Alternating Series :-

$$\sum_{n=}^{\infty} (-1)^{n} a_n \qquad \sum_{n=}^{\infty} (-1)^{n+1} a_n$$

$\boxed{1}$ $\lim_{n\to\infty} |a_n| \neq 0 \rightarrow$ div by D.T

$= 0$

$\boxed{2}$ a_n is decreasing $\rightarrow a_1 > a_2$

$\rightarrow a_n > a_{n+1}$

$\rightarrow (a_n)' < 0$

* Then the series is Convergence by A.S.T

Note : $(-1)^n = \cos(n\pi) = \cos^n(\pi)$

EX :- Test for Convergence or divergence ?

$\boxed{1}$ $\sum\limits_{n=}^{\infty} \frac{(-1)^n}{n}$ → Alternating series

↳ $\lim\limits_{n\to\infty} |a_n| = \lim\limits_{n\to\infty} \frac{1}{n} = \frac{1}{\infty} = \boxed{0}$ ✓

↳ a_n is decreasing $a_1 = \frac{1}{1}$, $a_2 = \frac{1}{2}$

$a_1 > a_2$ ✓

⇒ Conv by A.S.T

$\boxed{2}$ $\sum\limits_{n=1}^{\infty} \frac{\cos^n(\pi)}{n!} = \sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n!}$ A.S

↳ $\lim\limits_{n\to\infty} |a_n| = \lim\limits_{n\to\infty} \frac{1}{n!} = \frac{1}{\infty} = \boxed{0}$ ✓

↳ a_n is decreasing $a_1 = \frac{1}{1!} = 1$

$a_1 > a_2$ ✓ $a_2 = \frac{1}{2!} = \frac{1}{2}$

⇒ Conv by A.S.T

$\boxed{3}$ $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{3n-1}{2n+1}$ A.S

$\hookrightarrow \lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{3n-1}{2n+1} = \frac{\infty}{\infty}!!$

$\xrightarrow{C.R} \lim_{n\to\infty} \frac{3n}{2n} = \frac{3}{2} \neq 0$

The series is div by D.T

$\boxed{4}$ $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{5^n}$ A.S

$\hookrightarrow \lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{1}{5^n} = \frac{1}{5^\infty} = \frac{1}{\infty} = \boxed{0}$ ✓

$\hookrightarrow a_n$ is decreasing $a_1 = \frac{1}{5}$ $a_2 = \frac{1}{25}$

$a_1 > a_2$ ✓

$\Rightarrow$ Conv by A.S.T

$\boxed{5}$ $\displaystyle\sum_{n=1}^{\infty} \frac{n \cos(n\pi)}{2^n} = \sum_{n=1}^{\infty} \frac{n \cdot (-1)^n}{2^n}$ A.S

→ $\displaystyle\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{n}{2^n} \to \frac{\infty}{\infty} = \boxed{0}$ ✓

or → use L.R → $\displaystyle\lim_{n\to\infty} \frac{1}{2^n \cdot (1) \cdot \ln(2)} = \frac{1}{\infty} = \boxed{0}$ ✓

→ a_n is decreasing $\quad a_1 = \frac{1}{2} \quad a_2 = \frac{2}{4}$

✓ $\quad a_3 = \frac{3}{8} \quad a_4 = \frac{4}{16}$

⇒ The series is Conv by A.S.T

$\boxed{6}$ $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{n+1}\right)$ H.W

$\boxed{7}$ $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n \cdot n}{\ln(n)}$ A.S

$\hookrightarrow \displaystyle\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{n}{\ln(n)} = \frac{\infty}{\infty}!!$

$\xrightarrow{L.R} \displaystyle\lim_{n\to\infty} \frac{1}{\frac{1}{n}} = \lim_{n\to\infty} n = \infty \neq 0$

div by D.T ✓

$\boxed{8}$ $\displaystyle\sum_{n=2}^{\infty} (-1)^n \ln\left(1+\frac{1}{n}\right)$ H.W

Absolute Convergence

Conditional convergence or divergence

$\sum_{n=1}^{\infty} |a_n|$

- $\rightarrow$ Conv $\rightarrow$ Abs. Conv
- $\rightarrow$ div: find $\sum_{n=1}^{\infty} a_n$
  - $\rightarrow$ div $\rightarrow$ div
  - $\rightarrow$ Conv $\rightarrow$ Conditionally Conv

$\sum_{n=1}^{\infty} |a_n|$ $\rightarrow$

D.T

P.T

C.T

L.C.T

I.T

Radio T

Root T

$\sum_{n=1}^{\infty} a_n$ $\rightarrow$ A.S.T

Ex : Test the following series for

Abs. Conv, Conditionally Conv or div ?

$\boxed{1}$ $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$

$\Rightarrow \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{3}}}$ $\quad p = \frac{1}{3} < 1$

div by P-T

$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ A.S

$\lim_{n \to \infty} |a_n| = \lim_{n \to \infty} \frac{1}{\sqrt[3]{n}} = \frac{1}{\infty} = \boxed{0}$ ✓

a_n is decreasing $\quad a_1 = \frac{1}{\sqrt[3]{1}} = 1$

$a_1 > a_2$ ✓ $\quad a_2 = \frac{1}{\sqrt[3]{2}}$

$\Rightarrow$ Conv by A.S.T

So The series is Conditionally Conv ✓

$\boxed{2}$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+1}$

$\Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+1} \xrightarrow{C.T} \frac{1}{n^3+1} < \frac{1}{n^3}$ → $p = 3 > 1$ Conv by p-T

↳ is Conv by C.T

$\displaystyle\sum_{n=1}^{\infty} |a_n|$ is Conv <u>So</u> is Abs. Conv

$\boxed{3}$ $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{2^n}$

$\Rightarrow \displaystyle\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{n}{2^n}$ $\xrightarrow{C.T}$ ✓

↳ Root Test ✓

<u>Root test</u> $\displaystyle\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$

$$\lim_{n\to\infty} \left(\frac{n}{2^n}\right)^{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^{\frac{1}{n}}}{2} = \frac{1}{2} < 1$$

Conv by Root.T

$\Rightarrow$ The series is Abs. Conv.

$\boxed{4}$ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n}{2^n \cdot n^3}$

$\Rightarrow \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{3^n}{2^n \cdot n^3}$ $\rightsquigarrow$ Root test

$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(\frac{3^n}{2^n \cdot n^3}\right)^{\frac{1}{n}}$

$= \lim_{n\to\infty} \frac{3}{2 \cdot n^{\frac{3}{n}}} = \frac{3}{2 \cdot 1} = \frac{3}{2} > 1$

div by root Test

$\Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n}{2^n \cdot n^3}$ A.S.T

$\hookrightarrow \lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{3^n}{2^n \cdot n^3} \longrightarrow \infty \neq 0$

is div by A.S.T

$\Rightarrow$ The Series is div

$\boxed{5}$ $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+1}$

$\Rightarrow \sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$ use L.C.T

$b_n = \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}} = \frac{1}{n^{\frac{1}{2}}}$ $p = \frac{1}{2} < 1$
div by P-T

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\sqrt{n}}{n+1} * \frac{\sqrt{n}}{1}$

$= \lim_{n\to\infty} \frac{n}{n+1} = \frac{\infty}{\infty}!! \xrightarrow{\text{L.R}} \lim_{n\to\infty} \frac{n}{n} = 1$ نفس السلوك

$\sum_{n=1}^{\infty} |a_n|$ is div by L.C.T

$\Rightarrow \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+1}$ A.S.T

- $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{\sqrt{n}}{n+1} = \frac{N}{\infty} = \boxed{0}$ ✓
- a_n is decreasing $a_1 > a_2$

Conv by A.S.T

$\Rightarrow$ The series is Cond. Conv ✓

$\boxed{6}$ $$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^3}$$

$\boxed{7}$ $$\sum_{n=3}^{\infty} (-1)^n \cdot \frac{1}{\ln(n)}$$

$\boxed{8}$ $$\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{n^3}{e^n}$$

Test the series for Abs. Conv

Cond. Conv or div ?