# CHAPTER ONE
# KINEMATICS OF PARTICLES

## 1-1 Introduction to Dynamics

The term dynamic may be defined simply as time-varying
*Dynamics includes*:

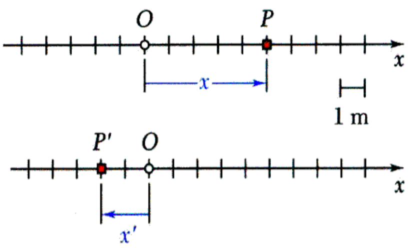
- Kinematics: study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time without reference to the cause of motion.
- Kinetics: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.

*The motion of particles include:*

- Rectilinear motion: position, velocity, and acceleration of a particle as it moves along a straight line.
- Curvilinear motion: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

## 1-2 Rectilinear Motion of Particals: Position, Velocity & Acceleration

Particle moving along a straight line is said to be in rectilinear motion. At any given instant t, the particle will occupy a certain position on the straight line. To define the position P of the particle, one can choose a fixed origin O on the straight line and a positive direction along the line. the distance x can be measured from O to P and record it with a plus or minus sign, according to

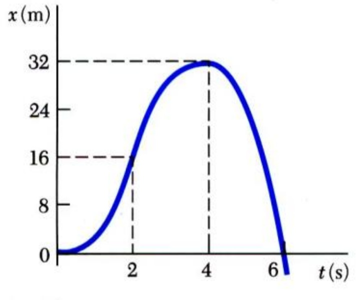



whether P is reached from O by moving along the line in the positive or the negative direction. The distance x, with the appropriate sign, completely defines the position of the particle; it is called the position coordinate of the particle considered.

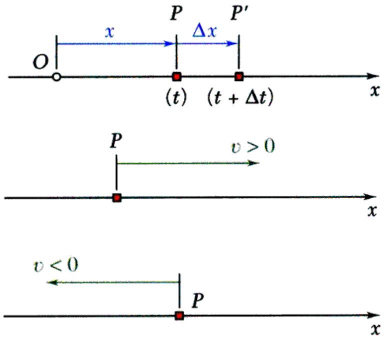
The motion of a particle is known if the position coordinate for particle is known for every value of time t. Motion of the particle may be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$

or in the form of a graph x vs. t.




Consider particle which occupies position *P* at time *t* and *P'* at *t+Dt*,




$$Average\ velocity = \frac{\Delta x}{\Delta t}$$

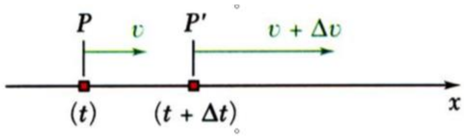
$$Instantaneous\ velocity = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*. From the definition of a derivative, e.g.,

$$x = 6t^2 - t^3$$

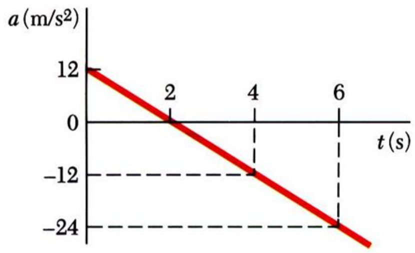
$$v = \frac{dx}{dt} = 12t - 3t^2$$

Now consider particle with velocity v at time t and v' at t+Dt;




$$Instantaneous\ acceleration = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d^2 x}{dt^2}$$

From the definition of a derivative, e.g.,




$$v = 12t - 3t^2$$

$$a = \frac{dv}{dt} = 12 - 6t$$

## *1-3 Uniform Rectilinear Motion*

For particle in uniform rectilinear motion, the acceleration is zero and the velocity is constant, then:

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

## *1-4 Uniformly Accelerated Rectilinear Motion*

For particle in uniformly accelerated rectilinear motion, the acceleration of the particle is constant.

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_{0}^{t} dt \qquad v - v_0 = at$$

$$v = v_0 + at$$

$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_{0}^{t} (v_0 + at)dt \qquad x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

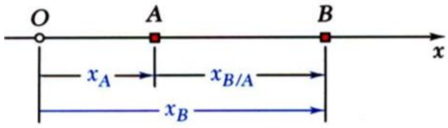
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \qquad \tfrac{1}{2}\left(v^2 - v_0^2\right) = a(x - x_0)$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

## 1-5 Motion of Several Particles: Relative Motion

For particles moving along the same line, time should be recorded from the same starting instant and displacements should be measured from the same origin in the same direction.




$x_{B/A} = x_B - x_A$ = relative position of B with respect to A

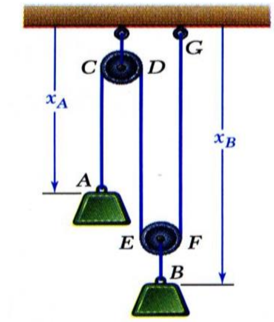
$x_B = x_A + x_{B/A}$

$v_{B/A} = v_B - v_A$ = relative velocity of B with respect to A ……. $v_B = v_A + v_{B/A}$

$a_{B/A} = a_B - a_A$ = relative acceleration of B with respect to A ........ $a_B = a_A + a_{B/A}$

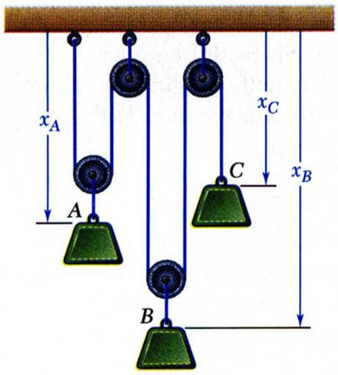
## 1-6 Motion of Several Particles: Dependent Motion

Position of a particle may depend on position of one or more other particles. Position of block *B* depends on position of block *A*. Since rope is of constant length, it follows that sum of lengths of segments must be constant.




$x_A + 2x_B =$ constant (one degree of freedom)

The positions of three blocks are also dependent as shown:

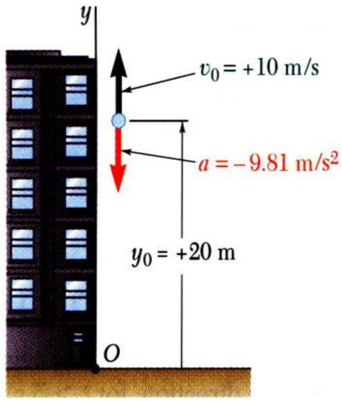



$2x_A + 2x_B + x_C =$ Positions of three blocks are dependent.

For linearly related positions, similar relations hold between velocities and accelerations.

$$2\frac{dx_A}{dt} + 2\frac{dx_B}{dt} + \frac{dx_C}{dt} = 0 \quad \text{or} \quad 2v_A + 2v_B + v_C = 0$$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \quad \text{or} \quad 2a_A + 2a_B + a_C = 0$$




***Example1:*** Ball tossed with 10 m/s vertical velocity from window 20 m above ground. Determine:

1 Velocity and elevation above ground at time t,
2 Highest elevation reached by ball and corresponding time, and
3 Time when ball will hit the ground and corresponding velocity.

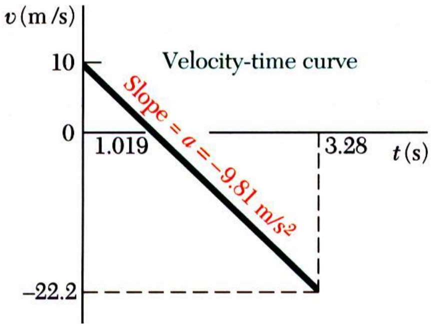
## *Solution:*

- Integrate twice to find v(t) and y(t).

$$\frac{dv}{dt} = a = -9.81\,\mathrm{m/s^2}$$

$$\int_{v_0}^{v(t)} dv = -\int_0^t 9.81\,dt \qquad v(t) - v_0 = -9.81t$$

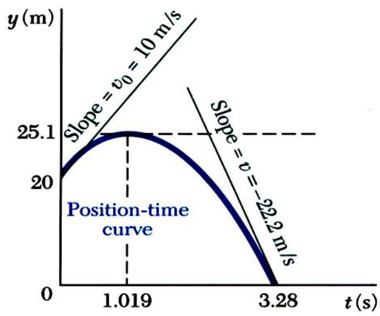
$$v(t) = 10\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)t$$




- Solve for $t$ at which velocity equals zero and evaluate corresponding altitude.

$$v(t) = 10\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)t = 0$$

$$t = 1.019\,\mathrm{s}$$




$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0}^{y(t)} dy = \int_0^t (10 - 9.81t)\,dt$$

$$y(t) - y_0 = 10t - \tfrac{1}{2}9.81t^2$$

$$y(t) = 20\,\mathrm{m} + \left(10\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^{\;2}$$

- Solve for $t$ at which altitude equals zero and evaluate corresponding velocity.

$$y = 20\,\mathrm{m} + \left(10\frac{\mathrm{m}}{\mathrm{s}}\right)(1.019\,\mathrm{s}) - \left(4.905\frac{\mathrm{m}}{\mathrm{s}^2}\right)(1.019\,\mathrm{s})^2 \;\;\ldots\ldots\ldots\;\; y = 25.1\,\mathrm{m}$$

- Solve for t at which altitude equals zero and evaluate corresponding velocity.

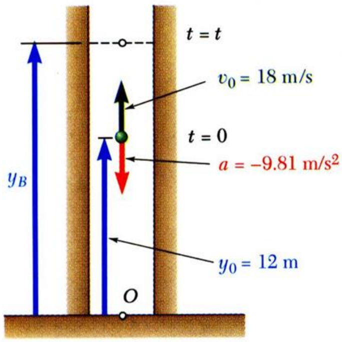
$$y(t) = 20\,\mathrm{m} + \left(10\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2 = 0$$

$$t = -1.243\,\mathrm{s}\ \left(\text{meaningles s}\right) \quad \text{so} \quad t = 3.28\,\mathrm{s}$$

$$v(t) = 10\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)t \quad \ldots\ldots.. \quad v(3.28\,\mathrm{s}) = 10\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)(3.28\,\mathrm{s}) = -22.2\frac{\mathrm{m}}{\mathrm{s}}$$

***Example 2:*** Ball thrown vertically from 12 m level in elevator shaft with initial velocity of 18 m/s. At same instant, open-platform elevator passes 5 m level moving upward at 2 m/s. Determine (a) when and where ball hits elevator and (b) relative velocity of ball and elevator at contact.

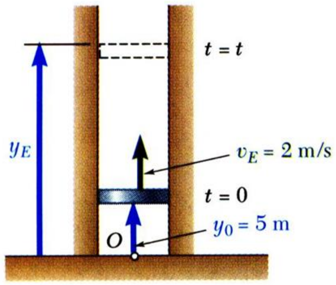
***Solution:***




1. Substitute initial position and velocity and constant acceleration of ball into general equations for uniformly accelerated rectilinear motion.

$$v_B = v_0 + at = 18\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)t$$

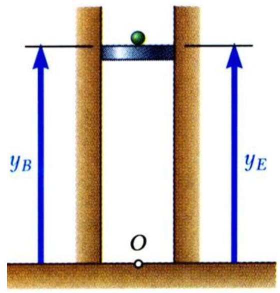
$$y_B = y_0 + v_0 t + \tfrac{1}{2}at^2 = 12\,\mathrm{m} + \left(18\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2$$




2. Substitute initial position and constant velocity of elevator into equation for uniform rectilinear motion.

$$v_E = 2\frac{\mathrm{m}}{\mathrm{s}}$$

$$y_E = y_0 + v_E t = 5\,\mathrm{m} + \left(2\frac{\mathrm{m}}{\mathrm{s}}\right)t$$




3. Write equation for relative position of ball with respect to elevator and solve for zero relative position, i.e., impact.

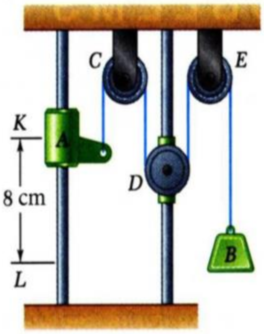
$$y_{B/E} = \left(12 + 18t - 4.905t^2\right) - (5 + 2t) = 0$$

$$t = -0.39\,\mathrm{s}\ \left(\text{meaningles s}\right) \ \ldots. \ \text{So}\ t = 3.65\,\mathrm{s}$$

$$y_E = 5 + 2(3.65) = 12.3\,\mathrm{m}$$

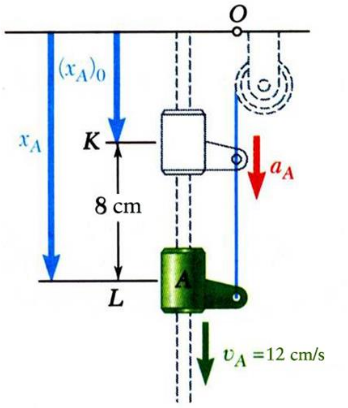
$$v_{B/E} = (18 - 9.81t) - 2 = 16 - 9.81(3.65) = -19.81\frac{\mathrm{m}}{\mathrm{s}}$$

***Example 3:*** Pulley D is attached to a collar which is pulled down at 3 cm/s. At t = 0, collar A starts moving down from K with constant acceleration and zero initial velocity. Knowing that velocity of collar A is 12 cm/s as it passes L, determine the change in elevation, velocity, and acceleration of block B when block A is at L.




## *Solution:*

1. Define origin at upper horizontal surface with positive displacement downward.
2. Collar *A* has uniformly accelerated rectilinear motion. Solve for acceleration and time *t* to reach *L*.



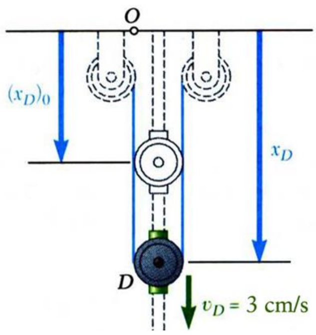

$$v_A^2 = (v_A)_0^2 + 2a_A\left[x_A - (x_A)_0\right]$$

$$\left(12\frac{\text{cm}}{\text{s}}\right)^2 = 2a_A(8\,\text{cm}) \qquad a_A = 9\frac{\text{cm}}{\text{s}^2}$$

$$v_A = (v_A)_0 + a_A t$$

$$12\frac{\text{cm}}{\text{s}} = 9\frac{\text{cm}}{\text{s}^2}t \qquad t = 1.333\,\text{s}$$

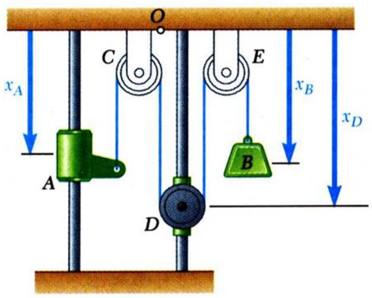
3. Pulley *D* has uniform rectilinear motion. Calculate change of position at time *t*.




$$x_D = (x_D)_0 + v_D t$$

$$x_D - (x_D)_0 = \left(3\frac{\text{cm}}{\text{s}}\right)(1.333\,\text{s}) = 4\,\text{cm}$$

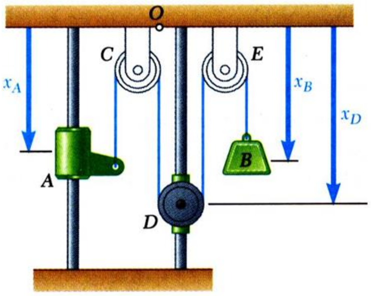
4. Block *B* motion is dependent on motions of collar *A* and pulley *D*. Write motion relationship and solve for change of block *B* position at time *t*. Since Total length of cable remains constant,




$$x_A + 2x_D + x_B = (x_A)_0 + 2(x_D)_0 + (x_B)_0$$
$$\left[x_A - (x_A)_0\right] + 2\left[x_D - (x_D)_0\right] + \left[x_B - (x_B)_0\right] = 0$$
$$(8\,\text{cm}) + 2(4\,\text{cm}) + \left[x_B - (x_B)_0\right] = 0$$

$$x_B - (x_B)_0 = -16\,\text{cm}$$

5. Differentiate motion relation twice to develop equations for velocity and acceleration of block *B*.




$$x_A + 2x_D + x_B = \text{constant}$$

$$v_A + 2v_D + v_B = 0$$

$$\left(12\frac{\text{cm}}{\text{s}}\right) + 2\left(3\frac{\text{cm}}{\text{s}}\right) + v_B = 0$$

$$v_B = 18\frac{\text{cm}}{\text{s}}$$

$$a_A + 2a_D + a_B = 0$$

$$\left(9\frac{\text{cm}}{\text{s}^2}\right) + v_B = 0 \qquad \ldots\ldots\ a_B = -9\frac{\text{cm}}{\text{s}^2}$$