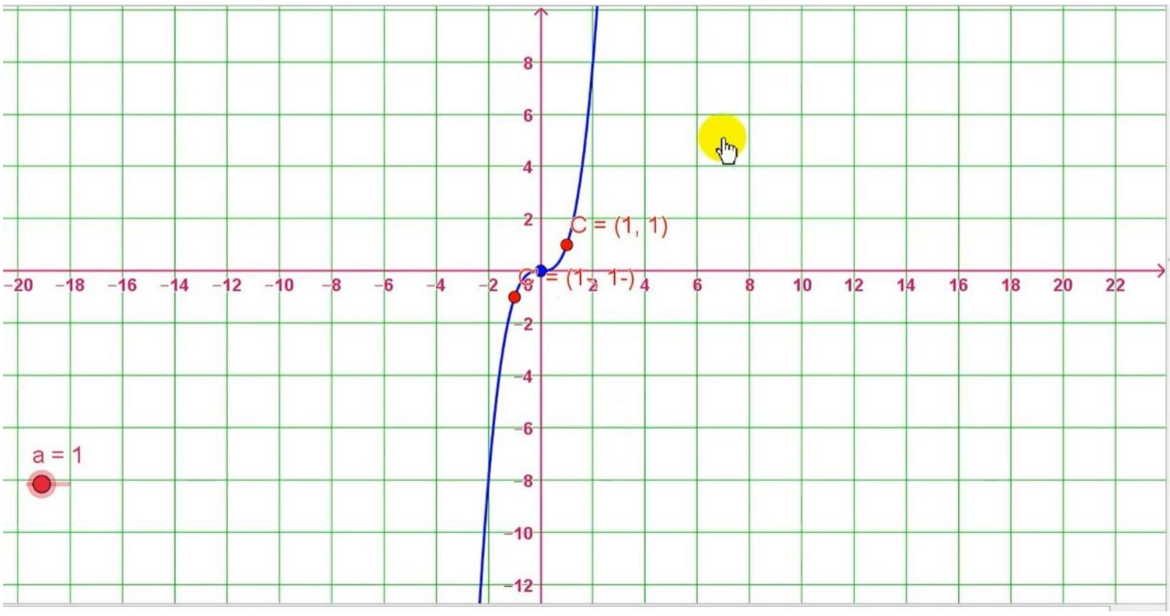
then (-*x*, *y*) is also on the graph.

DEFINITION 1.3.3 (Symmetry with Respect to Origin) **(التناظر حول نقطة الاصل)**

A graph is said to be symmetric with respect to the origin provided that whenever (x, y) is on the graph, then $(-x, -y)$ is also on the graph.

يقال المنحنى A متناظر حول نقطة الأصل إذا كان (x,y) عنصر في A فأن (-x,-y) عنصر في A





EXAMPLE 1.3.4 Show that the graph of the equation $y = 2x^3$ is symmetric with respect to the origin. **تحقق المنحنى الذي معادلته y =2 x³ متناظر حول نقطة الأصل**

Solution **If (*x*,*y*) is on the graph,**

$y = 2x^3$ **than** $-y = 2(-x)^3 = -2x^3$, $y = 2x^3$

then (-*x*, -*y*) is also on the graph.

SYMMETRY التناظر

(Symmetry with Respect to *x*-axis) (التناظر حول محور x)

whenever (*x*,*y*) is on the graph, then (*x*,-*y*) is also on the graph.

(Symmetry with Respect to *y*-axis) (التناظر حول محور y)

whenever (*x*,*y*) is on the graph, then (-*x*,*y*) is also on the graph.

(Symmetry with Respect to Origin) (التناظر حول نقطة الاصل)

whenever (*x*,*y*) is on the graph, then (-*x*,-*y*) is also on the graph.

Even and Odd Functions - الدوال الزوجية و الفردية

DEFINITION 1.3.4 (Even and Odd Functions) الدالة الزوجية و الفردية

- **a.** A function f is even if its graph is symmetric with respect to y – axis; that is,

$$f(-x) = f(x) \text{ for every } x \text{ in the function's domain.}$$

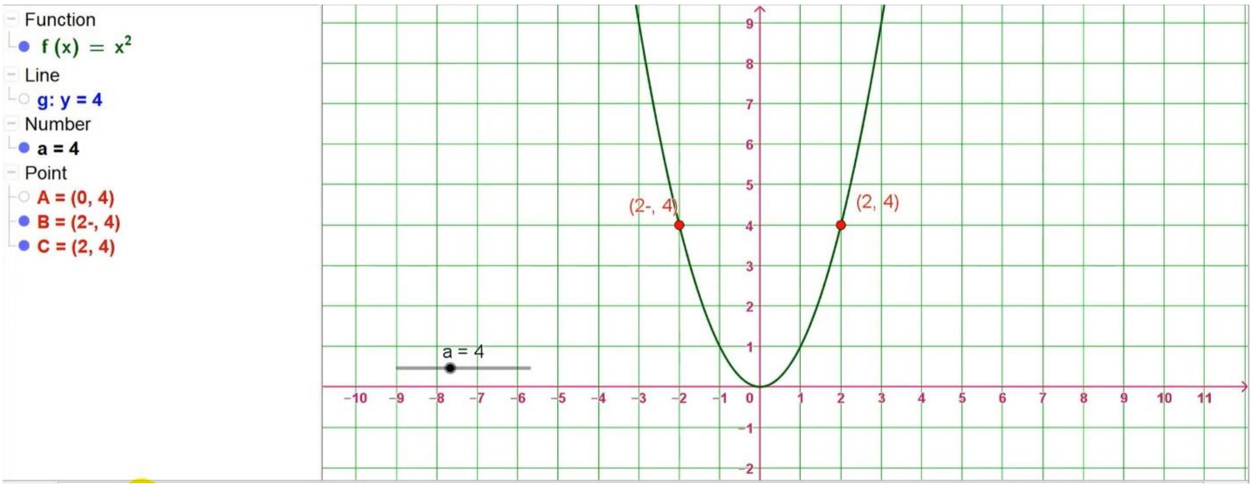
- **b.** A function f is odd if its graph is symmetric with respect to the origin; that is,

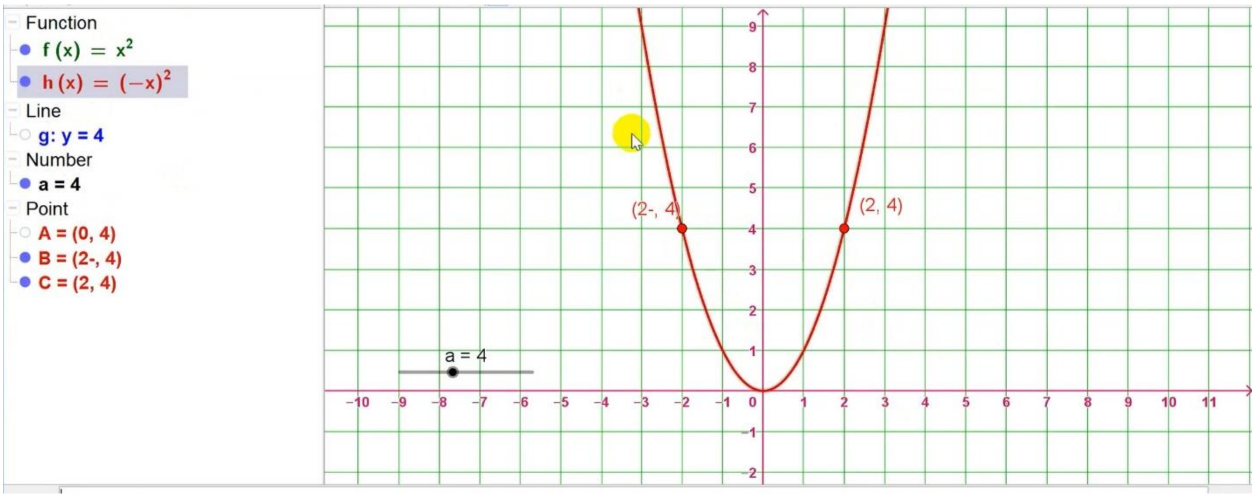
$$f(-x) = -f(x) \text{ for every } x \text{ in the function's domain.}$$

(Even Functions) الدالة الزوجية

- **a.** A function f is even if its graph is symmetric with respect to y – axis; that is,

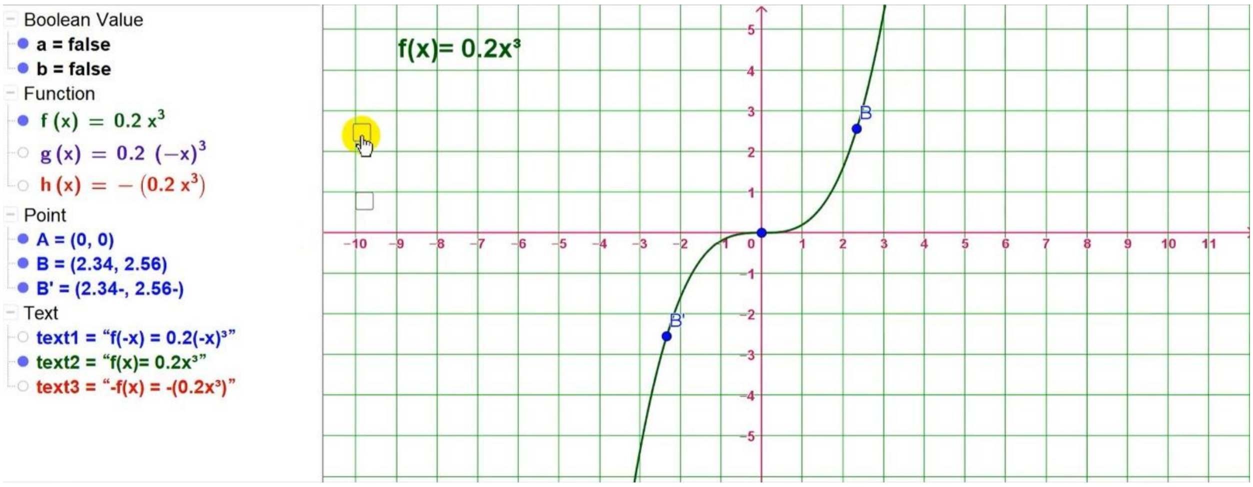
$$f(-x) = f(x) \text{ for every } x \text{ in the function's domain.}$$

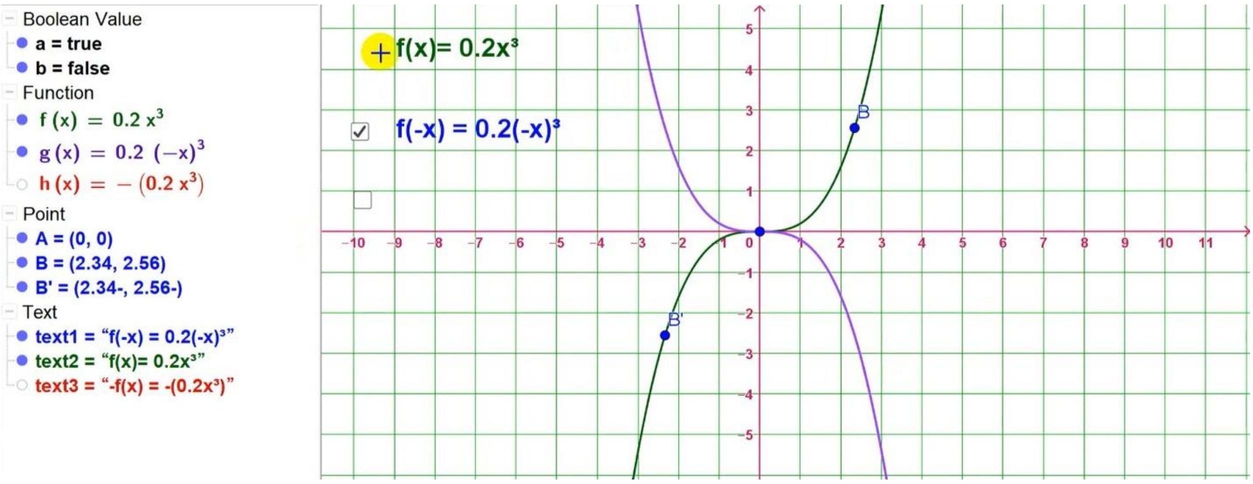


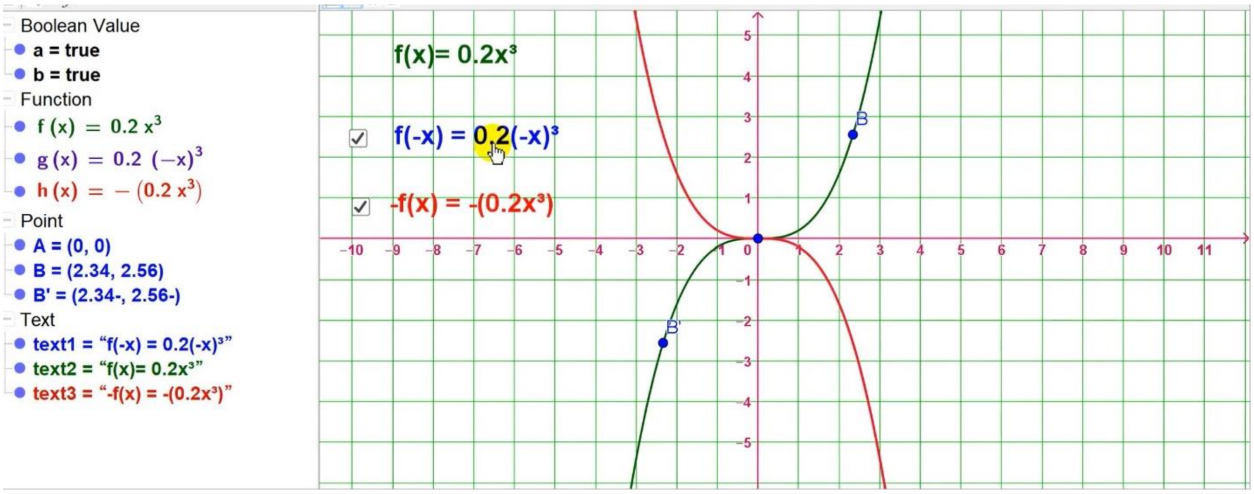


(Odd Functions) الدالة الفردية

- **b.** A function f is odd if its graph is symmetric with respect to the origin; that is, $f(-x) = -f(x)$ for every x in the function's domain.







EXAMPLE 1.3.5

Determine algebraically whether the following functions are even, odd, or nither.

حدد جبريا إذا كانت الدوال التالية زوجية أو فردية أو غير ذلك

a. $f(x) = x^4 - 3$

Solution

$f(-x) = (-x)^4 - 3 = x^4 - 3 = f(x)$. f is even.

b. $g(x) = \dfrac{x - 2x^3}{x^2 + 1}$

Solution

$$g(-x) = \frac{-x - 2(-x)^3}{(-x)^2 + 1} = \frac{-x + 2x^3}{x^2 + 1} = -\frac{x - 2x^3}{x^2 + 1} = -g(x).$$ g is odd.

c. $h(x) = x^3 + 1$.

Solution

$$h(-x) = (-x)^3 + 1 = -x^3 + 1 = -(x^3 - 1) \neq -h(x)$$

Since $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$

h is neither

A) Determine algebraically whether the function $f(x) = \dfrac{x^5 + 3x}{x^4 + x^2}$ is even, odd, or neither.

Solution

$$f(-x) = \frac{(-x)^5 + 3(-x)}{(-x)^4 + (-x)^2} = \frac{-x^5 - 3x}{x^4 + x^2} = \frac{-(x^5 + 3x)}{x^4 + x^2} = -\frac{(x^5 + 3x)}{x^4 + x^2} = -f(x)$$

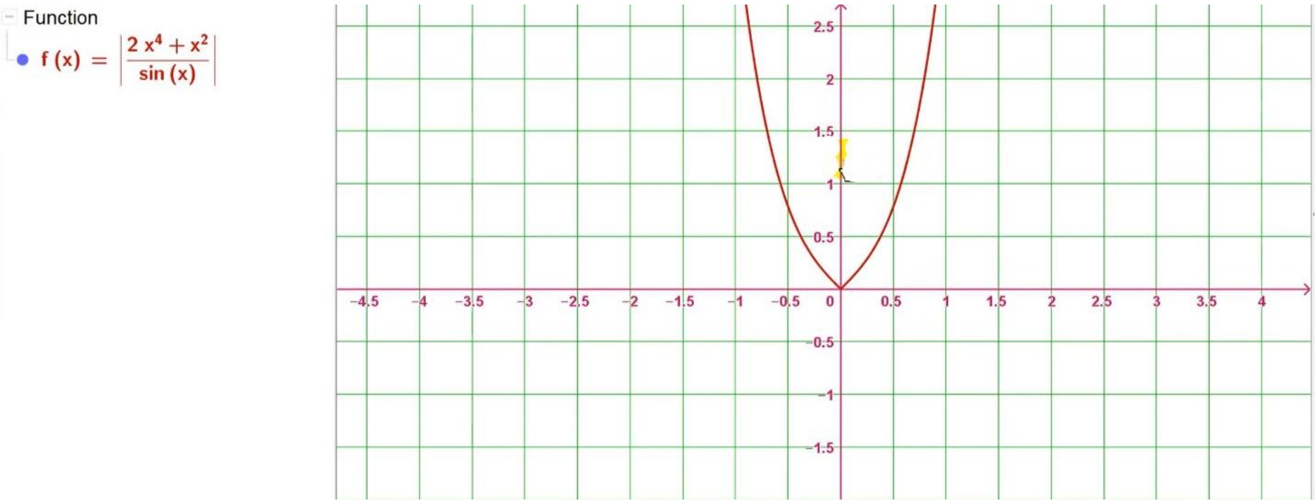
f *is odd*

A) Determine algebraically whether the function $f(x) = \left|\dfrac{2x^4 + x^2}{\sin x}\right|$ is even, odd, or neither.

Solution

$$f(-x) = \left|\frac{2(-x)^4 + (-x)^2}{\sin(-x)}\right| = \left|\frac{2x^4 + x^2}{-\sin x}\right| = \left|\frac{2x^4 + x^2}{\sin x}\right| = f(x)$$

f *is even*



B) Determine algebraically is the function $f(x) = \dfrac{x^4 + x^2}{|x|}$ **even, odd, or neither.**

f *is even*

RELATED PROBLEM 5 Determine algebraically whether the following functions are even, odd, or neither.

a. $f(x) = x^2 + 3$ **b.** $g(x) = \dfrac{2x - x^5}{x^4 + 1}$ **c.** $h(x) = x^3 + x^2$.

Answer

a. Even **b.** Odd **c.** Neither

INCREASING AND DECREASING FUNCTIONS - الدوال التزايدية و التناقصية

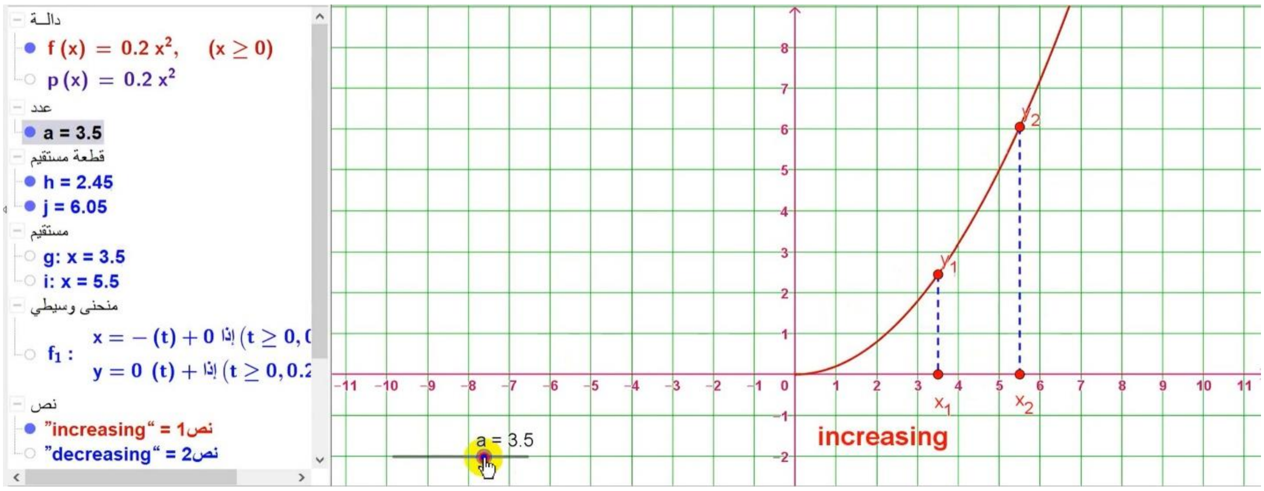
DEFINITION 1.3.5 (Increasing and Decreasing Functions) **الدوال التزايدية و التناقصية**

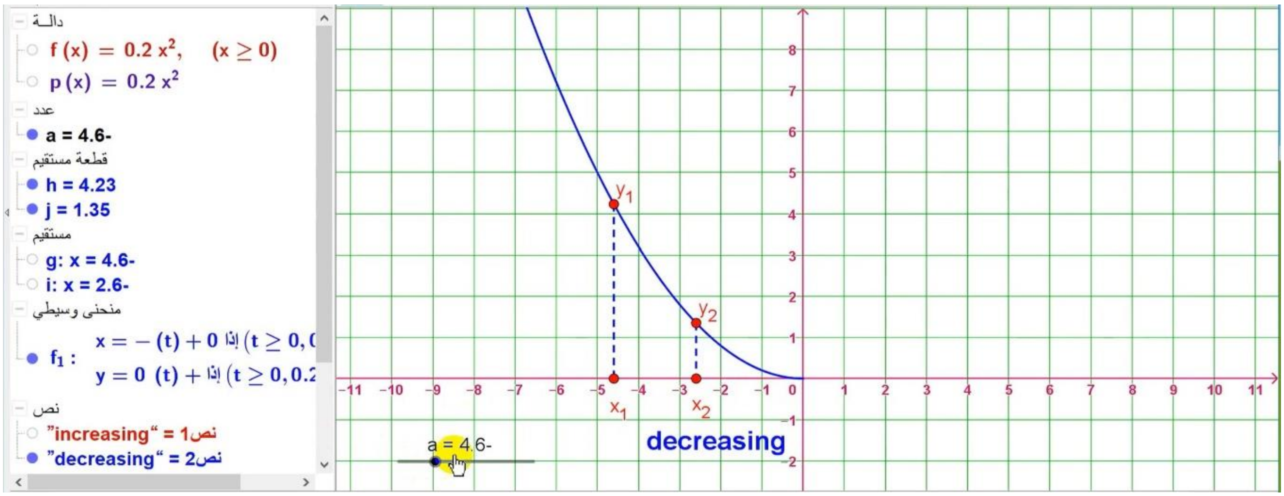
a. A function f defined on an interval I is said to be ***increasing*** on I if

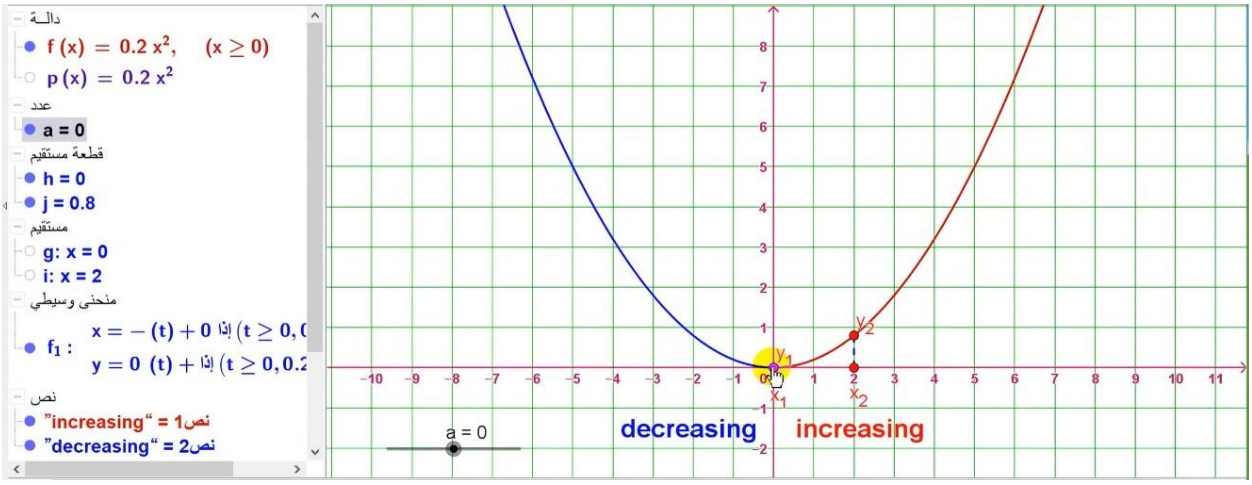
$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2, \text{ for all } x_1, x_2 \in I.$$

b. A function f is said to be ***decreasing*** on I if and only if

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2, \text{ for all } x_1, x_2 \in I.$$


دالــة
f (x) = 0.2 x², (x ≥ 0)
p (x) = 0.2 x²
عدد
a = 3.5
قطعة مستقيم
h = 2.45
j = 6.05
مستقيم
g: x = 3.5
i: x = 5.5
منحنى وسيطي
نص
"increasing" = نص1
"decreasing" = نص2
a = 3.5
increasing


دالــة
f (x) = 0.2 x², (x ≥ 0)
p (x) = 0.2 x²
عدد
a = 4.6-
قطعة مستقيم
h = 4.23
j = 1.35
مستقيم
g: x = 4.6-
i: x = 2.6-
منحنى وسيطي
نص
"increasing" = نص1
"decreasing" = نص2
a = 4.6-
decreasing


دالــة
f (x) = 0.2 x², (x ≥ 0)
p (x) = 0.2 x²
عدد
a = 0
قطعة مستقيم
h = 0
j = 0.8
مستقيم
g: x = 0
i: x = 2
منحنى وسيطي
نص
"increasing" = نص1
"decreasing" = نص2
a = 0
decreasing
increasing

EXAMPLE 1.3.6 Let $f(x) = x^2$. Determine:

a. The intervals on which f is increasing.

b. The intervals on which f is decreasing.

Solution

a. if $0 \le x_1 < x_2$ than $x_1^2 < x_2^2$ Thus $f(x_1)<f(x_2)$ f increasing interval $[0,\infty)$

a. if $x_1 < x_2 < 0$ than $x_1^2 > x_2^2$ Thus $f(x_1)>f(x_2)$ f decreasing interval $(-\infty , 0]$

RELATED PROBLEM 6 Let $f(x) = -x^2$. Determine:

a. The intervals on which f is increasing.

b. The intervals on which f is decreasing.

Answer

a. $(-\infty, 0]$ **b.** $[0, \infty)$

Basic Operations on Functions - العمليات الأساسية على الدوال

DEFINITION 1.3.7 (Basic Operations on Functions)

Let f and g be two functions. We define the sum $f+g$, the difference $f-g$, the product $f \cdot g$, and the quotient f/g as follows:

$$(f+g)(x) = f(x) + g(x)$$

$$(f-g)(x) = f(x) - g(x)$$

$$(f \cdot g)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad , \quad g(x) \neq 0$$

EXAMPLE 1.3.7 Let $f(x) = \sqrt{x+1}$ and $g(x) = \frac{1}{x+2}$. Find $f+g$ and its domain

Solution

$$(f+g)(x) = f(x) + g(x) = \sqrt{x+1} + \frac{1}{x+2}$$

$D_f = [-1, \infty)$

$D_g = R - \{-2\}$

$D_{f+g} = [-1, \infty)$

RELATED PROBLEM 7 Let $f(x) = \sqrt{2x-1}$ and $g(x) = \frac{1}{x-3}$. Find the domain and the rule of $f - g$.

Solution

$$(f-g)(x) = f(x) - g(x) = \sqrt{2x-1} - \frac{1}{x-3}$$

$D_f = [\frac{1}{2}, \infty)$

$D_g = R - \{3\}$

$D_{f-g} = [\frac{1}{2}, \infty) - \{3\}$

EXAMPLE 1.3.8 Let $f(x) = x - 7$ and $g(x) = x^2 - 16$. Find the domain and the rule of $\frac{f}{g}$.

Solution

$$(\frac{f}{g})(x) = \frac{f(x)}{g(x)} = \frac{x-7}{x^2-16}$$

$$x^2 - 16 = 0$$

$$x^2 = 16$$

$$x = \pm 4$$

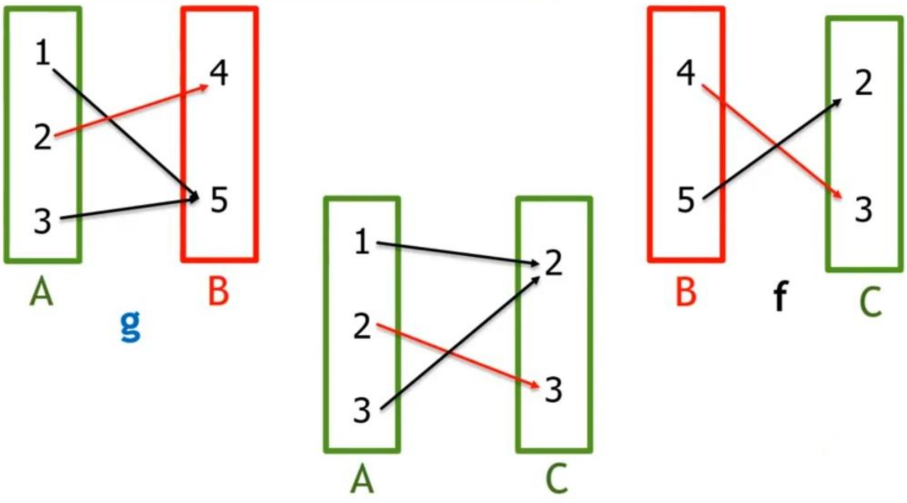
تنبيه مهم	الصحيح
$x^2 = 16$	$x^2 = 16$
$\sqrt{x^2} = \sqrt{16}$	$\sqrt{x^2} = \sqrt{16}$
$x = \pm 4$	$\pm x = 4$
	$x = \pm 4$

$D_{f/g} = R - \{4, -4\}$

RELATED PROBLEM 8 Let $f(x) = 2x - 3$ and $g(x) = x^2 - 5x + 6$. Find the domain and rule of $\frac{f}{g}$.

Answer Domain: $\mathbb{R} - \{2,3\}$, Rule: $\left(\frac{f}{g}\right)(x) = \frac{2x-3}{x^2-5x+6}$.

composition of functions - تركيب الدوال



$$(f \circ g)(x) = f(g(x))$$

DEFINITION 1.3.8

Let f and g be two functions, we define the composition $f \circ g$ of f and g as the function

$$(f \circ g)(x) = f(g(x))$$

The domain of $f \circ g$ consists of all numbers x in the domain of g for which the number $g(x)$ is in the domain of f.

EXAMPLE Let $f(x) = 2x$, $g(x) = x^2$ find $f \circ g$, $g \circ f$, $(f \circ g)(3)$

Solution $(f \circ g)(x) = f(g(x)) = f(x^2) = 2x^2$

$(g \circ f)(x) = g(f(x)) = g(2x) = 4x^2$

$(f \circ g)(3) = 2 . 3^2 = 18$

EXAMPLE 1.3.9 Let $f(x) = \sqrt{x+1}$ and $g(x) = \dfrac{1}{x+2}$. Determine the functions $g \circ f$, $f \circ g$ and their domains, then find $(g \circ f)(3)$ and $(f \circ g)\left(\dfrac{1}{2}\right)$.

Solution

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+1}) = \frac{1}{\sqrt{x+1}+2}$$

$D_f = [-1, \infty)$

$D_g = (-\infty, -2) \cup (-2, \infty)$

$in(-1, \infty)$ $\quad \sqrt{x+1} > 0$

$D_{(g \circ f)} = [-1, \infty)$

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x+2}\right) = \sqrt{\frac{1}{x+2}+1}$$

$D_f = [-1, \infty)$

$D_g = (-\infty, -2) \cup (2, \infty)$

$in(2, \infty)$ $\quad \dfrac{1}{x+2} \geq -1 \quad , 1 \geq -x-2 \quad , \quad 3 \geq -x \quad , \quad -3 \leq x$

$in(-\infty, -2)$ $\quad \dfrac{1}{x+2} \geq -1 \quad , 1 \leq -x-2 \quad , 3 \leq -x \quad , \quad -3 \geq x$

$D_{(f \circ g)} = (2, \infty) \cup [-3, \infty)$

$$(f \circ g)\left(\tfrac{1}{2}\right) = \sqrt{\frac{1}{\frac{1}{2}+2}+1} = \sqrt{\frac{1}{\frac{5}{2}}+1} = \sqrt{\frac{2}{5}+1} = \sqrt{\frac{7}{5}}$$

للعلم

$\frac{1}{2} + 2 = 2\frac{1}{2} = \frac{5}{2}$

RELATED PROBLEM 9 Let $f(x) = \sqrt{1-x}$ and $g(x) = \dfrac{1}{2-x}$. Determine the functions $g \circ f$ and $f \circ g$ and their domains, and then find $(g \circ f)(-8)$ and $(f \circ g)\left(\dfrac{1}{2}\right)$.

Answer

$(g \circ f)(x) = \dfrac{1}{2-\sqrt{1-x}}$, Domain: $(-\infty, 1] - \{-3\}$.

$(f \circ g)(x) = \sqrt{1 - \dfrac{1}{2-x}}$, Domain: $(-\infty, 1] \cup (2, \infty)$.

$(g \circ f)(-8) = -1$, $(f \circ g)\left(\dfrac{1}{2}\right) = \dfrac{\sqrt{3}}{3}$.

Question 2:

A) Let $f(x) = x^2$, $g(x) = \sqrt{x}$. Find:

1) $(f \circ g)(x)$.

2) D_f, D_g, and $D_{f \circ g}$.

Solution

$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$

$D_f = R$

$D_g = [0, \infty)$

$D_{(f \circ g)} = [0, \infty)$

Question 2:

A) Let $f(x) = \dfrac{3}{\sqrt{x-4}}$, $g(x) = x^2 + 4$. Find:

1) $(f \circ g)(x)$.

2) D_f, D_g, and $D_{f \circ g}$.

Solution

$(f \circ g)(x) = f(g(x)) = f(x^2 + 4) = \dfrac{3}{\sqrt{x^2+4-4}} = \dfrac{3}{\sqrt{x^2}} = \dfrac{3}{|x|}$

$D_f = (4, \infty)$

$D_g = R$

$D_{(f \circ g)} = R - \{0\}$

Question 2:

A) Let $f(x) = \dfrac{7}{4 - x^2}$, $g(x) = \sqrt{x}$. Find:

1) $(f \circ g)(x)$.

2) D_f, D_g, and $D_{f \circ g}$.

Solution

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x}) = \frac{7}{4 - (\sqrt{x})^2} = \frac{7}{4 - x}$$

$D_f = R - \{-2 , 2\}$

$D_g = [\,0 , \infty\,)$

$D_{(f \circ g)} = [\,0 , \infty\,) - \{4\}$

EXERCISES 1.3

In Exercises 1 – 12, find the domain of each function

1. $f(x) = x^3 - 4x + 1$
2. $f(x) = \sqrt{1 - 7x}$
3. $f(x) = \sqrt[3]{2x^2 - 3x + 1}$
4. $f(x) = \dfrac{1}{\sqrt{x - 5}}$
5. $f(x) = \sqrt{\dfrac{2x + 1}{x + 2}}$
6. $f(t) = \sqrt{3 - \dfrac{1}{t^2}}$
7. $g(w) = \dfrac{2}{w - 1}$
8. $g(t) = \dfrac{2t - 8}{t^2 - 16}$
9. $g(r) = \dfrac{r - 1}{r^2 - r - 6}$
10. $f(x) = \dfrac{2x^4 - 3x + 1}{|2x - 4| + 1}$
11. $f(x) = \dfrac{2x - 5}{|x + 1| - 3}$
12. $f(x) = \dfrac{3x^2 - x + 4}{\sqrt{2x - 4} - 3}$

In Exercises 12-18, determine all intercepts of the graph of the equation. Then decide whether the graph is symmetric with respect to the x –axis, the y – axis, or the origin.

13. $x = 3y^2 - 2$
14. $x^2 - y^2 = 1$
15. $x^4 = 3y^3$
16. $x^2y^4 - 2x^4 = 1$
17. $y = x - \dfrac{1}{x}$
18. $y = \sqrt{9 - x^2}$

In Exercises 19-21, List the intercepts and describe the symmetry (if any) of the graph.

19. $y = \dfrac{1}{3}x$
20. $2x = -y^2$
21. $y = x^3 - 3$

In Exercises 22-27, determine which of the following functions are odd, even, or neither

22. $f(x) = 5x^3 - 3$
23. $f(x) = (x - 2)^2$
24. $f(x) = \dfrac{x}{x^2 + 4}$
25. $f(x) = (x^2 + 2)^3$
26. $y = \dfrac{|x|}{x}$
27. $x(x^2 + 1)^3$

In Exercises 28 – 33, determine the intervals on which each of the following functions are increasing and the intervals on which they are decreasing.

28. $f(x) = 1 - 3x$
29. $f(x) = 4$
30. $f(x) = x^2 - 8$
31. $f(x) = 2 - x^2$
32. $f(x) = x^3$
33. $f(x) = -x^3$

In Exercises 34-42, let $f(x) = x^2 + 4x - 2$ and $g(x) = 2 - x^2$. Find the specified values

34. $(f + g)(-1)$
35. $(f - g)(2)$
36. $(f - g)(a), a \in \mathbb{R}$
37. $(f \cdot g)(0)$
38. $\left(\dfrac{f}{g}\right)(1)$
39. $(f \circ g)(3)$
40. $(g \circ f)(3)$
41. $(f \circ f)(-2)$
42. $(g \circ g)(2)$

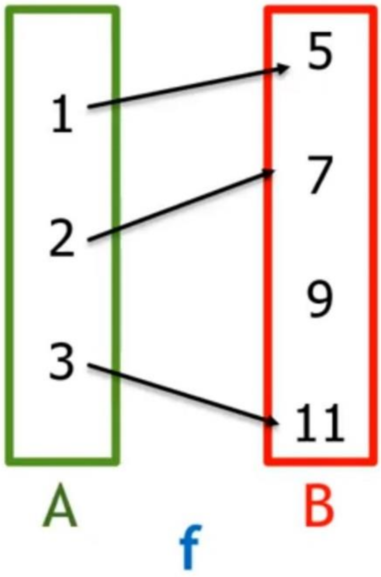
In Exercises 43 –48, find $f + g, f \cdot g$, and $\dfrac{f}{g}$ and their domains.

43. $f(x) = 2x + 1;\ g(x) = 3 - x$
44. $f(x) = x - 2;\ g(x) = x^2 - 2$

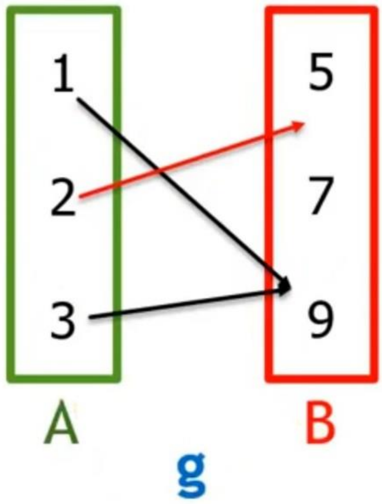
SECTION 1.4 INVERSE FUNCTIONS

معكوس الدوال

One-to-One Function - الدالة المتباين



one-to-one



not one-to-one

DEFINITION 1.4.1 (One-to-One Function) التطبيق المتباين

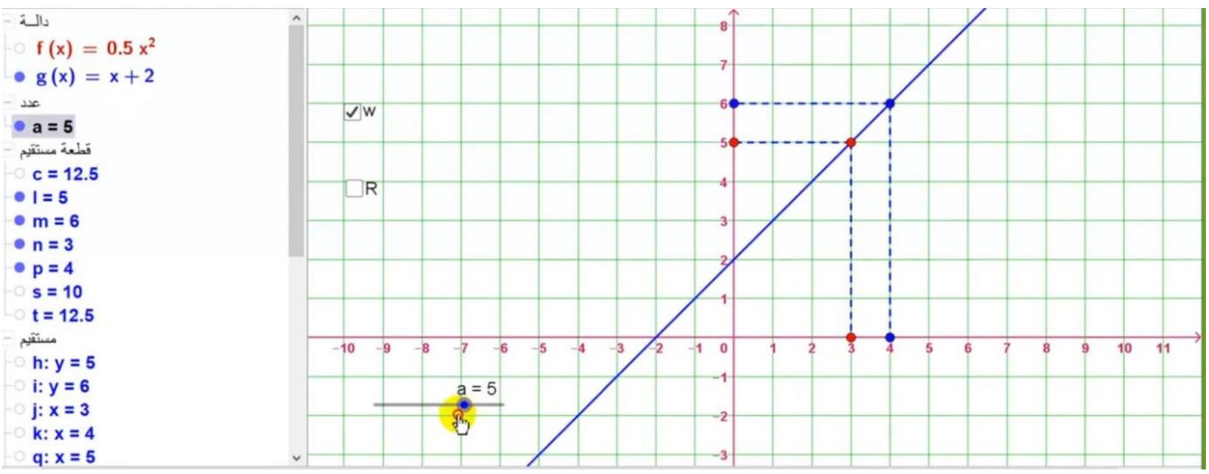
A function f is said to be ***one-to-one*** (often written $1-1$) if every element in its range corresponds to exactly one element in its domain.

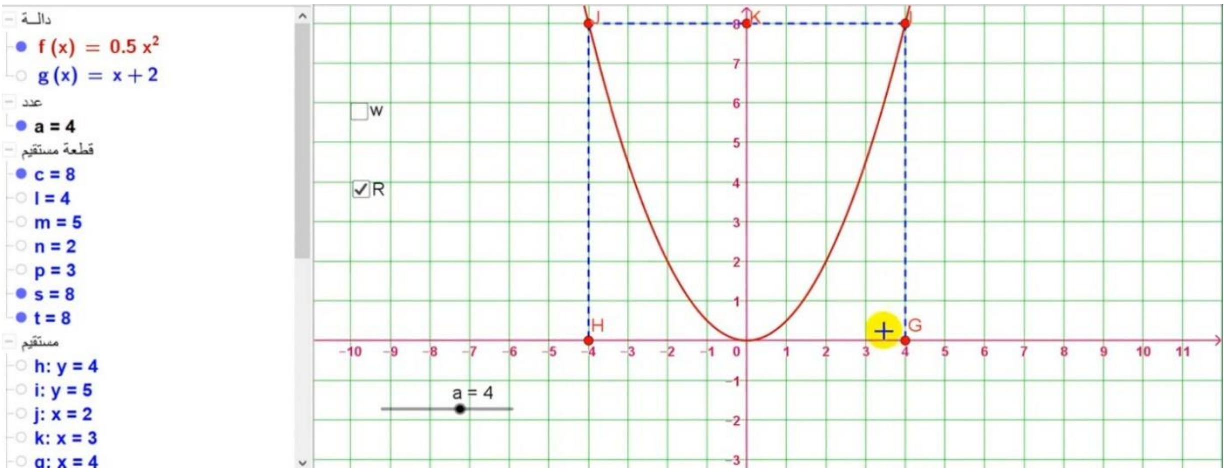
That is, for all x_1 and x_2 in the domain of f

$$\text{if } x_1 \neq x_2 \text{, then } f(x_1) \neq f(x_2),$$

which is equivalent to, for all x_1 and x_2 in the domain of f

$$\text{if } f(x_1) = f(x_2) \text{, then } x_1 = x_2 .$$





EXAMPLE 1.4.1 Show that $f(x) = 2x + 5$ is a one-to-one function.

Solution

$$f(x_1) = f(x_2)$$

$$2x_1 + 5 = 2x_2 + 5$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

one to one

REMARK one to one ملاحظة : الدالة الخطية

In fact all linear functions are one to one, because the linear function can be written in the form $f(x) = ax + b$ and if $f(x_1) = f(x_2)$, then

$$ax_1 + b = ax_2 + b\,,$$

Simplifying the two terms, we have

$$x_1 = x_2\,.$$

EXAMPLE 1.4.2 Determine whether each of the following functions is one to one.

a. $f(x) = 2x^2 + 1$

Solution

$$f(x_1) = f(x_2)$$

$$2x_1^2 + 1 = 2x_2^2 + 1$$

$$2x_1^2 = 2x_2^2$$

$$x_1^2 = x_2^2$$

$$\sqrt{x_1^2} = \sqrt{x_2^2}$$

$$|x_1| = |x_2|$$

$$x_1 = \pm x_2$$

Not one to one

EXAMPLE 1.4.2 Determine whether each of the following functions is one to one.

b. $f(x) = x^2 + 1, \ x \geq 0$

Solution

$$f(x_1) = f(x_2)$$

$$x_1^2 + 1 = x_2^2 + 1$$

$$x_1^2 = x_2^2$$

$$\sqrt{x_1^2} = \sqrt{x_2^2}$$

$$|x_1| = |x_2|$$

$x \geq 0$ $\quad$ $x_1 = x_2$

one to one

EXAMPLE 1.4.2 Determine whether each of the following functions is one to one.

c. $f(x) = 2 + \sqrt[3]{2x + 1}$

Solution

$$f(x_1) = f(x_2)$$

$$2 + \sqrt[3]{2x_1 + 1} = 2 + \sqrt[3]{2x_2 + 1}$$

$$\sqrt[3]{2x_1 + 1} = \sqrt[3]{2x_2 + 1}$$

$$2x_1 + 1 = 2x_2 + 1$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

one to one

B) Show that $f(x) = x^2 - 4x - 5, \ x > 2$ is a one-to-one function.

Solution

$$f(x_1) = f(x_2)$$

$$x_1^2 - 4x_1 - 5 = x_2^2 - 4x_2 - 5$$

$$x_1^2 - 4x_1 = x_2^2 - 4x_2$$

$$x_1^2 - 4x_1 + 4 = x_2^2 - 4x_2 + 4$$

$$(x_1 - 2)^2 = (x_2 - 2)^2$$

$$\sqrt{(x_1 - 2)^2} = \sqrt{(x_2 - 2)^2}$$

$$|x_1 - 2| = |x_2 - 2|$$

$x > 2$ $\quad$ $x_1 - 2 = x_2 - 2$

$$x_1 = x_2$$

one to one

RELATED PROBLEM 2 Determine whether each of the following functions is one to one.

a. $f(x) = 1 - 3x^2$ **b.** $f(x) = x^2 + 2x - 1,\ x \geq -1$ **c.** $f(x) = 6 + \sqrt[5]{7x+2}$

Answer

a. Not one-to-one **b.** One-to-one **c.** One-to-one

Let $f(x) = \dfrac{2x+1}{x-1}$,

a. Show that $f(x)$ is one-to-one function on its domain.

Solution

$$f(x_1) = f(x_2)$$

$$\frac{2x_1+1}{x_1-1} = \frac{2x_2+1}{x_2-1}$$

$$2x_1x_2 + x_2 - 2x_1 - 1 = 2x_2x_1 + x_1 - 2x_2 - 1$$

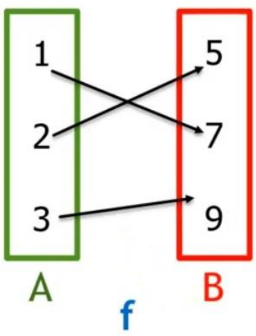
$$x_2 - 2x_1 = x_1 - 2x_2$$

$$3x_2 = 3x_1$$

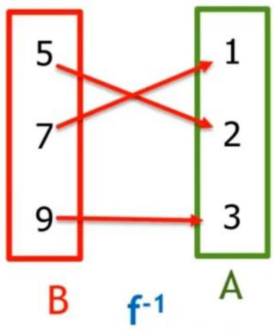
$$x_2 = x_1$$

one to one

Inverse Function - الدالة العكسية



one-to-one



$f^{-1}(f(1)) = f^{-1}(7) = 1$ $f^{-1}(f(x)) = x$ $(f^{-1} \circ f)(x)) = x$

$f(f^{-1}(7)) = f(1) = 7$ $f(f^{-1}(x)) = x$ $(f \circ f^{-1})(x) = x$

DEFINITION 1.4.2 (Inverse Function)

If f is a one-to-one function, then there is a function f^{-1}, called the inverse of f, such that $y = f(x)$ if and only if $x = f^{-1}(y)$. The domain of f^{-1} is the range of f and the range of f^{-1} is the domain of f.

THEOREM 1.4.1

If f is a one-to-one function and if f^{-1} is its inverse function, then f^{-1} is a one-to–one having f as its inverse. Furthermore

$$f^{-1}(f(x)) = x \quad \text{for } x \text{ in the domain of } f$$

and

$$f(f^{-1}(x)) = x \quad \text{for } x \text{ in the domain of } f^{-1}$$

EXAMPLE 1.4.3 Determine whether the functions f and g are inverses of each other.

a. $f(x) = 2x + 1$ and $g(x) = \frac{x-1}{2}$.

Solution

$$(f \circ g)(x) = f(g(x)) = f(\frac{x-1}{2}) = 2\,\frac{x-1}{2} + 1 = x$$

$$(g \circ f)(x) = g(f(x)) = g(2x+1) = \frac{2x+1-1}{2} = x$$

Then f is the inverse of g and vice versa

b. $f(x) = 2x$ and $g(x) = x + 1$.

Solution

$$(f \circ g)(x) = f(g(x)) = f(x+1) = 2(x+1) = 2x+2 \neq x$$

then f and g are not inverses of each other.

To obtain f^{-1}, we follow the next procedure | للحصول على f^{-1} نتبع الخطوات التالية

Step 1: We prove *f* is one - to – one. This proves that f^{-1} exists.

Step 2: Substitute *y* for *f*(*x*) and solve the resulting equation for *x*. This gives the equation $x = f^{-1}(y)$

Step 3: We obtain $f^{-1}(x)$ from the definition of $f^{-1}(y)$.

EXAMPLE 1.4.4 Find f^{-1} for the function $f(x) = 2x + 5$.

Solution $f(x) = 2x + 5$ is one-to-one

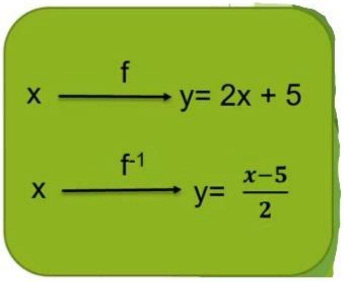
$$y = 2x + 5$$

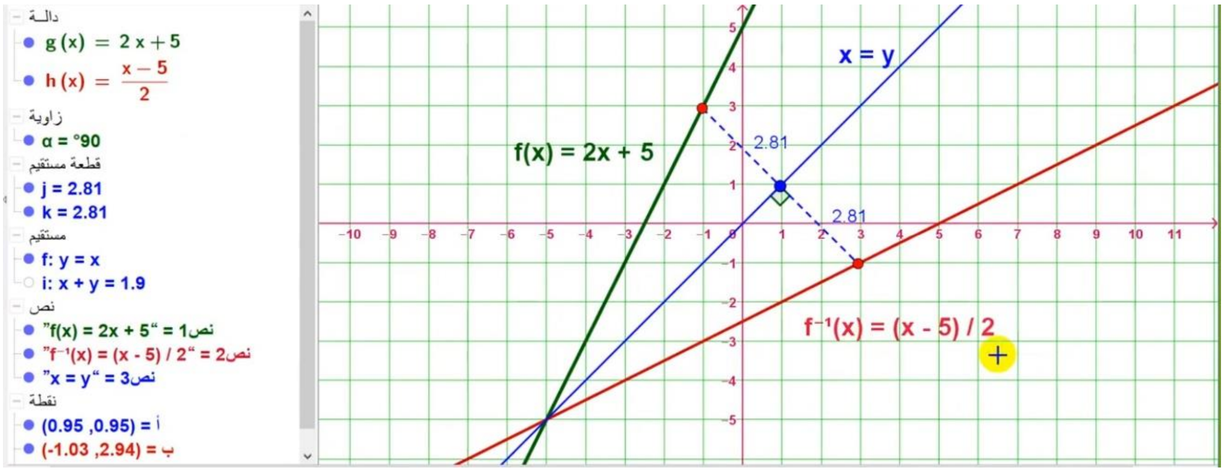
$$y - 5 = 2x$$

$$x = \frac{y-5}{2}$$

$$y = \frac{x-5}{2}$$

$$f^{-1}(x) = \frac{x-5}{2}$$





RELATED PROBLEM 4 Assume that the following two functions are 1 -1. Find their inverse functions.

a. $f(x) = \left(2x^3 - 5\right)^{1/7}$.

Solution $y = \left(2x^3 - 5\right)^{1/7}$

$$y^7 = 2x^3 - 5$$

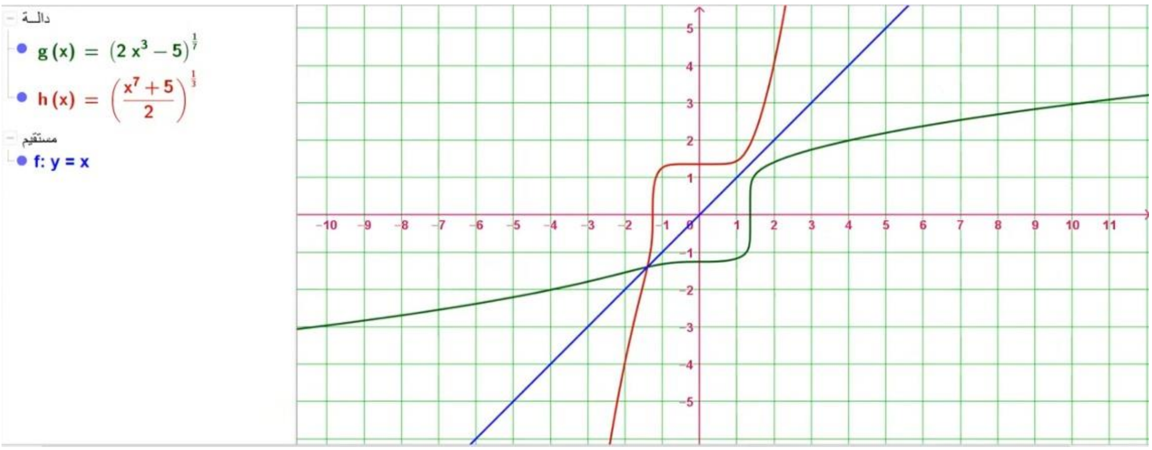
$$y^7 + 5 = 2x^3$$

$$\frac{y^7+5}{2} = x^3$$

$$x = \sqrt[3]{\frac{y^7+5}{2}}$$

$$y = \sqrt[3]{\frac{x^7+5}{2}}$$

$$f^{-1}(x) = \sqrt[3]{\frac{x^7+5}{2}}$$



B) Show that $f(x) = 3x + 2$ is a one-to-one function, and find $f^{-1}(x)$.

Solution

$$y = 3x+2$$

$$\frac{y-2}{3} = x$$

$$y = \frac{x-2}{3}$$

$$f^{-1}(x) = \frac{x-2}{3}$$

B) Given that $f(x) = \dfrac{1-2x}{3x+2}$ is a one-to-one function, find $f^{-1}(x)$.

معطى

$$y = \frac{1-2x}{3x+2}$$

$$3xy+2y = 1-2x$$

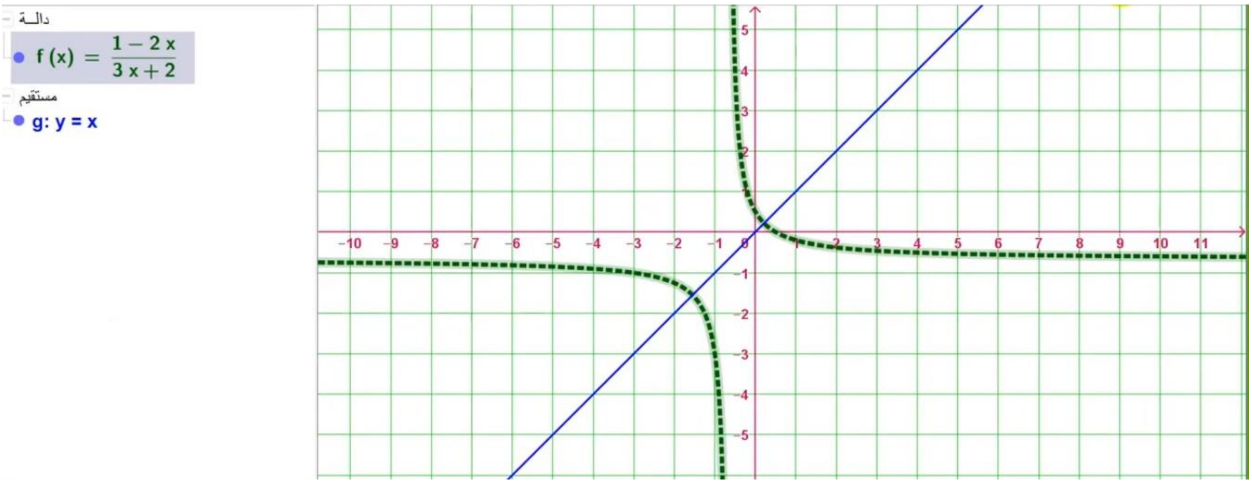
$$3xy+2x = 1-2y$$

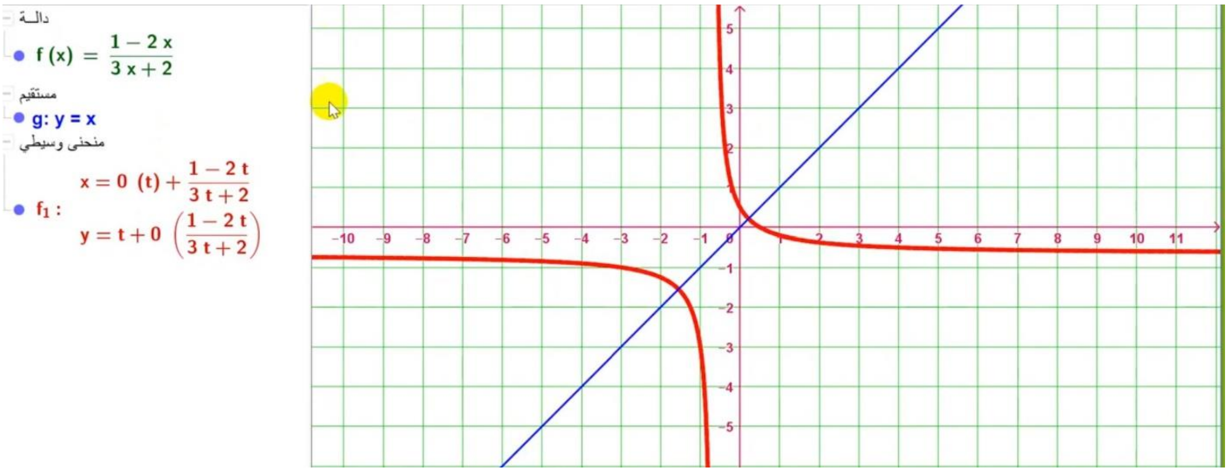
$$x(3y+2) = 1-2y$$

$$x = \frac{1-2y}{3y+2}$$

$$y = \frac{1-2x}{3x+2}$$

$$f^{-1}(x) = \frac{1-2x}{3x+2}$$

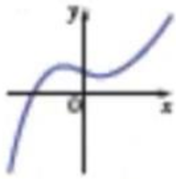

دالـــة
f (x) = (1 − 2 x)/(3 x + 2)
مستقيم
g: y = x


دالـــة
f (x) = (1 − 2 x)/(3 x + 2)
مستقيم
g: y = x
منحنى وسيطي
x = 0 (t) + (1 − 2 t)/(3 t + 2)
f1 :
y = t + 0 ((1 − 2 t)/(3 t + 2))

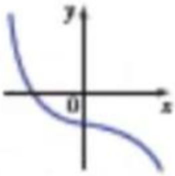
EXERCISES 1.4

In Exercises 1-5, Use the horizontal line test to determine whether the given function is one-to-one.

1.



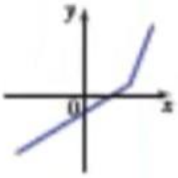
2.



3.

4.

5.



In Exercises 6-16, Determine whether the given function is one-to-one. If it is one-to-one, find its inverse.

6. $f = \{(12,2),(15,4),(19,-1),(25,6),(78,0)\}$
7. $g = \{(-1,2),(0,4),(9,-4),(18,6),(23,-4)\}$
8. $h(x) = x^3 + 2$.
9. $I(x) = \dfrac{1}{2x-4},\ x \neq 2$.
10. $J(x) = -5x + \dfrac{5}{3}$.
11. $K(x) = |5x - 4|$.
12. $f(x) = -\dfrac{11}{x+3},\ x \neq -3$.
13. $f(x) = \sqrt{x+5}$.
14. $f(x) = x\sqrt{9-x^2}$.
15. $g(x) = \sqrt[3]{x} + 4$
16. $g(x) = 2 - (3-x)^{1/3}$

In Exercises 17-22, Assume the functions are one-to-one. Find the requested inverse.

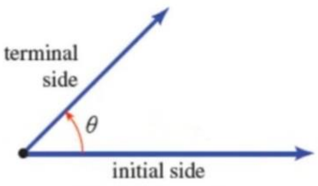
17. If $f(4) = 3$, find $f^{-1}(3)$.
18. If $f(2) = 4$, find $f^{-1}(4)$.
19. If $g(-5) = 6$, find $g^{-1}(6)$.

Section 1.5

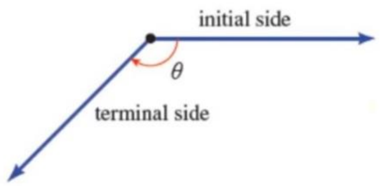
TRIGONOMETRIC FUNCTIONS الدوال المثلثية

DEGREES/RADIANS CONVERSION FACTORS - التحويل بين الدرجات و الراديان

ANGLES

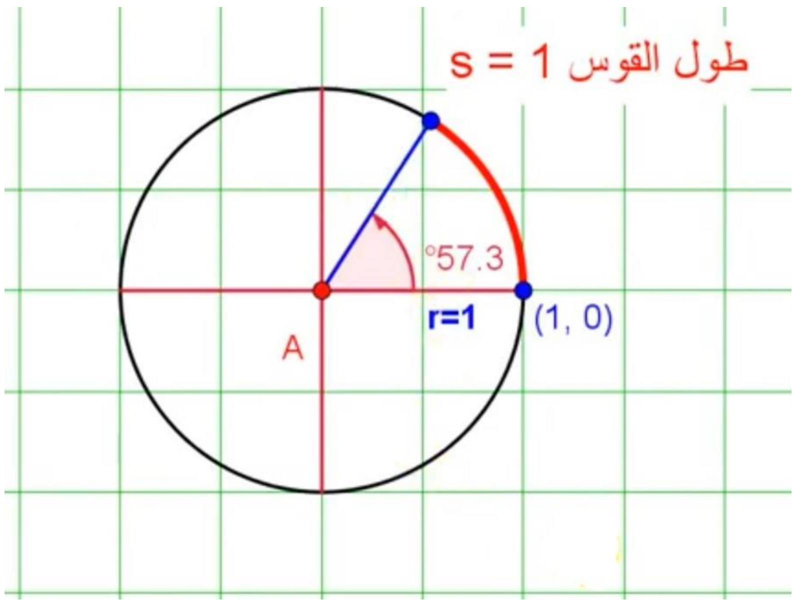


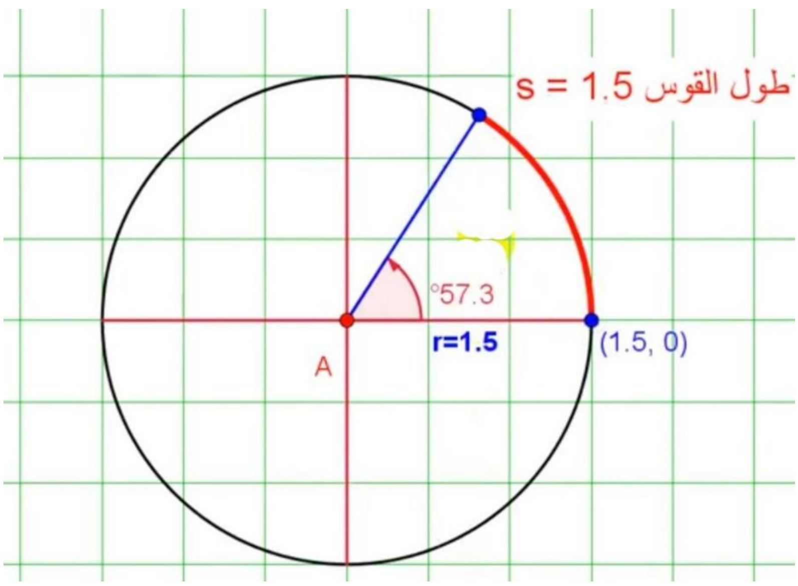
Positive angle

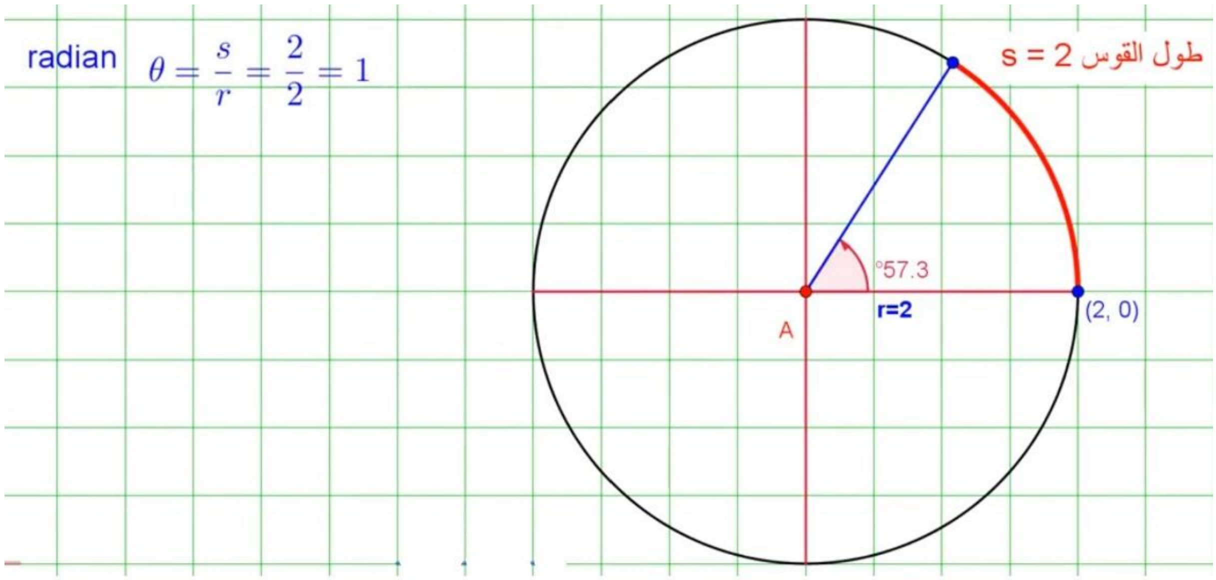


Negative angle

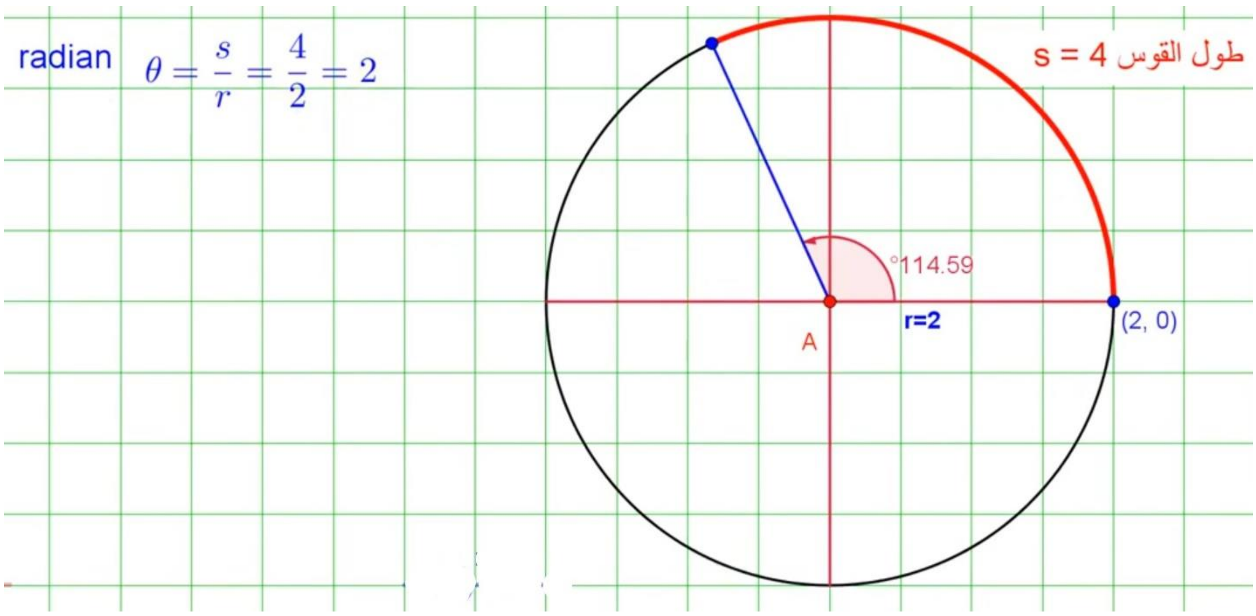
Radian Measured القياس بالراديان



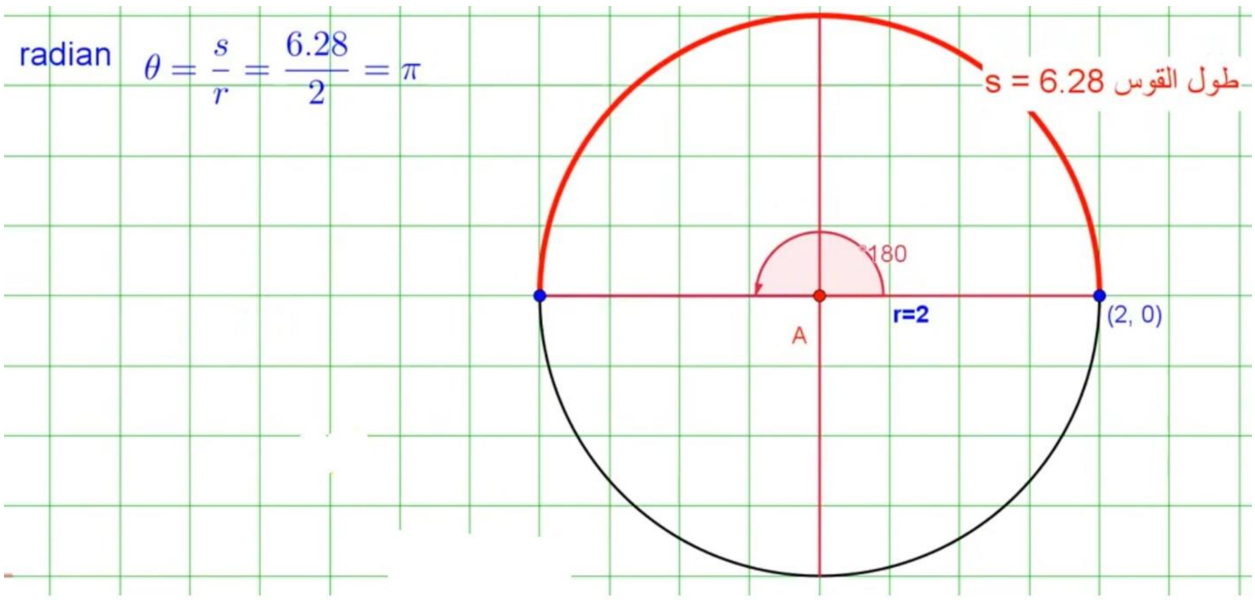




radian
$\theta = \frac{s}{r} = \frac{2}{2} = 1$
طول القوس s = 2
°57.3
r=2
(2, 0)
A



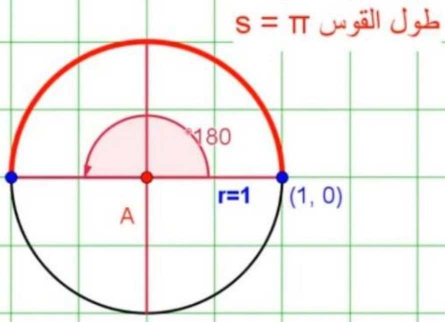
radian
$\theta = \frac{s}{r} = \frac{4}{2} = 2$
طول القوس s = 4
°114.59
r=2
(2, 0)
A



radian
$\theta = \frac{s}{r} = \frac{6.28}{2} = \pi$
طول القوس s = 6.28
°180
r=2
(2, 0)
A

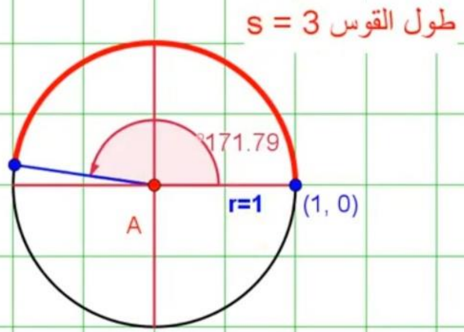
radian $\theta = \frac{s}{r} = \frac{\pi}{1} = \pi$

π = 180°



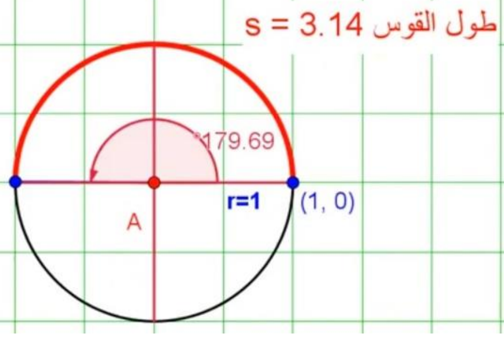
radian $\theta = \frac{s}{r} = \frac{3}{1} = 3$

π = 180°



radian $\theta = \frac{s}{r} = \frac{3.14}{1} = 3.14$

π = 180°



radian $\theta = \frac{s}{r} = \frac{\pi}{1} = \pi$

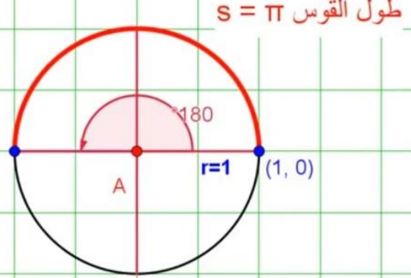
π = 180°

$1 radian = \frac{180°}{\pi} degrees$

$\frac{\pi}{4} radian = \frac{\pi}{4} . \frac{180°}{\pi} = \frac{180°}{4} = 45°$

$1 degrees = \frac{\pi}{180°} radian$

$60° = 60° . \frac{\pi}{180°} = \frac{\pi}{3} radians$



EXAMPLE 1.5.1 Convert the following degree measures to radians.

a. $75°$ **b.** $-225°$

Solution

a. $75° = 75 \times \frac{\pi}{180} = \frac{5\pi}{12}$ radians.

b. $-225° = (-225) \times \frac{\pi}{180} = -\frac{5\pi}{4}$ radians.

RELATED PROBLEM 1 Convert the following degree measures to radians.

a. $60°$ **b.** $-200°$

Answers

a. $\frac{\pi}{3}$ **b.** $-\frac{10\pi}{9}$

EXAMPLE 1.5.2 Convert the following radian measures to degrees.

a. $\frac{5\pi}{9}$ **b.** $\frac{17\pi}{36}$

Solution

a. $\frac{5\pi}{9}$ radian $= \frac{5\pi}{9} \times \frac{180}{\pi} = 100°$. **b.** $\frac{17\pi}{36}$ radian $= \frac{17\pi}{36} \times \frac{180}{\pi} = 85°$

RELATED PROBLEM 2 Convert the following radian measures to degrees.

a. $\frac{\pi}{10}$ **b.** $-\frac{13\pi}{12}$

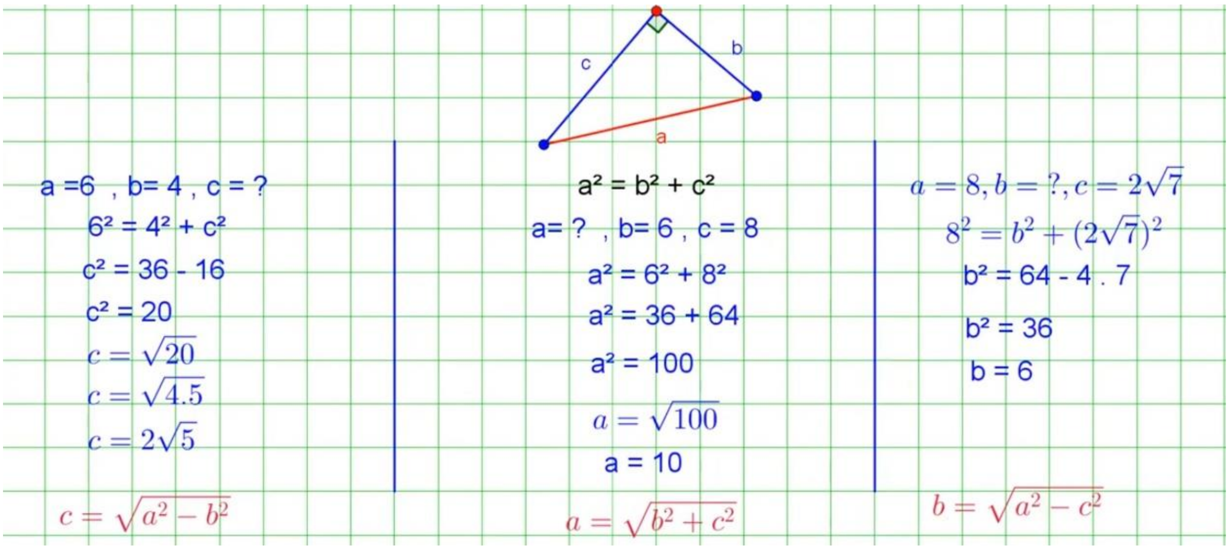
Answers

a. $18°$ **b.** $-195°$

Degrees	0	30^0	45^0	60^0	90^0	120^0	135^0	150^0	180^0	270^0	360^0
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

TRIGONOMETRIC FUNCTIONS – الدوال المثلثية

تذكير نظرية فيثاغورس

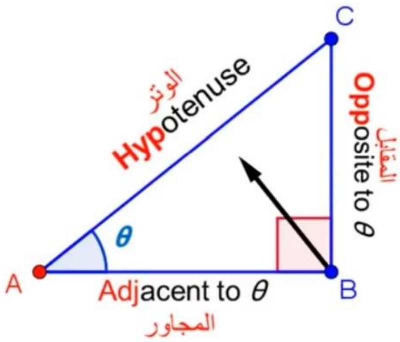


If θ is an *acute* angle $\left(0 < \theta < \frac{\pi}{2}\right)$

$\sin\theta = \frac{\text{المقابل}}{\text{الوتر}}$

$\cos\theta = \frac{\text{المجاور}}{\text{الوتر}}$

$\tan\theta = \frac{\text{المقابل}}{\text{المجاور}}$



$\csc\theta = \frac{\text{الوتر}}{\text{المقابل}}$

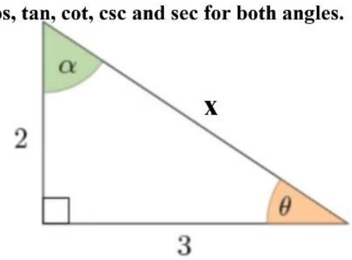
$\sec\theta = \frac{\text{الوتر}}{\text{المجاور}}$

$\cot\theta = \frac{\text{المجاور}}{\text{المقابل}}$

EXAMPLE **Given the figure below; Find x, sin, cos, tan, cot, csc and sec for both angles.**

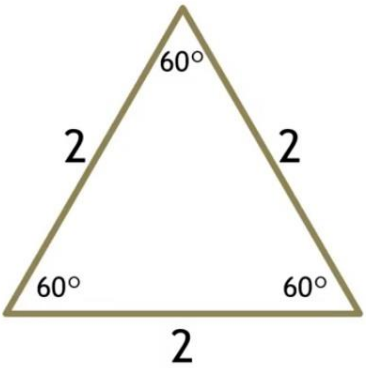
$x^2 = 4 + 9$

$x = \sqrt{13}$

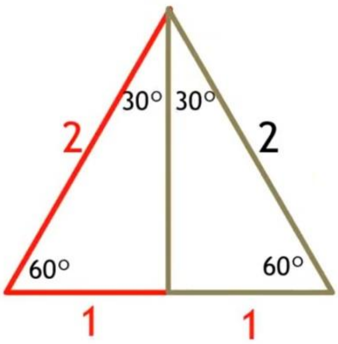


Answers

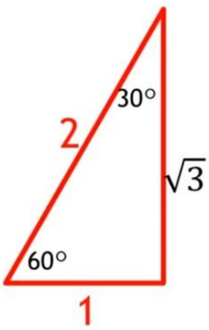
	sin	cos	tan	cot	csc	sec
α	$\frac{3}{\sqrt{13}}$	$\frac{2}{\sqrt{13}}$	$\frac{3}{2}$	$\frac{2}{3}$	$\frac{\sqrt{13}}{3}$	$\frac{\sqrt{13}}{2}$
θ	$\frac{2}{\sqrt{13}}$	$\frac{3}{\sqrt{13}}$	$\frac{2}{3}$	$\frac{3}{2}$	$\frac{\sqrt{13}}{2}$	$\frac{\sqrt{13}}{3}$

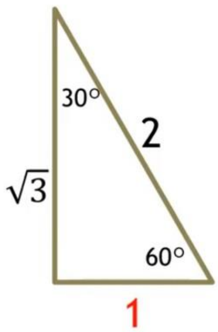


	sin	cos	tan	cot	csc	sec
30°						
60°						

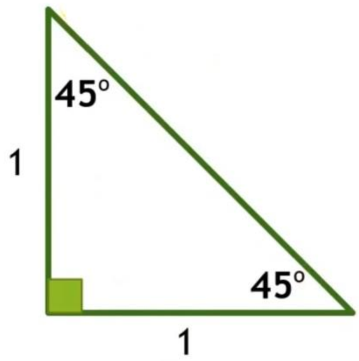


	sin	cos	tan	cot	csc	sec
30°						
60°						

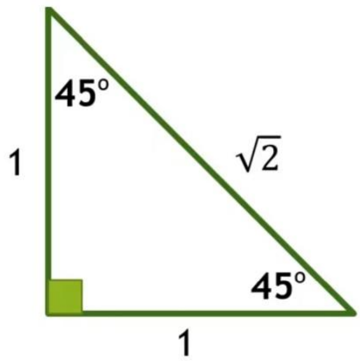




	sin	cos	tan	cot	csc	sec
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	2	$\frac{2\sqrt{3}}{3}$
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$	$\frac{2\sqrt{3}}{3}$	2



	sin	cos	tan	cot	csc	sec
45°						



	sin	cos	tan	cot	csc	sec
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$

trigonometric functions using the unit circle– الدوال المثلثية باستخدام دائرة الوحدة

DEFINITION 1.5.3

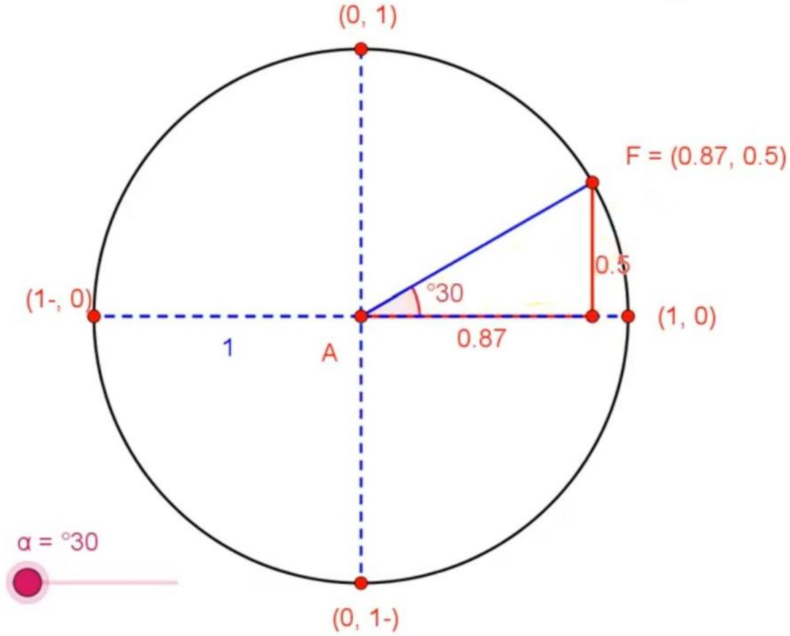
a. Let $x^2 + y^2 = r^2$ be a circle centered at the origin and an angle with θ radians in standard form. If P is the point (x, y) as shown in Figure 1.5.11, then the trigonometric functions are defined by:

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}, y \neq 0$$

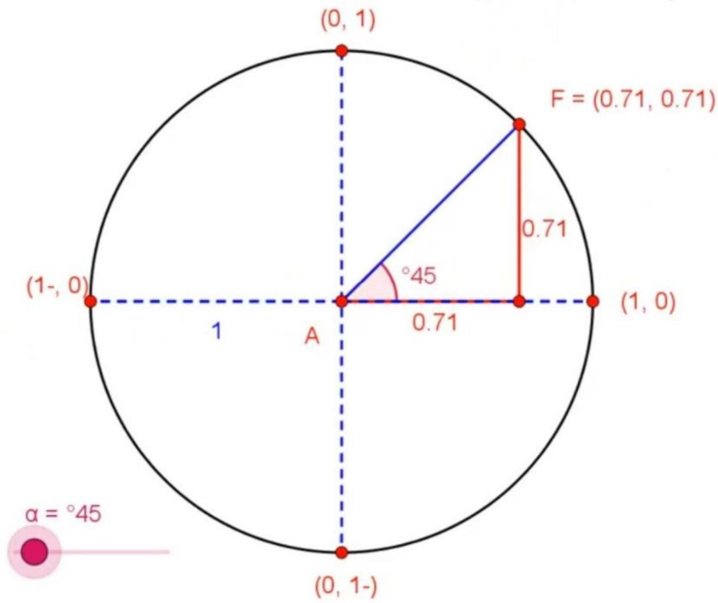
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}, x \neq 0$$

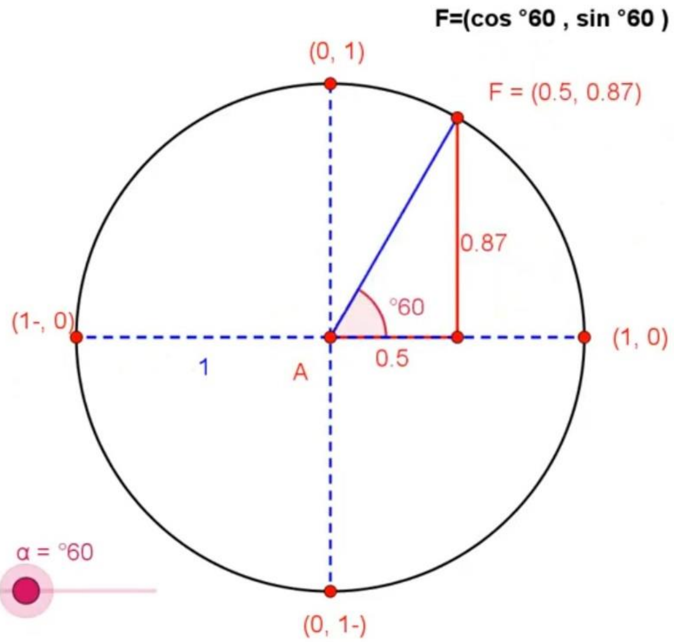
$$\tan\theta = \frac{y}{x}, x \neq 0 \qquad \cot\theta = \frac{x}{y}, y \neq 0$$

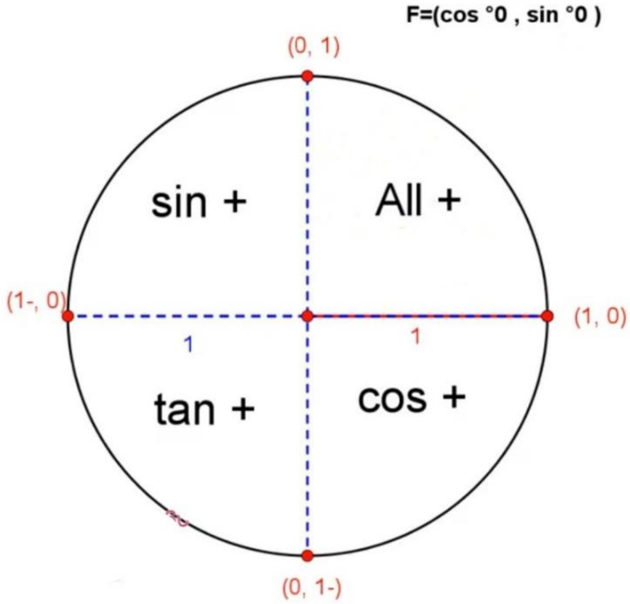
F=(cos °30 , sin °30)



F=(cos °45 , sin °45)

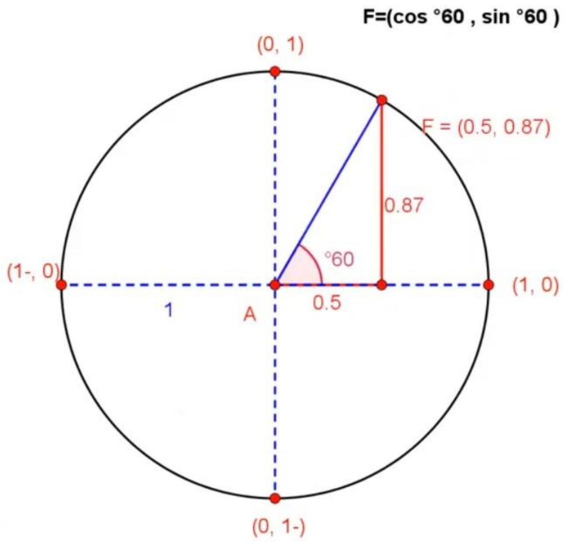






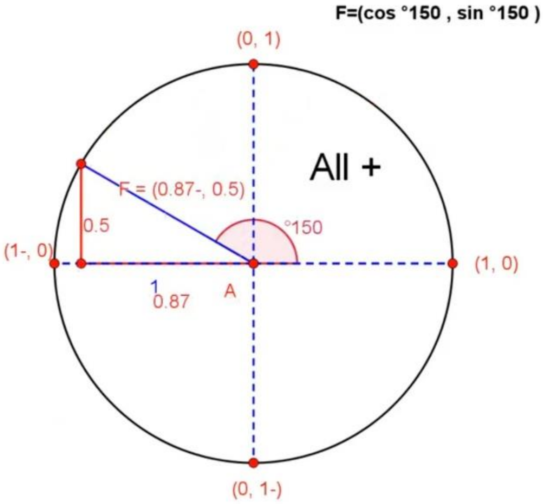
$$x^2 + y^2 = (0.5)^2 + (0.87)^2$$

$$= 0.25 + 0.75 = 1$$



$x^2 + y^2 = (\ 0.87-)^2+(0.5)^2$

$= 0.75 + 0.25 = 1$

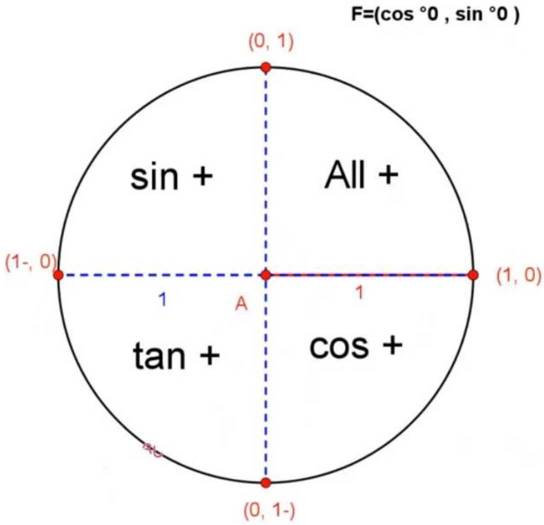


$x^2 + y^2 = (\ 1)^2+(0)^2$

$= 1 + 0 = 1$

$x^2 + y^2 = 1$

$\sin^2\theta + \cos^2\theta = 1$



$r = 3$

$x^2 + y^2 = (\ 1.5\ )^2 + (\ 2.6\ )^2$

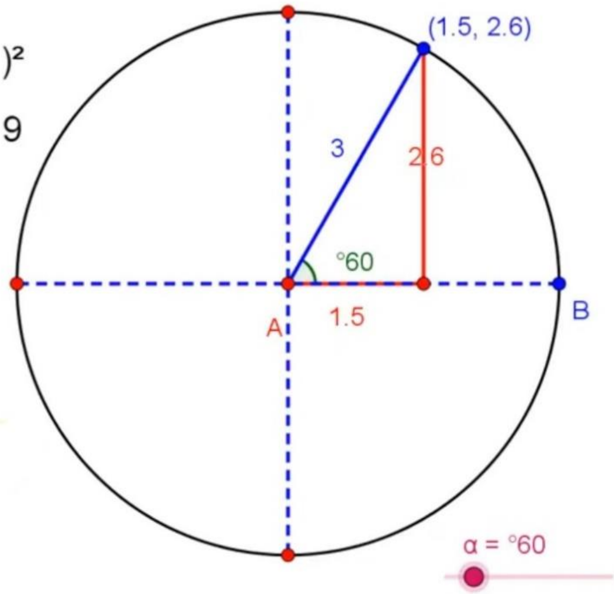
$= 2.25 + 6.75 = 9$

$x^2 + y^2 = r^2$

$sin°60 = \frac{y}{r} = \frac{2.6}{3}$

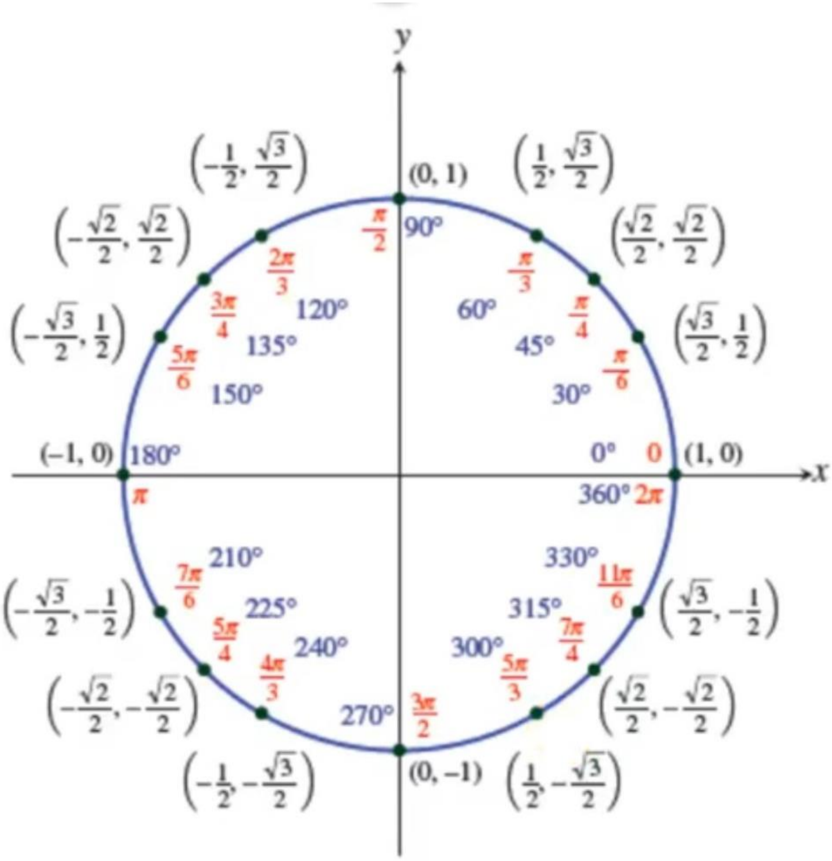
$cos°60 = \frac{x}{r} = \frac{1.5}{3}$

$tan°60 = \frac{y}{x} = \frac{2.6}{1.5}$



VALUES OF SINE AND COSINE

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	0	$\frac{1}{2}$	1



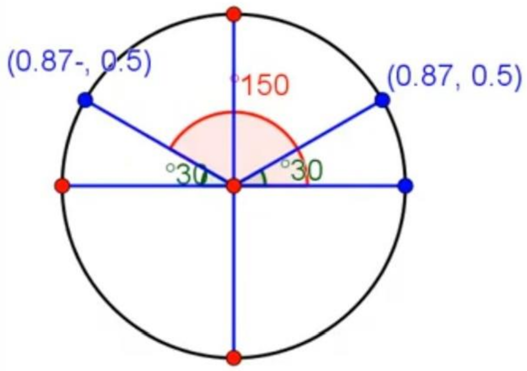
EXAMPLE 1.5.4 If θ is in standard position and $Q(4,-3)$ is on the terminal side of θ. Use Definition 1.5.3 to find the values of all six trigonometric functions for θ.

Solution Notice that the point is on a circle of radius $r=\sqrt{16+9}=5$. Thus, we obtain

$$\sin\theta=\frac{-3}{5},\quad \cos\theta=\frac{4}{5},\quad \tan\theta=\frac{-3}{4},$$

$$\csc\theta=\frac{5}{-3},\quad \sec\theta=\frac{5}{4},\quad \cot\theta=\frac{4}{-3}.$$

VALUES OF SINE AND COSINE - قيم الجيب و الجيب تمام



$$sin°150 = sin(\pi -° 150)$$

$$= \sin°30 = 0.5$$

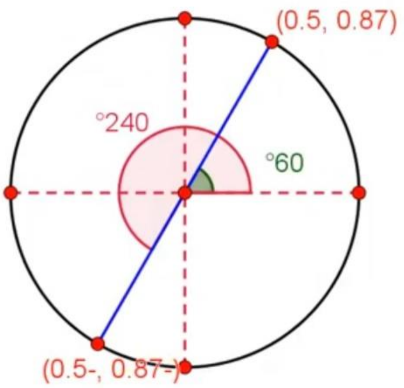
$$sin\theta = sin(\pi - \theta)$$

$$cos°150 = -cos(\pi -° 150)$$

$$= -\cos°30 = -0.87$$

$$cos\theta = -cos(\pi - \theta)$$

$$tan\theta = -(\pi - \theta)$$



$$\sin°240 = -\sin(°240 - \pi)$$

$$= - \sin °60 = -0.87$$

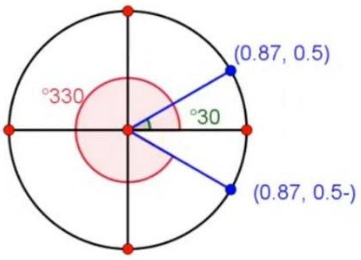
$$sin\theta = -sin(\theta - \pi)$$

$$\cos °240 = - \cos(°240 - \pi)$$

$$= - \cos °60 = -0.5$$

$$\cos\theta = -\cos(\theta - \pi)$$

$$\tan \theta = \tan (\theta - \pi)$$



$$sin^{\circ}330 = -sin(2\pi -^{\circ} 330)$$

$= - \sin {}^{\circ}30 = -0.5$

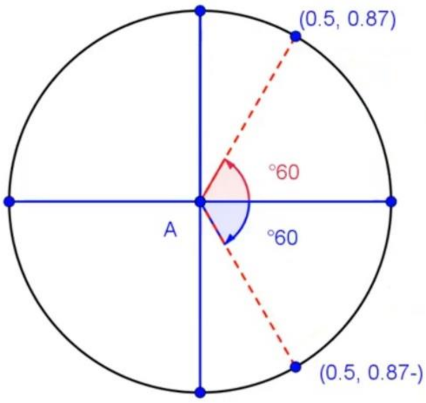
$$\sin\theta = -\sin(2\pi - \theta)$$

$$cos^{\circ}330 = cos(2\pi -^{\circ} 330)$$

$= \cos^{\circ}30 = 0.87$

$$\cos\theta = \cos(2\pi - \theta)$$

$$\tan\theta = -\tan(2\pi - \theta)$$



$$\sin(-^{\circ}60) = -\sin^{\circ}60 = -0.87$$

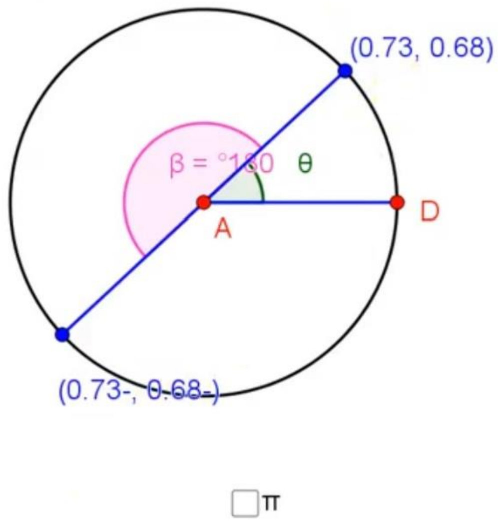
$$\sin(-\theta) = -\sin\theta$$

$\cos(-^{\circ}60) = \cos {}^{\circ}60 = 0.5$

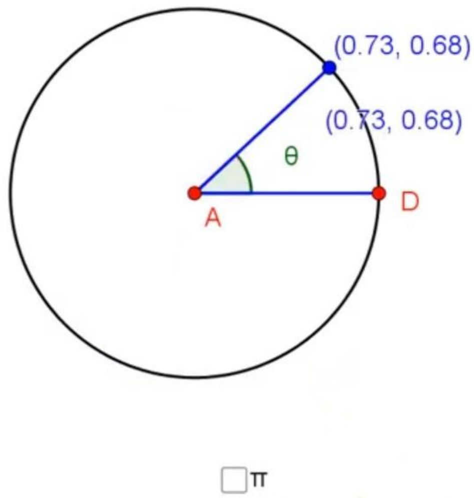
$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

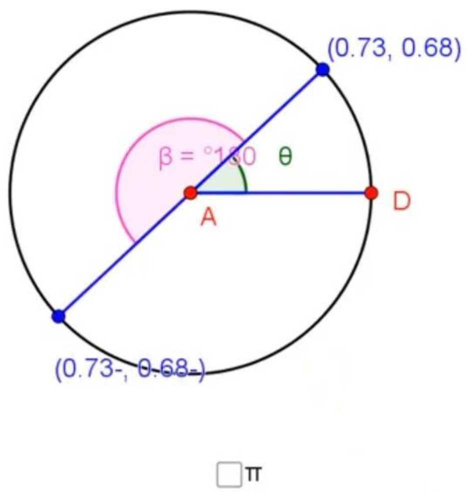
tan (θ +1 π) = tan θ



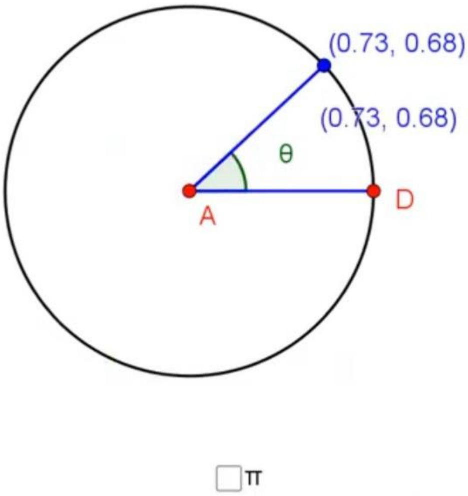
tan (θ +2 π) = tan θ



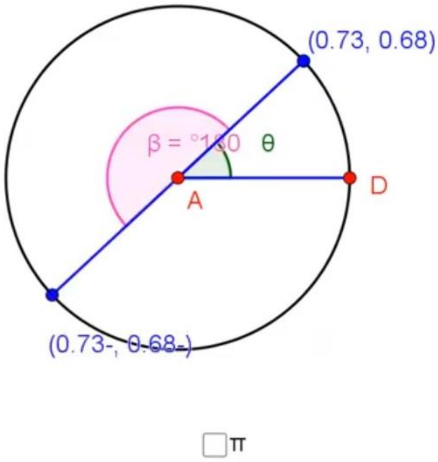
tan (θ +3 π) = tan θ



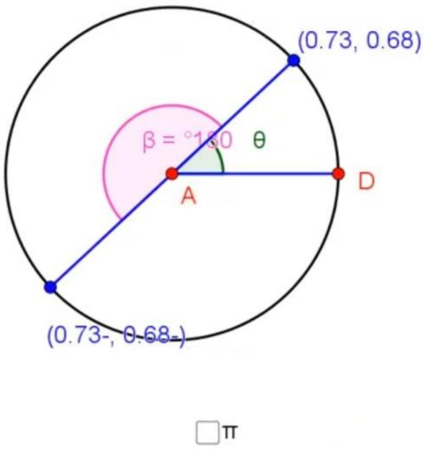
$\tan(\theta + 4\pi) = \tan\theta$



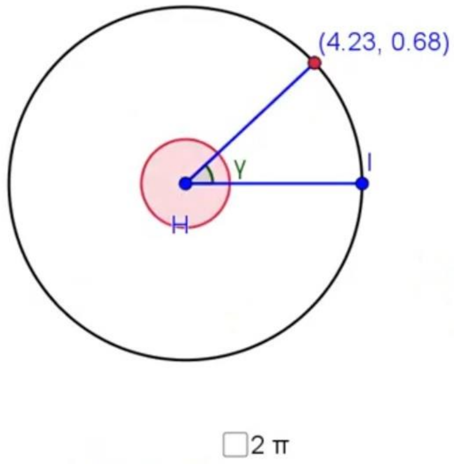
$\tan(\theta + 5\pi) = \tan\theta$



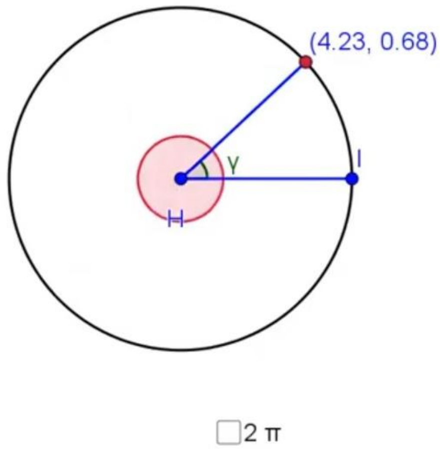
$\tan(\theta + 9\pi) = \tan\theta$



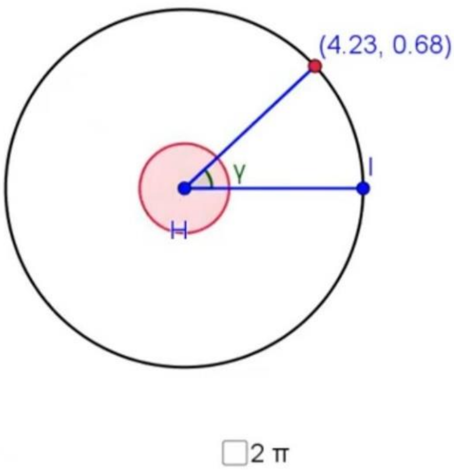
sin(θ+1.2π)=sin θ



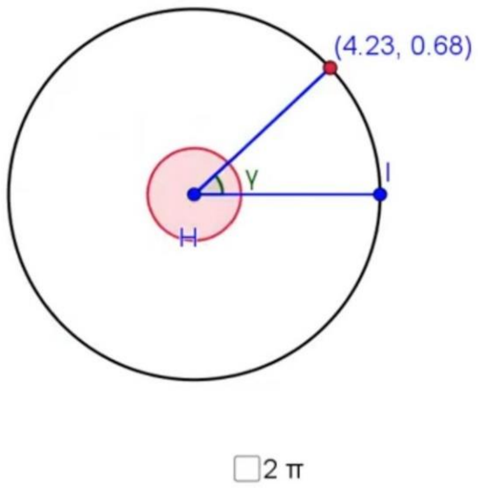
sin(θ+2.2π)=sin θ



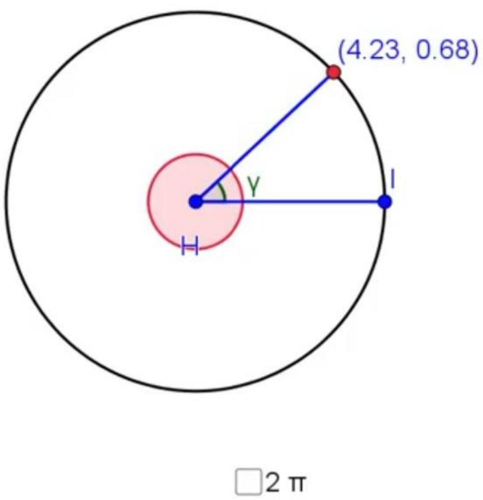
sin(θ+3.2π)=sin θ



$\sin(\theta+4.2\pi)=\sin\theta$



$\sin(\theta+7.2\pi)=\sin\theta$



EXAMPLE

$$\sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{2\pi}{3}\right) = \sin\left(\frac{3\pi}{3} - \frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{7\pi}{6} - \pi\right) = -\cos\left(\frac{7\pi}{6} - \frac{6\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(-\frac{7\pi}{6}\right) = \cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(\frac{16\pi}{3}\right) = \cos\left(\frac{\pi}{3} + \frac{15\pi}{3}\right) = \cos\left(\frac{\pi}{3} + 5\pi\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

$$\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

EXAMPLE 1.5.6 Find the value of each of the following:

a. $\sin\left(\frac{17\pi}{4}\right)$ **b.** $\cos\left(-\frac{7\pi}{6}\right)$ **c.** $\cos\left(\frac{4\pi}{3}\right)$

Solution

a. $\sin\left(\frac{17\pi}{4}\right) = \sin\left(\frac{\pi}{4} + \frac{16}{4}\pi\right) = \sin\left(\frac{\pi}{4} + 4\pi\right) = \sin\left(\frac{\pi}{4} + 2 \cdot 2\pi\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}.$

c. $\cos\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{4\pi}{3} - \pi\right) = -\cos\left(\frac{4\pi}{3} - \frac{3\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$

EXAMPLE 1.5.7 Use the periodicity of tangent and secant as well as their values when $0 \le x \le 2\pi$ to find the exact value of each of the following.

a. $\sec\left(\frac{9\pi}{4}\right)$ **b.** $\sin\left(\frac{15\pi}{2}\right)$ **c.** $\sec\left(-\frac{2\pi}{3}\right)$ **d.** $\tan\left(-\frac{5\pi}{6}\right)$

Solution

a. $\sec\left(\frac{9\pi}{4}\right) = \sec\left(\frac{\pi}{4} + \frac{8\pi}{4}\right) = \sec\left(\frac{\pi}{4} + 2\pi\right) = \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$

b. $\sin\left(\frac{15\pi}{2}\right) = \sin\left(\frac{\pi}{2} + \frac{14\pi}{2}\right) = \sin\left(\frac{\pi}{2} + \frac{14\pi}{2}\right) = \sin\left(\frac{\pi}{2} + 7\pi\right) = \sin\left(\frac{3\pi}{2}\right) = -1$

c. $\sec\left(-\frac{2\pi}{3}\right) = \sec\left(\frac{2\pi}{3}\right) = -\sec\left(\frac{\pi}{3}\right) = -2$

d. $\tan\left(-\frac{5\pi}{6}\right) = -\tan\left(\frac{5\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right) =$

$$\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

RELATED PROBLEM 5 Find the value of each of the following:

a. $\tan\left(\frac{17\pi}{4}\right)$ b. $\cot\left(-\frac{5\pi}{6}\right)$ c. $\cos\left(\frac{-55\pi}{6}\right)$ d. $\sin\left(\frac{2\pi}{3}-\frac{27\pi}{4}\right)$

Answers

a. 1 b. $\sqrt{3}$ c. $\frac{-\sqrt{3}}{2}$ d. $-\frac{\sqrt{6}-\sqrt{2}}{4}$

TRIGONOMETRIC IDENTITIES - المتطابقات المثلثية

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1 + \cot^2\theta = cec^2\,\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\tan^2\theta + 1 = \sec^2\theta$$

The **sum and difference identities** متطابقات المجموع و الفرق

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}.$$

$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$

sin(x + y) = sin x cos y + sin y cos x

sin(x + x) = sin x cos x + sin x cos x

sin(2x) =2 sin x cos x

$\sin^2\theta + \cos^2\theta = 1$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

cos(x + y) = cos x cos y - sin x sin y

cos(x + x) = cos x cos x - sin x sin x

cos(2x) = $\cos^2 x - \sin^2 x$

cos(2x) = $(1 - \sin^2 x) - \sin^2 x$

cos(2x) = $1 - 2\sin^2 x$

cos(2x) = $\cos^2 x - (1 - \cos^2 x)$

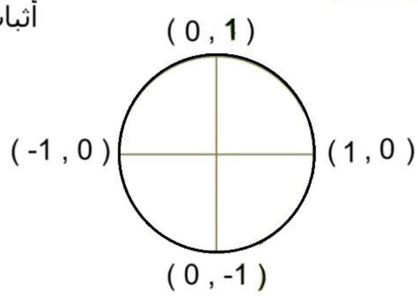
cos(2x) = $2\cos^2 x - 1$

prove some of the identities أثبات صحة بعض المتطابقات

$$\sin\left(\frac{\pi}{2} - x\right) = \sin\left(\frac{\pi}{2} + x\right) = \cos x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \sin\frac{\pi}{2}\cos x - \sin x \cos\frac{\pi}{2} = \cos x$$

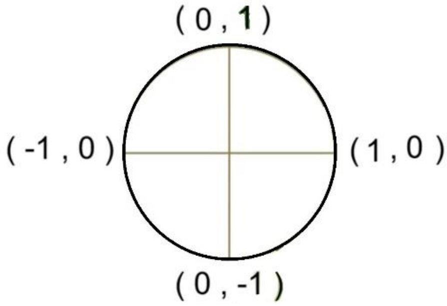
$$\sin\left(\frac{\pi}{2} + x\right) = \sin\frac{\pi}{2}\cos x + \sin x \cos\frac{\pi}{2} = \cos x$$



5. $\cos(\pi - x) = \cos(\pi + x) = -\cos x$

$$\cos(\pi - x) = \cos\pi\cos x + \sin\pi\sin x = -\cos x$$

$$\cos(\pi + x) = \cos\pi\cos x - \sin\pi\sin x = -\cos x$$



Prove this identity:

$$\sin(x + y) + \sin(x - y) = 2\sin x \cos y.$$

$\sin(x + y) = \sin x \cos y + \sin y \cos x$

$\sin(x - y) = \sin x \cos y - \sin y \cos x$ بالجمع

$\sin(x + y) + \sin(x - y) = 2\sin x \cos y$

The ***half-angle*** formula is

$\sin(2x) = 2\sin x \cos x$

$\sin(x) = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(x) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$

$\cos(2x) = 1 - 2\sin^2 x$

$\cos(x) = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$\cos(2x) = 2\cos^2 x - 1$

$\cos(x) = 2\cos^2\left(\frac{x}{2}\right) - 1$

verify the identity $2\sin^2(2t) + \cos(4t) = 1$

$2\sin^2(2t) + \cos(4t) = 2\sin^2(2t) + \cos(2.2t) = 2\sin^2(2t) + 1 - 2\sin^2(2t) = 1$

verify the identity $\dfrac{1+\csc\alpha}{\sec\alpha}-\cot\alpha=\cos\alpha$

Solution

$$\frac{1+\csc\alpha}{\sec\alpha}-\cot\alpha=\frac{1+\frac{1}{\sin\alpha}}{\frac{1}{\cos\alpha}}-\frac{\cos\alpha}{\sin\alpha}=\frac{\cos\alpha}{1}\cdot\frac{1+\frac{1}{\sin\alpha}}{1}-\frac{\cos\alpha}{\sin\alpha}$$

$$=\cos\alpha+\frac{\cos\alpha}{\sin\alpha}-\frac{\cos\alpha}{\sin\alpha}=\cos\alpha$$

The **sum and difference identities** متطابقات المجموع و الفرق

$$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}\qquad\qquad\tan(2x)=\frac{2\tan x}{1-\tan^2 x}$$

EXAMPLE 1.5.6 Find the value of each of the following:

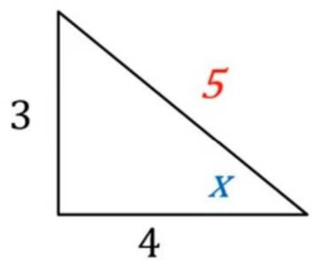
$$\tan\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\frac{\tan\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{4}\right)}{1-\tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}=\frac{\sqrt{3}+1}{1-\sqrt{3}}$$

$$=\frac{\sqrt{3}+1}{1-\sqrt{3}}\cdot\boxed{\frac{1+\sqrt{3}}{1+\sqrt{3}}}=\frac{4+2\sqrt{3}}{-2}=-2-\sqrt{3}.$$

GRAPHS OF THE TRIGONOMETRIC FUNCTIONS - تمثيل الدوال المثلية

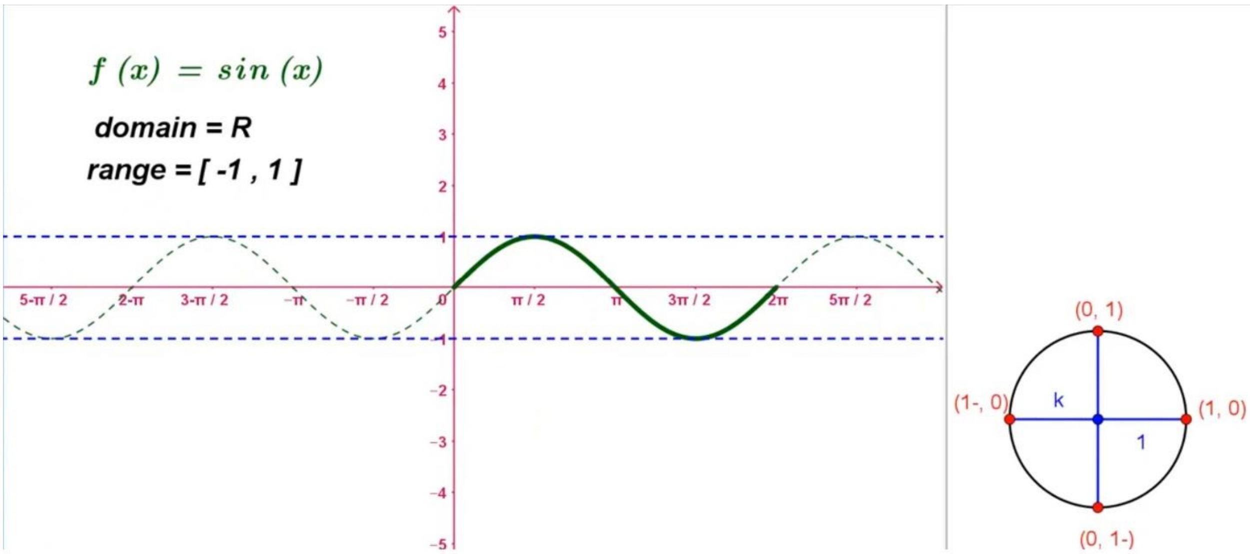
EXAMPLE

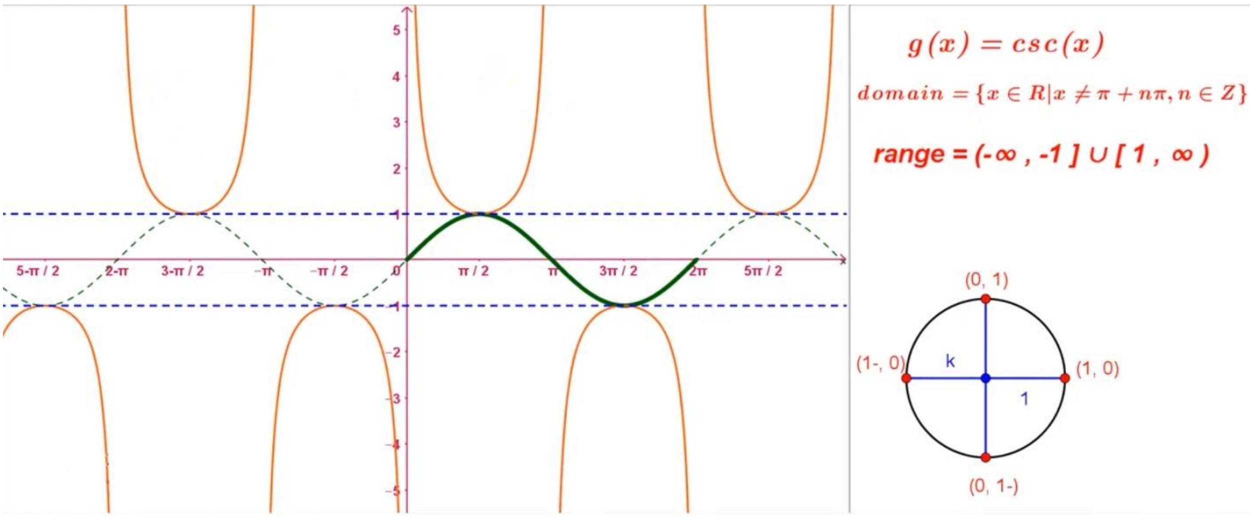
$\tan x=\dfrac{3}{4}$ $\Longrightarrow$ Sin x = 3 , cos x = 4 ? F

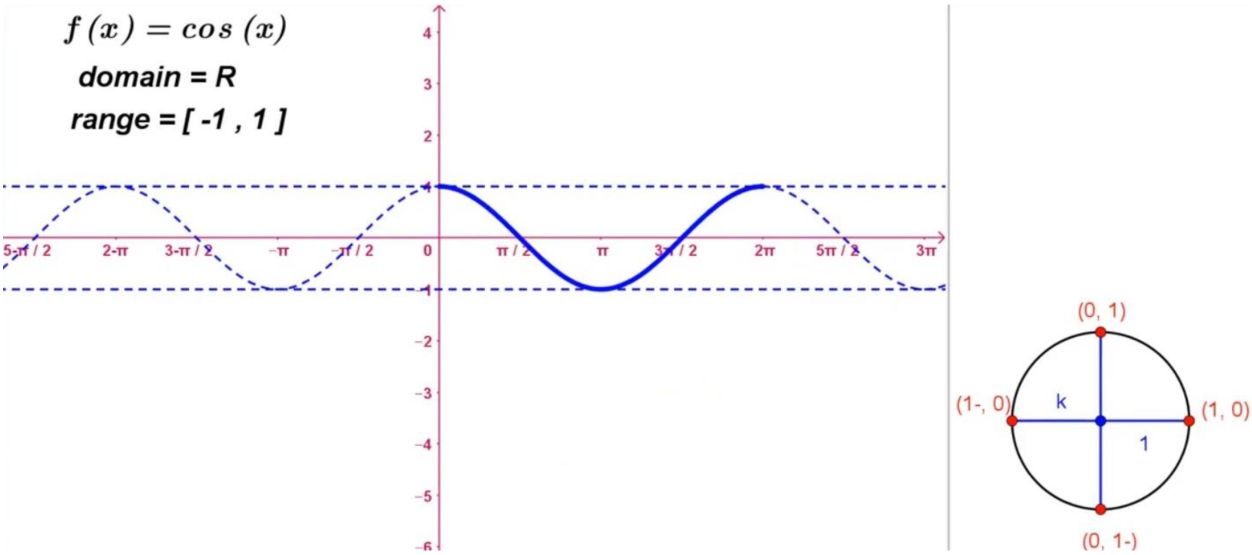


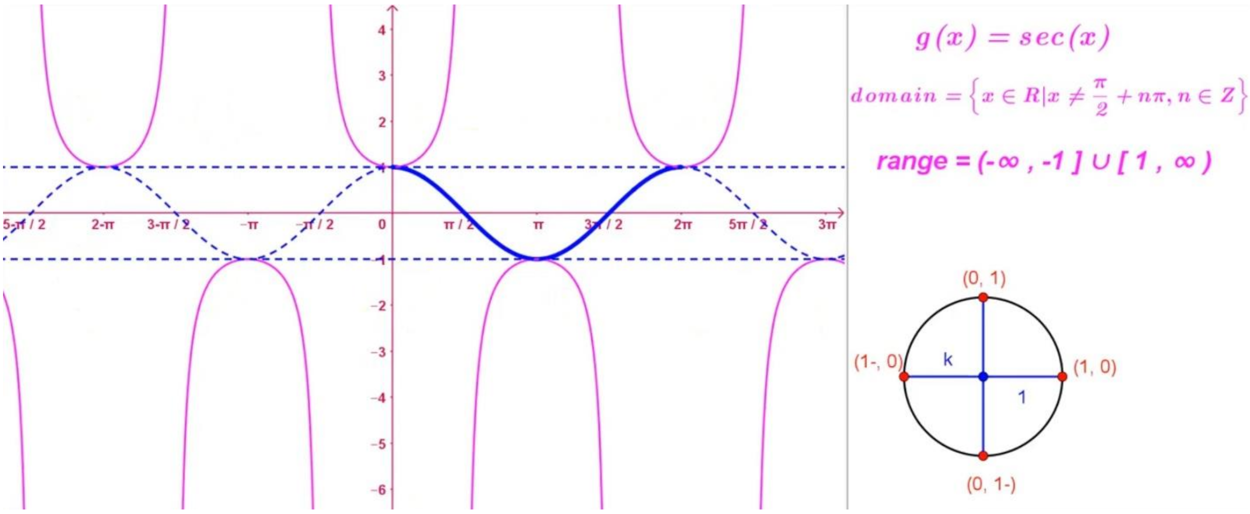
$\sin x=\dfrac{3}{5}$ $\qquad\qquad$ $\cos x=\dfrac{4}{5}$

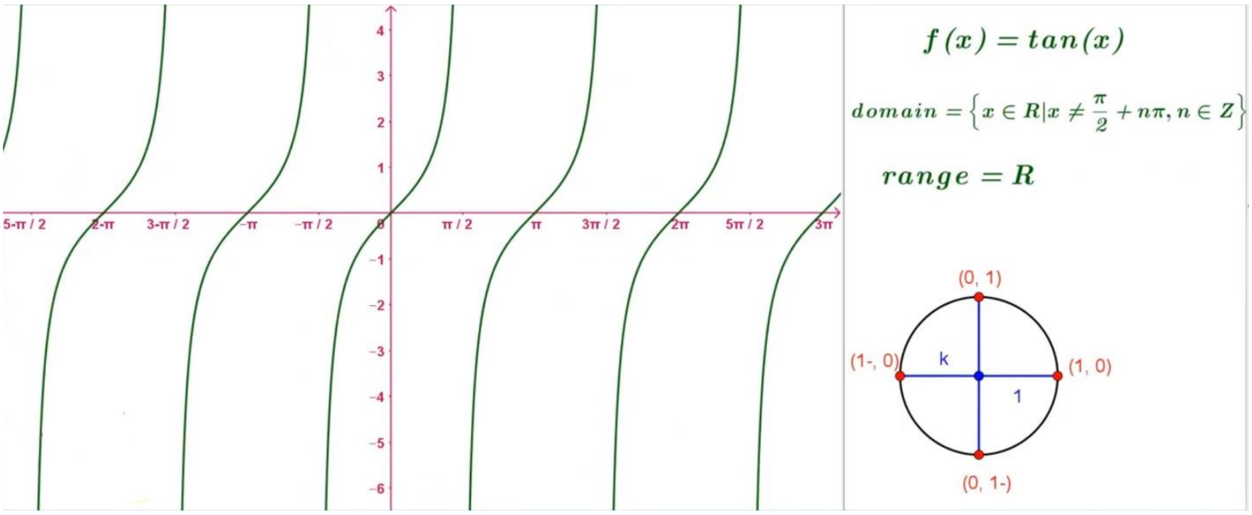
$-1\le\sin x\le 1$ $\qquad\qquad$ $-1\le\cos x\le 1$

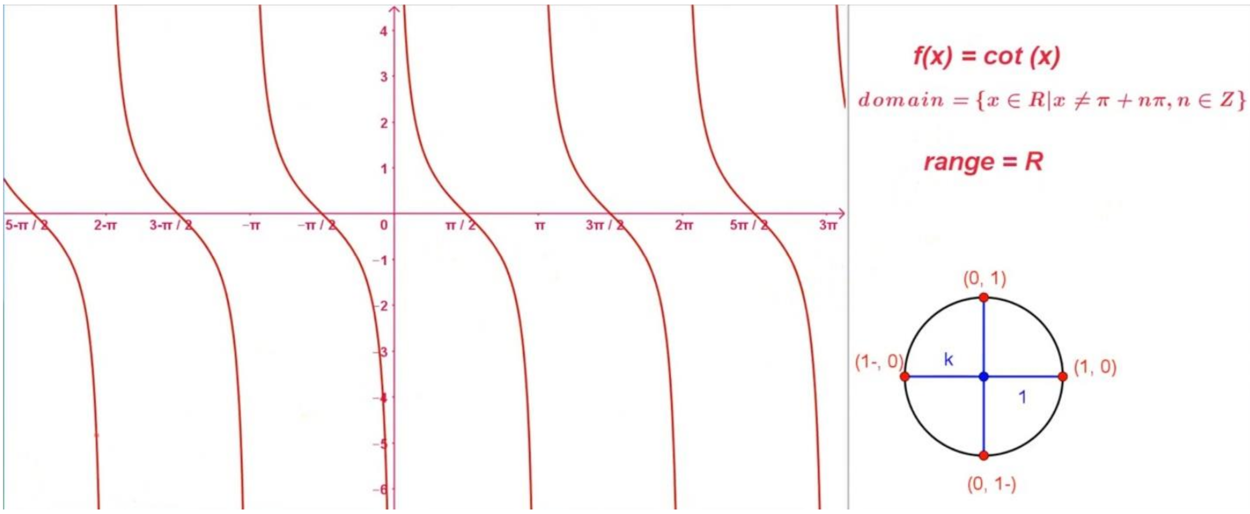

f (x) = sin (x)
domain = R
range = [-1 , 1]
5-π / 2
2-π
3-π / 2
−π
−π / 2
0
π / 2
π
3π / 2
2π
5π / 2
(0, 1)
(1-, 0)
(1, 0)
k
1
(0, 1-)


g (x) = csc (x)
domain = {x ∈ R|x ≠ π + nπ, n ∈ Z}
range = (-∞ , -1] ∪ [1 , ∞)
5-π / 2
2-π
3-π / 2
−π
−π / 2
0
π / 2
π
3π / 2
2π
5π / 2
(0, 1)
(1-, 0)
(1, 0)
k
1
(0, 1-)


f (x) = cos (x)
domain = R
range = [-1 , 1]
5-π / 2
2-π
3-π / 2
−π
−π / 2
0
π / 2
π
3π / 2
2π
5π / 2
3π
(0, 1)
(1-, 0)
(1, 0)
k
1
(0, 1-)


g(x) = sec(x)
domain = {x ∈ R | x ≠ π/2 + nπ, n ∈ Z}
range = (-∞ , -1] ∪ [1 , ∞)
(0, 1)
(1-, 0)
(1, 0)
(0, 1-)
k
1


f(x) = tan(x)
domain = {x ∈ R | x ≠ π/2 + nπ, n ∈ Z}
range = R
(0, 1)
(1-, 0)
(1, 0)
(0, 1-)
k
1

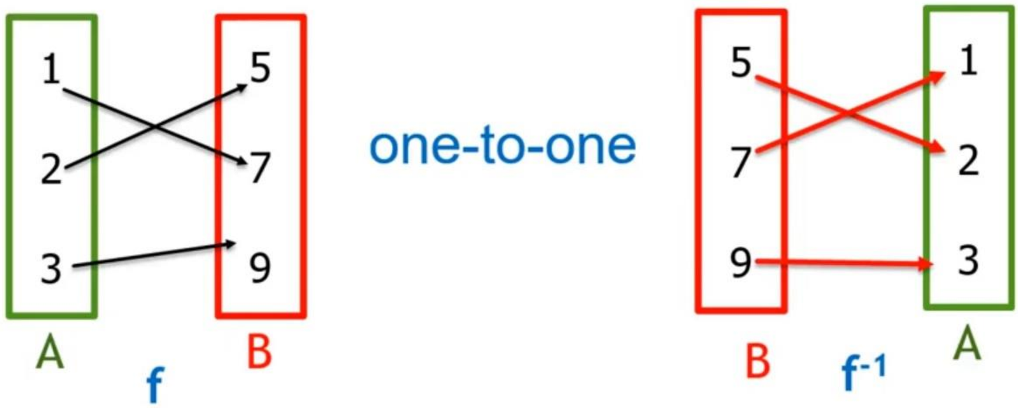

f(x) = cot (x)
domain = {x ∈ R | x ≠ π + nπ, n ∈ Z}
range = R
(0, 1)
(1-, 0)
(1, 0)
(0, 1-)
k
1

Section 1.6

INVERSE TRIGONOMETRIC FUNCTIONS معكوس الدوال المثلثية

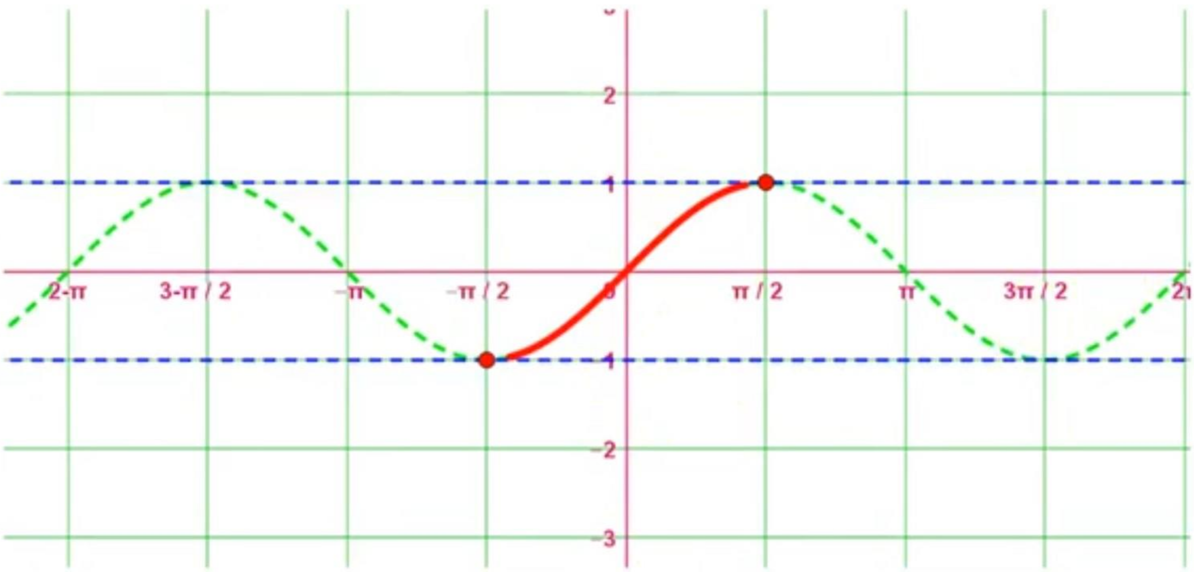
INVERSRE OF SINE AND COSIN FUNCTIONS - معكوس الجيب و الجيب تمام

Inverse Function الدالة العكسية مراجعة

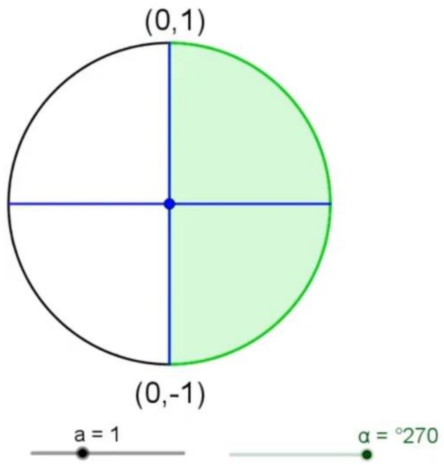


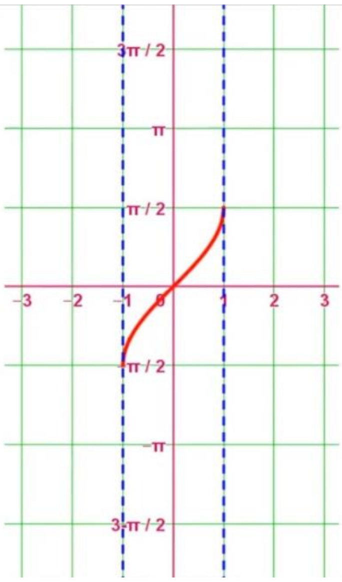
$f^{-1}(f(1)) = f^{-1}(7) = 1$ $\quad$ $f^{-1}(f(x)) = x$

$f(f^{-1}(7)) = f(1) = 7$ $\quad$ $f(f^{-1}(x)) = x$



y = sin x

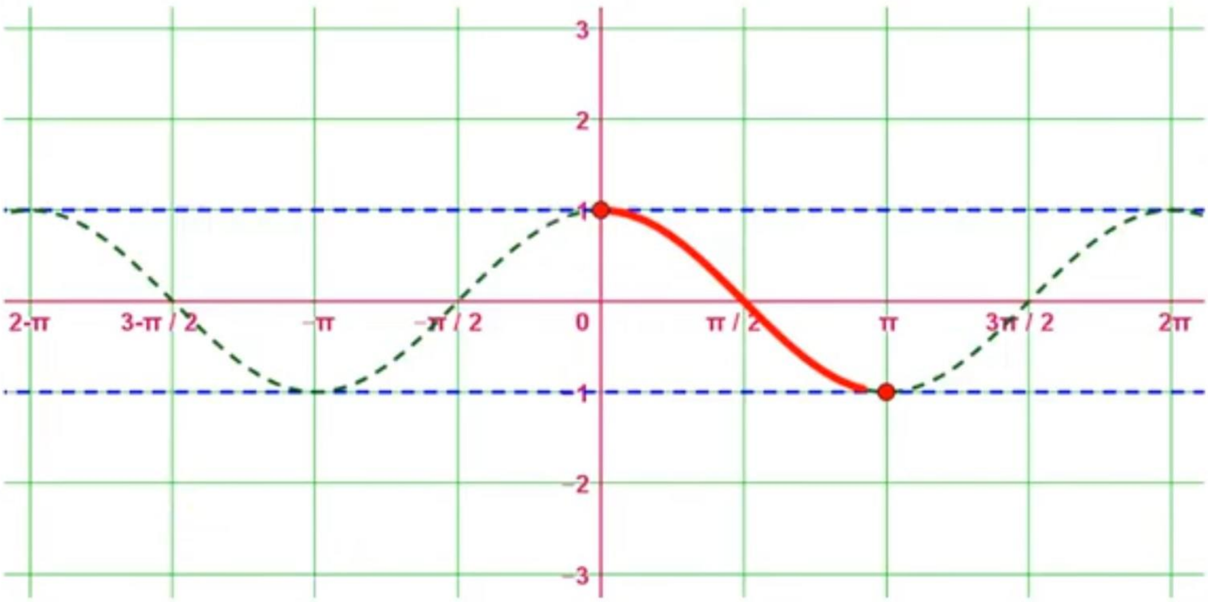




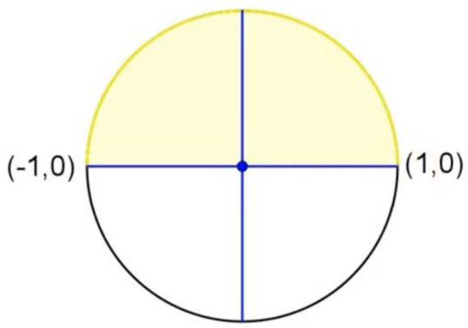
domain =[-1 , 1]

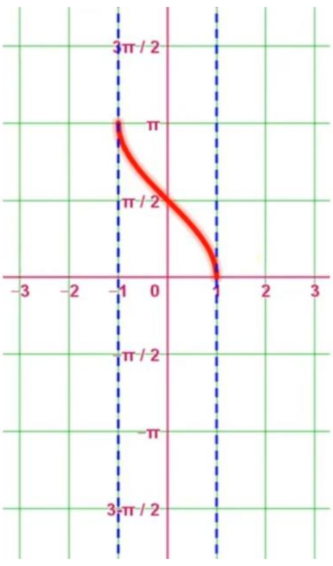
range $= [-\frac{\pi}{2}, \frac{\pi}{2}]$

$y = \sin^{-1} x$



y = cos x

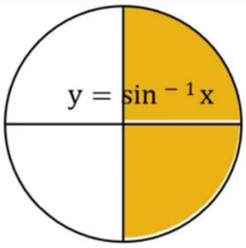


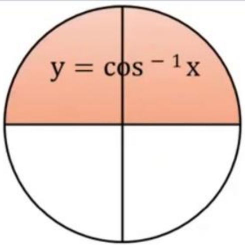


domain = [-1 , 1]

range = [0 , π]

$y = \cos^{-1} x$



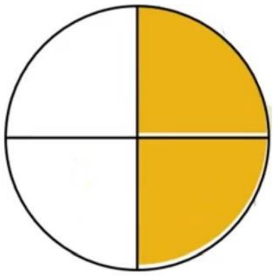


determine the exact function value.

a. $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ **b.** $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ **c.** $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$ **d.** $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

EXAMPLE 1.6.1 Find the following:

a. $\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$



Solution

$$\cos\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \cos\left(-\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

b. $\arcsin\left(\cos\left(\frac{2\pi}{3}\right)\right)$

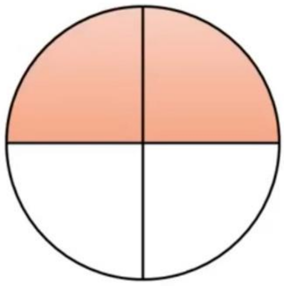
Note: arcsin (which can also be written as sin^{-1}) is the inverse function of the sine function. i.e., If $y = sin^{-1}x$ then sin y = x.

Solution

$$\arcsin\left(\cos\left(\frac{2\pi}{3}\right)\right) = \arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

EXAMPLE 1.6.2 Find the following:

a. $\sin\left(\arccos\left(-\frac{1}{2}\right)\right)$



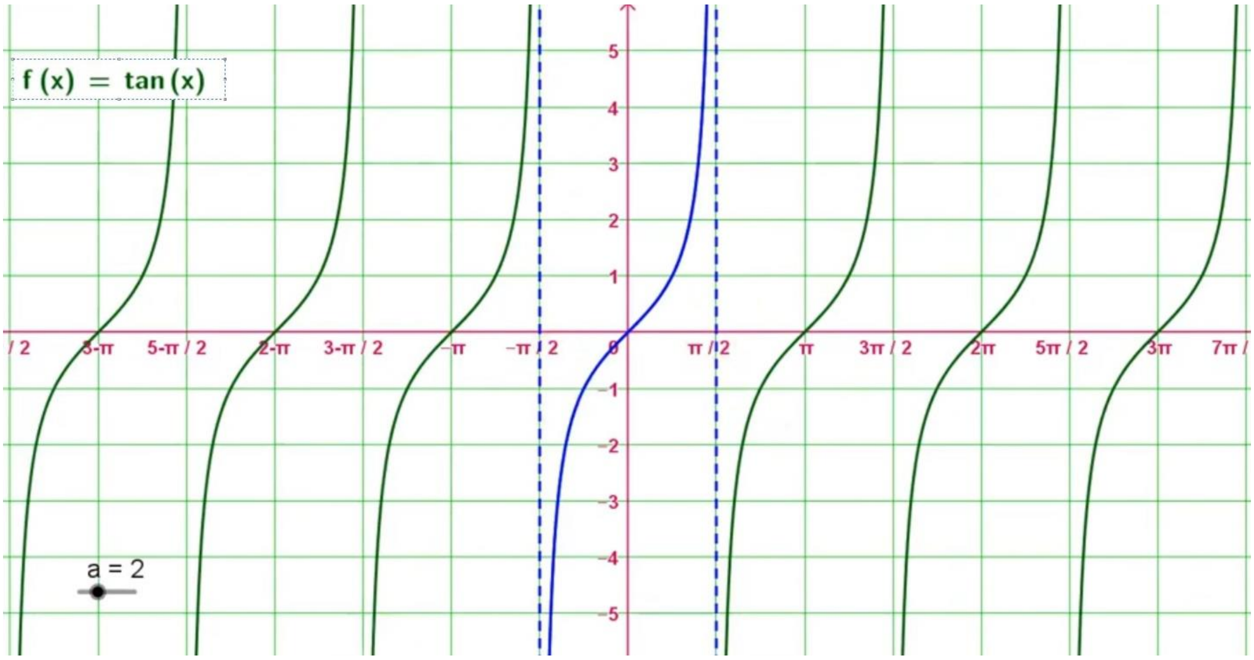
Solution

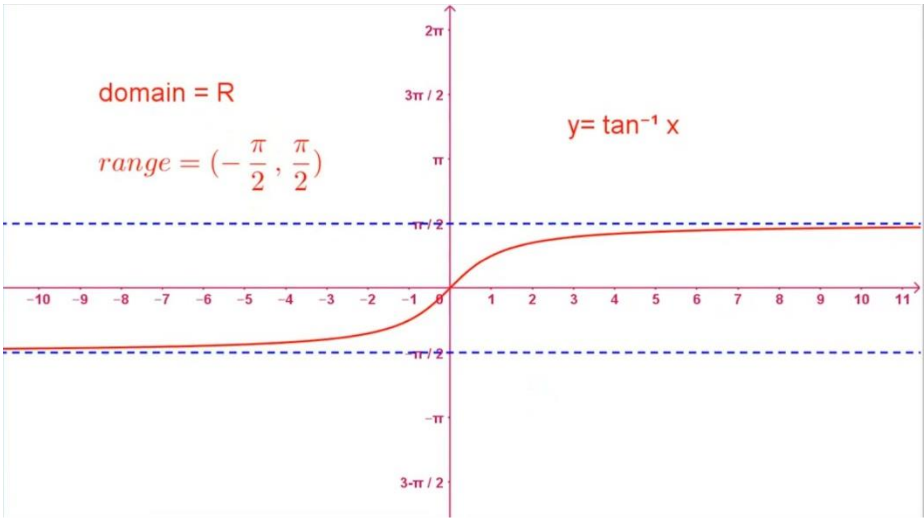
$$\sin\left(\arccos\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{2\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

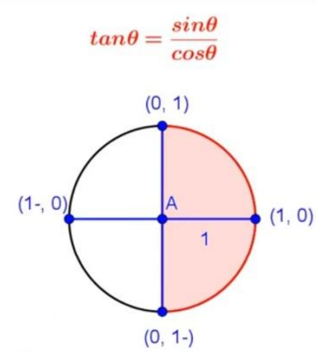
b. $\cos^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

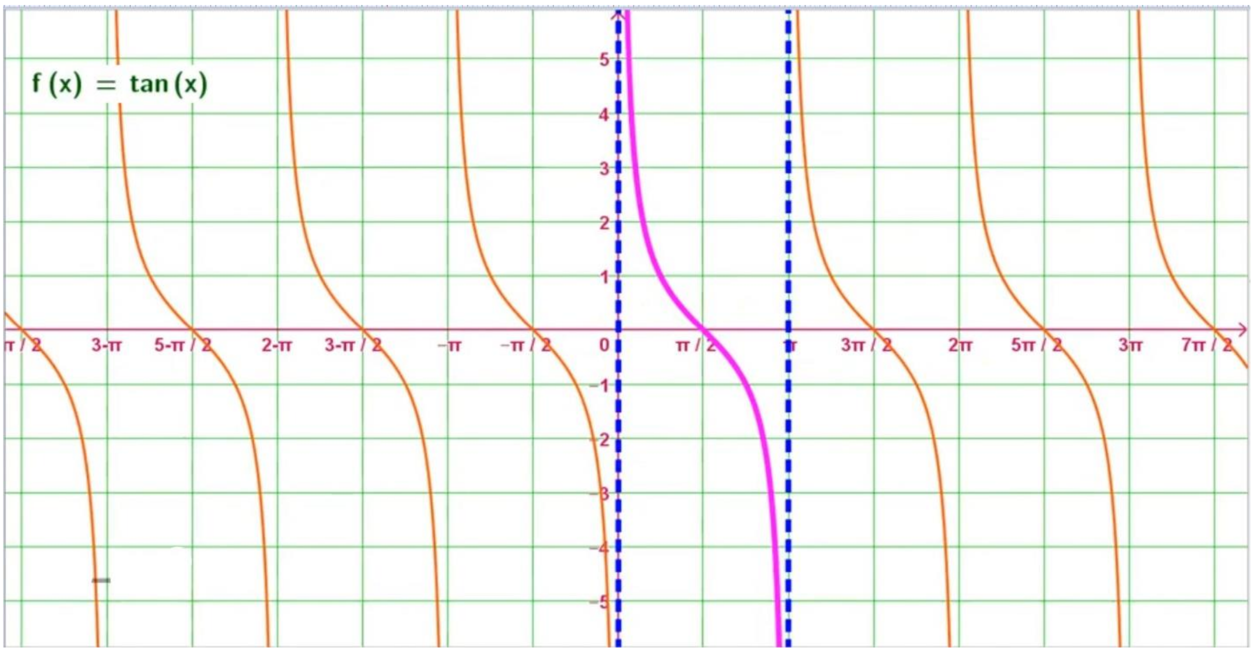
Solution

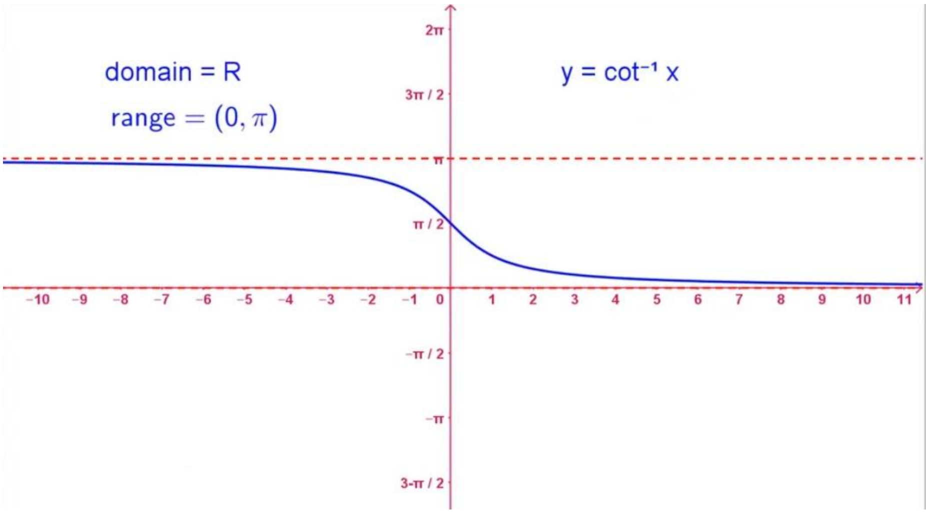
$$\cos^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

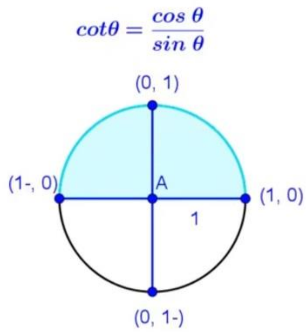

f (x) = tan (x)
a = 2

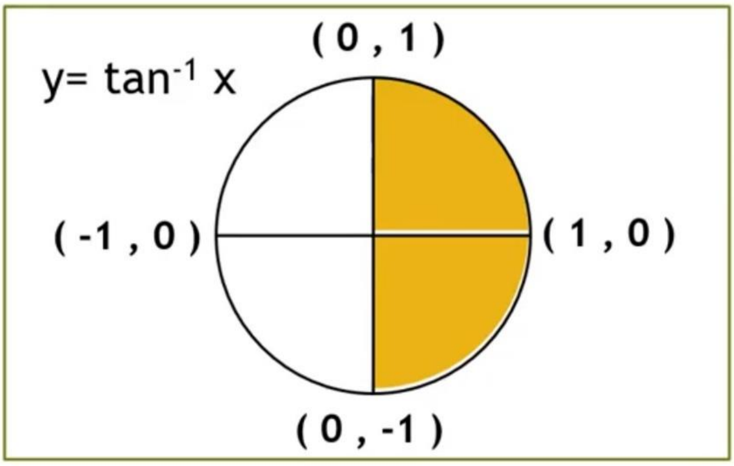

domain = R
$range = (-\frac{\pi}{2}, \frac{\pi}{2})$
y= tan⁻¹ x


$tan\theta = \frac{sin\theta}{cos\theta}$
(0, 1)
(1-, 0)
A
(1, 0)
1
(0, 1-)


f (x) = tan (x)





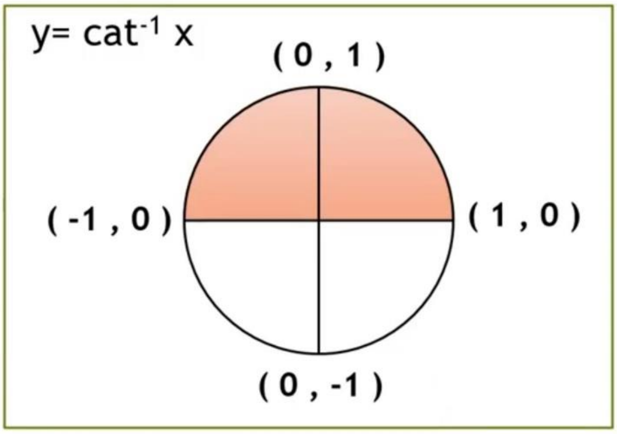


$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

3.

a. $\tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$ **b.** $\tan^{-1}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$



$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

4.

a. $\cot^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{3}$ **b.** $\cot^{-1}\left(-\sqrt{3}\right) = \frac{5\pi}{6}$

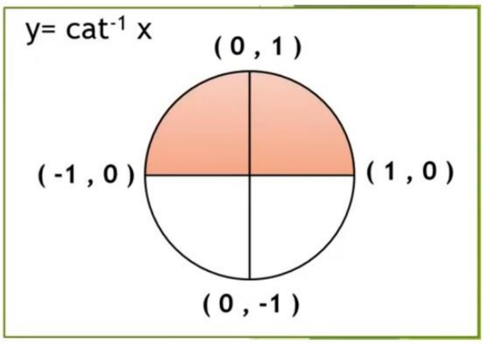
EXAMPLE 1.6.3 Find the exact value of the following

a. $\cos\left(\tan^{-1}(-1)\right)$ **b.** $\sin\left(\cot^{-1}(1)\right)$

Solution

$\cos(\tan^{-1}(-1)) = \cos(-\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$\sin(\cot^{-1}(1)) = \sin((\frac{\pi}{4})) = \frac{\sqrt{2}}{2}$



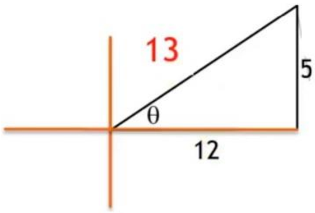
EXAMPLE 1.6.4 Find the exact value of $\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$

Solution

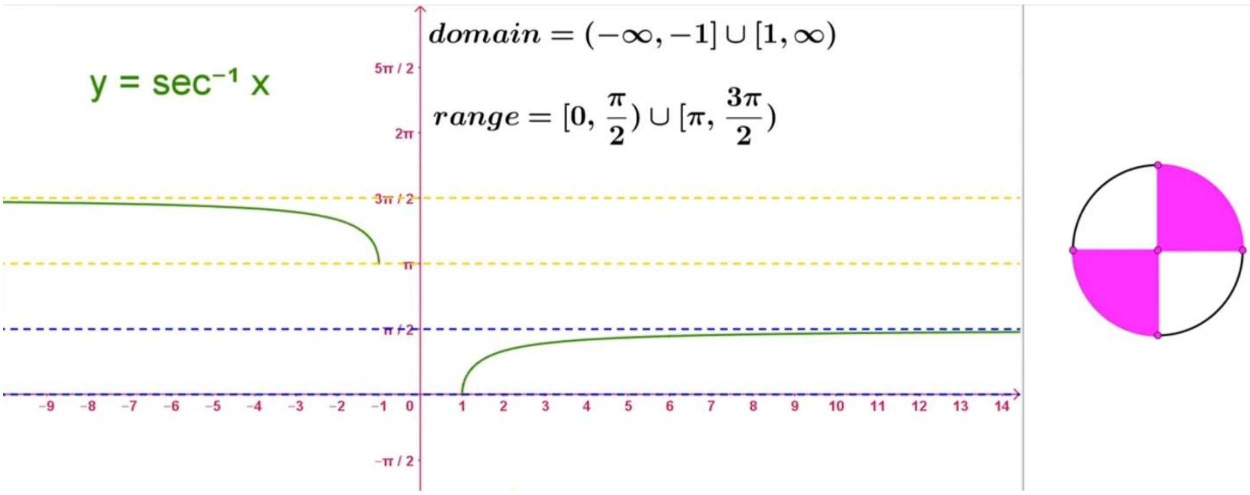
Let $\tan^{-1}\left(\frac{5}{12}\right) = \theta$ ⟹ $\tan\theta = \frac{5}{12}$

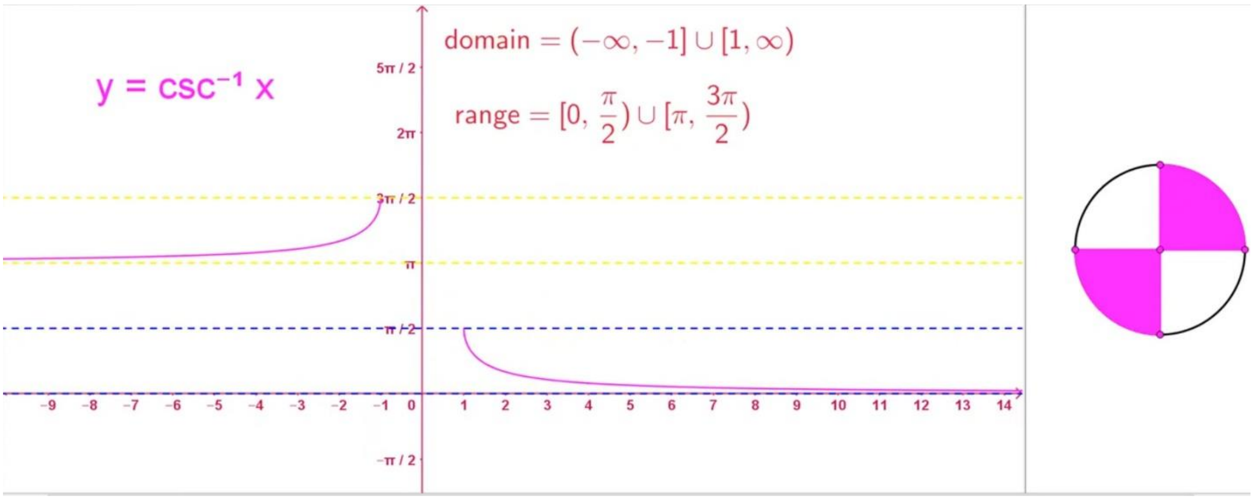
$x^2 = 25 + 144 = 169$

$x = 13$

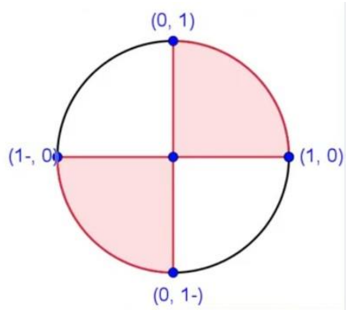


$\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \cos\theta = \frac{12}{13}$





determine the exact function value.



c. $\sec^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{6}$

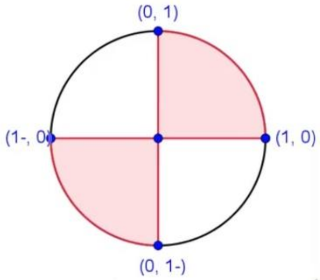
d. $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right) = \frac{7\pi}{6}$

c. $\csc^{-1}\frac{2}{\sqrt{3}} = \frac{\pi}{3}$

d. $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right) = \frac{4\pi}{3}$

EXAMPLE 1.6.5 Find the exact value of the following

a. $\sin\left(\sec^{-1}(2)\right)$ **b.** $\sin\left(\sec^{-1}\left(-\frac{3}{2}\right)\right)$



Solution

a. $\sin\left(\sec^{-1}(2)\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

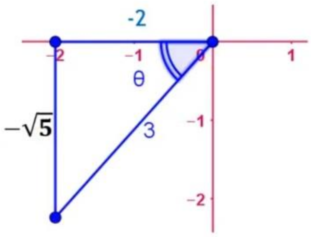
b. $\sin\left(\sec^{-1}\left(-\frac{3}{2}\right)\right)$

let $\sec^{-1}\left(-\frac{3}{2}\right) = \theta$ ⟹ $\text{Sec}\ \theta = -\frac{3}{2}$

$x^2 = 9 - 4 = 5$

$x = \pm\sqrt{5}$

$\sin\theta = \frac{-\sqrt{5}}{3}$



EXERCISES 1.6

In Exercises 1-4, determine the exact function value.

1.

a. $\sin^{-1}\frac{1}{2}$ **b.** $\sin^{-1}\left(-\frac{1}{2}\right)$ **c.** $\cos^{-1}\frac{1}{2}$ **d.** $\cos^{-1}\left(-\frac{1}{2}\right)$

2.

a. $\sin^{-1}\frac{\sqrt{3}}{2}$ **b.** $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ **c.** $\cos^{-1}\frac{\sqrt{3}}{2}$ **d.** $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

3.

a. $\tan^{-1}\frac{1}{\sqrt{3}}$ **b.** $\tan^{-1}\left(-\sqrt{3}\right)$ **c.** $\sec^{-1}\frac{2}{\sqrt{3}}$ **d.** $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

4.

a. $\cot^{-1}\frac{1}{\sqrt{3}}$ **b.** $\cot^{-1}\left(-\sqrt{3}\right)$ **c.** $\csc^{-1}\frac{2}{\sqrt{3}}$ **d.** $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

In Exercises 5-10, find the exact value of the quantity.

5.

a. $\sin^{-1}\left(\sin\left(\frac{1}{6}\pi\right)\right)$ **b.** $\sin^{-1}\left(\sin\left(-\frac{1}{6}\pi\right)\right)$

c. $\sin^{-1}\left(\sin\left(\frac{5}{6}\pi\right)\right)$ **d.** $\sin^{-1}\left(\sin\left(-\frac{5}{6}\pi\right)\right)$

6.

a. $\cos^{-1}\left(\cos\left(\frac{1}{3}\pi\right)\right)$ **b.** $\cos^{-1}\left(\cos\left(-\frac{1}{3}\pi\right)\right)$

c. $\cos^{-1}\left(\cos\left(\frac{2}{3}\pi\right)\right)$ **d.** $\cos^{-1}\left(\cos\left(\frac{4}{3}\pi\right)\right)$

7.

a. $\tan^{-1}\left(\tan\left(\frac{1}{6}\pi\right)\right)$ **b.** $\tan^{-1}\left(\tan\left(-\frac{1}{3}\pi\right)\right)$

c. $\tan^{-1}\left(\tan\left(\frac{7}{6}\pi\right)\right)$ **d.** $\tan^{-1}\left(\tan\left(-\frac{4}{3}\pi\right)\right)$

8.

a. $\cot^{-1}\left(\cot\left(\frac{1}{6}\pi\right)\right)$ **b.** $\sec^{-1}\left(\sec\left(\frac{1}{3}\pi\right)\right)$