11.2 :- Series :- "المتسلسلات"

$$a_1 + a_2 + a_3 + \cdots = \sum_{n=1}^{\infty} a_n = S_n$$

مجموع

* تعريفها :- مجموعة من القيم بينها عمليات جمع وتتبع قاعدة معينة تسمى a_n .

$$\Rightarrow S_n = S_{n-1} + a_n \Leftarrow$$ قانون المجموع *

Ex :- If $S_n = \frac{2n-1}{n+1}$; find a_n ?

find S_{n-1} $\Rightarrow S_{n-1} = \frac{2(n-1)-1}{(n-1)+1} = \frac{2n-3}{n}$

Now $S_n = S_{n-1} + a_n$

$$\frac{2n-1}{n+1} = \frac{2n-3}{n} + a_n$$

$$a_n = \frac{2n-1}{n+1} - \frac{2n-3}{n}$$

$$a_n = \frac{2n^2 - n - 2n^2 - 2n + 3n + 3}{(n+1)n} = \frac{3}{(n+1)n}$$

Scanned with CamScanner

partial sum (telescoping test) :-

الشكل العام $\rightsquigarrow \sum_{n=1}^{\infty}$ اقتران − اقتران

حالاته → $\boxed{1}$ مباشر → $\sum$ اقتران - اقتران

$\boxed{2}$ خواص ال ln → $\sum \ln\left(\frac{A}{B}\right) = \sum \ln(A) - \ln(B)$

$\boxed{3}$ كسور جزئية → $\sum \frac{\quad}{(A)(B)} = \sum \frac{\quad}{A} - \frac{\quad}{B}$

Ex :- Find the sum for $\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+2}$

$$S_n = \left(\boxed{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \cancel{\dots} \left(\cancel{\frac{1}{n}} - \cancel{\frac{1}{n+1}}\right) + \left(\cancel{\frac{1}{n+1}} \boxed{- \frac{1}{n+2}}\right)$$

$$S_n = \frac{1}{2} - \frac{1}{n+2}$$

$$\rightarrow \lim_{n \to \infty} S_n = \lim_{n \to \infty} \frac{1}{2} - \frac{1}{n+2}$$

$$= \frac{1}{2} - 0 = \boxed{\frac{1}{2}} \rightsquigarrow \text{Conv}$$

past paper 2020 : The sum of $\sum_{n=1}^{\infty}\left(11^{\frac{1}{n}} - 11^{\frac{1}{n+2}}\right)$?

$$S_n = \left(\boxed{11^{\frac{1}{1}}} - \cancel{11^{\frac{1}{3}}}\right) + \left(\boxed{11^{\frac{1}{2}}} - \cancel{11^{\frac{1}{4}}}\right) + \left(\cancel{11^{\frac{1}{3}}} - \cancel{11^{\frac{1}{5}}}\right)$$

$$+ \cdots \left(\cancel{11^{\frac{1}{n-1}}} \boxed{- 11^{\frac{1}{n+1}}}\right) + \left(\cancel{11^{\frac{1}{n}}} \boxed{- 11^{\frac{1}{n+2}}}\right)$$

$$S_n = 11 + \sqrt{11} - 11^{\frac{1}{n+1}} - 11^{\frac{1}{n+2}}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} 11 + \sqrt{11} - 11^0 - 11^0$$

$$= 11 + \sqrt{11} - 1 - 1$$

$$= 9 + \sqrt{11}$$

past paper 2020 : $\sum_{n=1}^{\infty}\left(e^{\frac{4}{n}} - e^{\frac{4}{n+1}}\right)$ equals ?

Ⓐ $e-1$ Ⓑ e^2-1 Ⓒ e^3-1

Ⓓ e^4-1 Ⓔ e^5-1

Ex :- $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$\Rightarrow 1 = A(n+1) + B(n)$

Try

$n=0 \rightarrow 1 = A(1) + 0 \rightarrow A = 1$

$n=-1 \rightarrow 1 = 0 + B(-1) \rightarrow B = -1$

$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$

$S_n = (\boxed{\frac{1}{1}} - \cancel{\frac{1}{2}}) + \cancel{\cdots} (\cancel{\frac{1}{n}} \boxed{- \frac{1}{n+1}})$

$S_n = 1 - \frac{1}{n+1}$

$\lim_{n \to \infty} S_n = \lim_{n \to \infty} 1 - \frac{1}{n+1} = 1 - 0 = \boxed{1}$

Conv ✓

---

* Home work :-

find the sum if exist for

$\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$

Geometric series : $\sum_{n=0}^{\infty} (r)^n$ اي عدد

$$\sum_{n=0}^{\infty} a.(r)^n = \begin{cases} |r| \geq 1 \rightarrow \text{diverge} \rightarrow \text{No Sum} \\ |r| < 1 \rightarrow \text{Converge} \rightarrow S_n = \dfrac{a_1}{1-r} \end{cases}$$

Ex : find the sum if exist for the following series :-

$\boxed{1}$ $\sum_{n=0}^{\infty} \frac{2}{3^n} \rightarrow \sum_{n=0}^{\infty} 2.\left(\frac{1}{3^n}\right) \rightarrow \sum_{n=0}^{\infty} 2.\left(\frac{1}{3}\right)^n$

$S_n = \frac{a_1}{1-r} = \frac{2}{1-\frac{1}{3}} = \boxed{3}$ ✓

$\left|\frac{1}{3}\right| < 1$ ✓ So Conv

$\boxed{2}$ $\sum_{n=1}^{\infty} 2^{2n+1} * 8^{-n}$

$\Rightarrow \sum_{n=1}^{\infty} \frac{(2^2)^n * (2)^1}{(8)^n} = \sum_{n=1}^{\infty} 2\left(\frac{4}{8}\right)^n = 2\left(\frac{1}{2}\right)^n$

$r = \frac{1}{2} < 1 \rightarrow$ Conv ✓

$S_n = \frac{a_1}{1-r} = \frac{1}{1-\frac{1}{2}} = \boxed{2}$

$\boxed{\text{Note}}$:-

$$\text{Conv} \pm \text{Conv} = \text{Conv}$$
$$\text{Conv} \pm \text{div} = \text{div}$$
$$\text{div} + \text{div} = \text{div}$$
$$\text{div} - \text{div} = \text{"الله اعلم"}$$

EX :- Find the sum if exist for the following series :

$\boxed{1}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}+5^{n}}{4^{2n}}$, $\boxed{2}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n}+6^{n}}{5^{n+1}}$

* نوزع المقام ع البسط *

$$\sum_{n=1}^{\infty} \frac{(3)^n.(3)^{-1}}{(4^2)^n} + \frac{5^n}{(4^2)^n}$$

$$\sum_{n=1}^{\infty} \frac{1}{3} * \left(\frac{3}{16}\right)^n + \left(\frac{5}{16}\right)^n$$

$r = \frac{3}{16} < 1$ Conv ✓ $\quad r = \frac{5}{16} < 1$ Conv ✓

$$= S_{n_1} + S_{n_2}$$

$$= \boxed{\frac{76}{143}}$$

* اهانة
طلع معك
الرقم نفسه؟

$$\sum_{n=1}^{\infty} \frac{3^n}{5^n.5'} + \frac{6^n}{5^n.5'}$$

$$\sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n \cdot \left(\frac{1}{5}\right) + \left(\frac{6}{5}\right)^n \left(\frac{1}{5}\right)$$

$r = \frac{3}{5} < 1 \quad r = \frac{6}{5} > 1$

Conv ✓ + div ✗

= div ✓

past paper 2020 : $\sum_{n=1}^{\infty} \frac{1+5^n}{7^n} = ?$

* نوزع البسط على المقام *

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{7^n} + \frac{5^n}{7^n}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{7}\right)^n + \left(\frac{5}{7}\right)^n$$

$r = \frac{1}{7} < 1$ Conv ✓

$r = \frac{5}{7} < 1$ Conv ✓

$$S_n = \frac{\frac{1}{7}}{1-\frac{1}{7}} + S_n = \frac{\frac{5}{7}}{1-\frac{5}{7}}$$

$$= \frac{1}{6} + \frac{5}{2} = \boxed{\frac{8}{3}}$$

So Converge with Sum $= \frac{8}{3}$ ✓

Ex: Write the number $0.\overline{4}$ as a ratio of integers :-

* فكرة تحويل الأعداد الدورية الى اعداد كسرية *

تحل عن طريق ← Geometric Series

$$0.\overline{4} = 0.44444444444\ldots$$

$$= 0.4 + 0.04 + 0.004 + 0.004 + \cdots$$

$$= \underbrace{\frac{4}{10}}_{a_1} + \underbrace{\frac{4}{(10)^2}}_{a_2} + \underbrace{\frac{4}{(10)^3}}_{a_3} + \underbrace{\frac{4}{(10)^4}}_{a_4} + \cdots$$

So $a_n = \dfrac{4}{(10)^n}$

له القاعدة العامة

→ Geometric Series

$$\Rightarrow 0.\overline{4} = \sum_{n=1}^{\infty} 4\cdot\underbrace{\left(\frac{1}{10}\right)^n}_{r=\frac{1}{10}<1 \to \text{Conv}}$$

$$S_n = \frac{a_1}{1-r}$$

$$= \frac{4/10}{1-1/10} = \frac{4/10}{9/10} = \boxed{\frac{4}{9}} \ \#$$