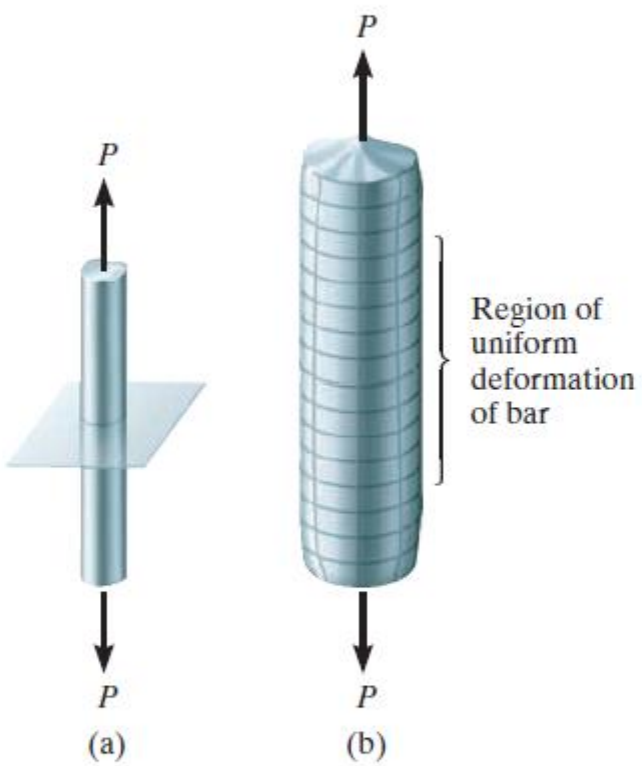
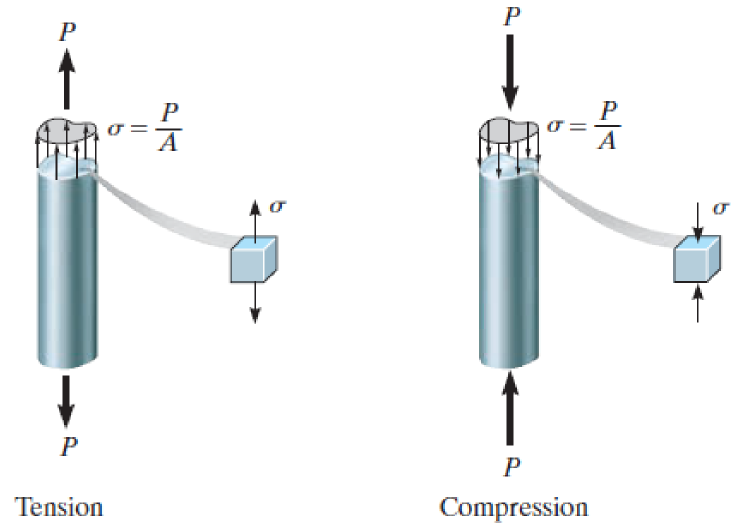
Chapter 2-- Stresses

Average Normal Stress in an Axially Loaded Bar

This bar is **prismatic** since all cross sections are the same

Homogeneous material has the same physical and mechanical properties throughout its volume, and ***isotropic material*** has these same properties in all directions.

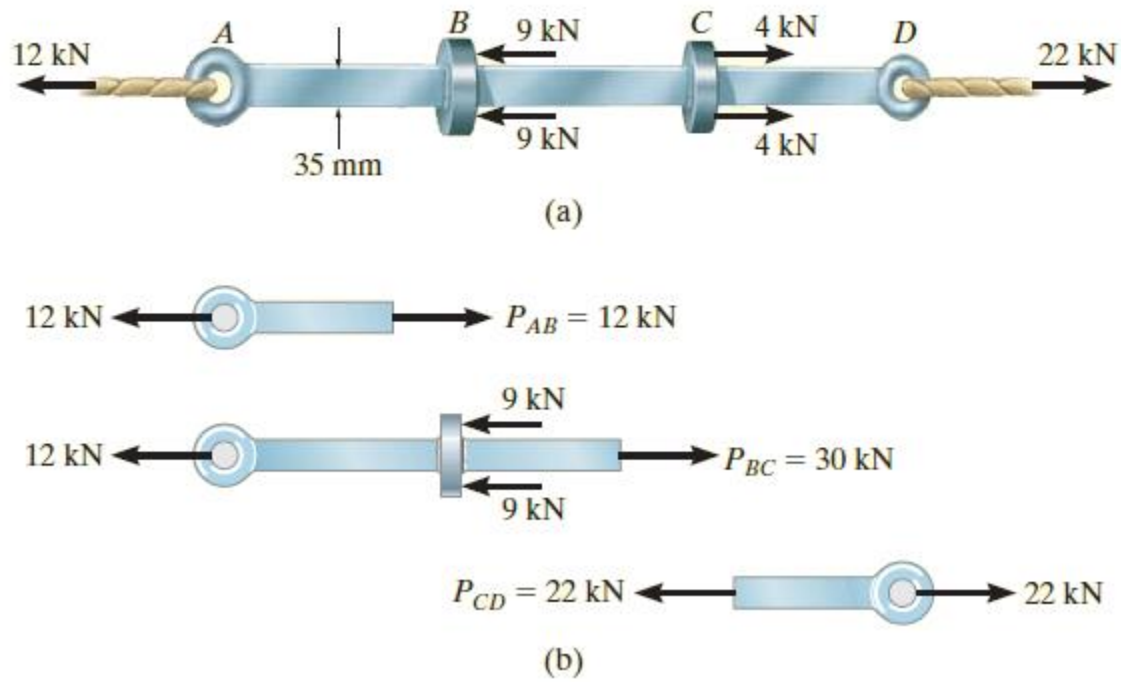




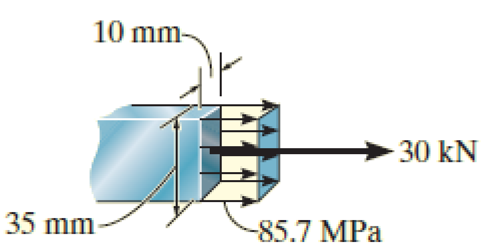
$$\sigma = \frac{P}{A}$$

Examples

The bar in Fig. 1–16*a* has a constant width of 35 mm and a thickness of 10 mm. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{ N}}{(0.035\text{ m})(0.010\text{ m})} = 85.7\text{ MPa} \qquad \textit{Ans.}$$



The casting shown in Fig. 1–18*a* is made of steel having a specific weight of $\gamma_{st} = 490\ \text{lb/ft}^3$. Determine the average compressive stress acting at points *A* and *B*.

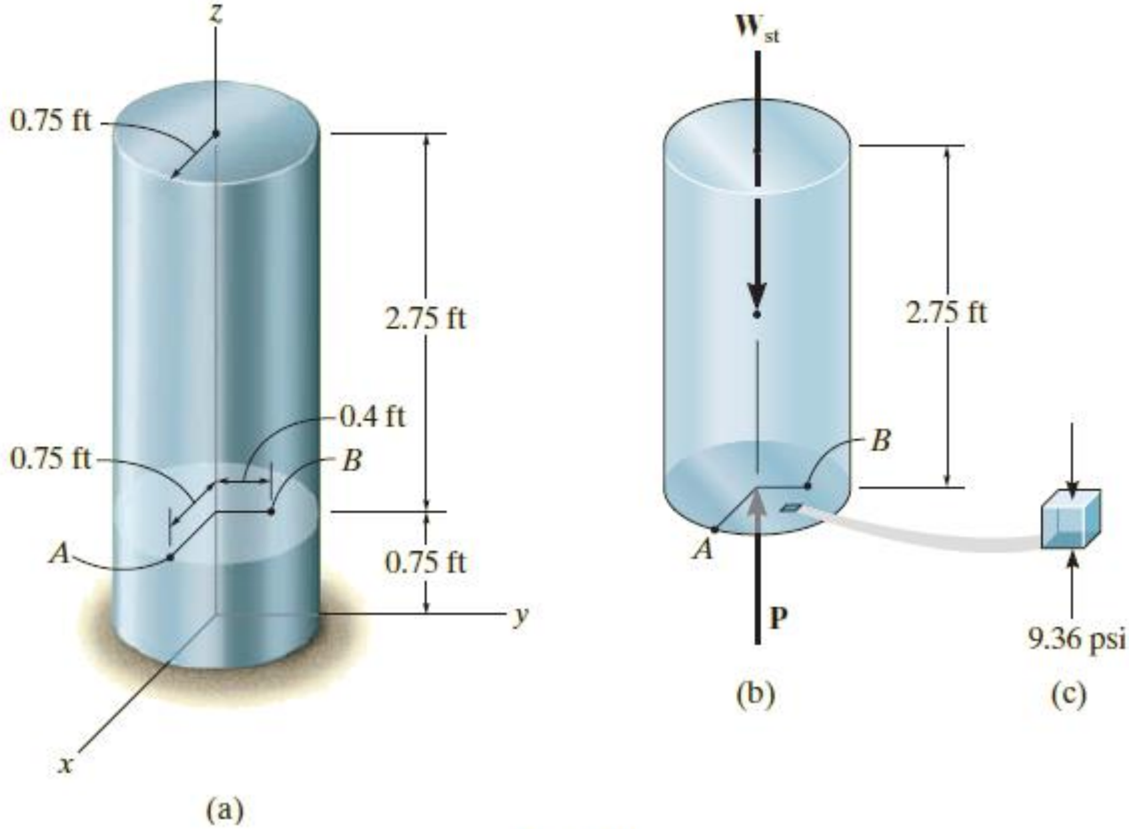


Fig. 1–18

SOLUTION

Internal Loading. A free-body diagram of the top segment of the casting where the section passes through points *A* and *B* is shown in Fig. 1–18*b*. The weight of this segment is determined from $W_{st} = \gamma_{st} V_{st}$. Thus the internal axial force *P* at the section is

$$+\uparrow \Sigma F_z = 0; \qquad P - W_{st} = 0$$

$$P - (490\ \text{lb/ft}^3)(2.75\ \text{ft})[\pi(0.75\ \text{ft})^2] = 0$$

$$P = 2381\ \text{lb}$$

Average Compressive Stress. The cross-sectional area at the section is $A = \pi(0.75\ \text{ft})^2$, and so the average compressive stress becomes

$$\sigma = \frac{P}{A} = \frac{2381\ \text{lb}}{\pi(0.75\ \text{ft})^2} = 1347.5\ \text{lb/ft}^2$$

$$\sigma = 1347.5\ \text{lb/ft}^2\ (1\ \text{ft}^2/144\ \text{in}^2) = 9.36\ \text{psi} \qquad \textit{Ans.}$$

Member AC shown in Fig. 1–19a is subjected to a vertical force of 3 kN. Determine the position x of this force so that the average compressive stress at the smooth support C is equal to the average tensile stress in the tie rod AB. The rod has a cross-sectional area of 400 mm^2 and the contact area at C is 650 mm^2.

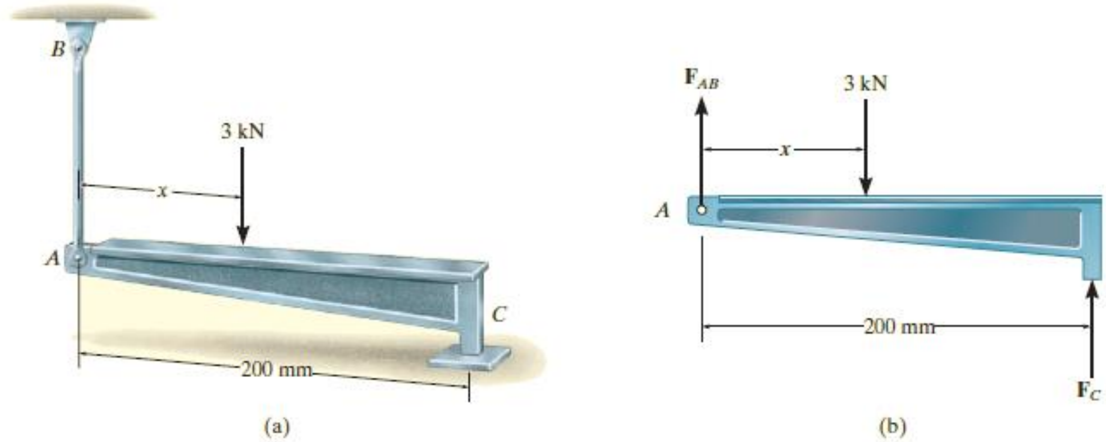


Fig. 1–19

SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} + F_C - 3000 \text{ N} = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad -3000 \text{ N}(x) + F_C(200 \text{ mm}) = 0 \qquad (2)$$

Average Normal Stress. A necessary third equation can be written that requires the tensile stress in the bar AB and the compressive stress at C to be equivalent, i.e.,

$$\sigma = \frac{F_{AB}}{400 \text{ mm}^2} = \frac{F_C}{650 \text{ mm}^2}$$

$$F_C = 1.625 F_{AB}$$

Substituting this into Eq. 1, solving for F_{AB}, then solving for F_C, we obtain

$$F_{AB} = 1143 \text{ N}$$

$$F_C = 1857 \text{ N}$$

The position of the applied load is determined from Eq. 2,

$$x = 124 \text{ mm} \qquad \textit{Ans.}$$

NOTE: $0 < x < 200$ mm, as required.

Average Shear Stress

Shear stress has been defined in Section 1.3 as the stress component that acts *in the plane* of the sectioned area.

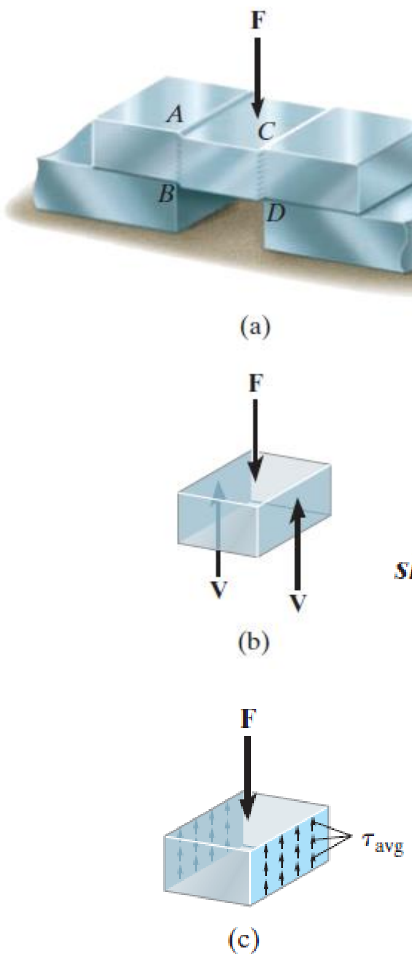


Fig. 1–20

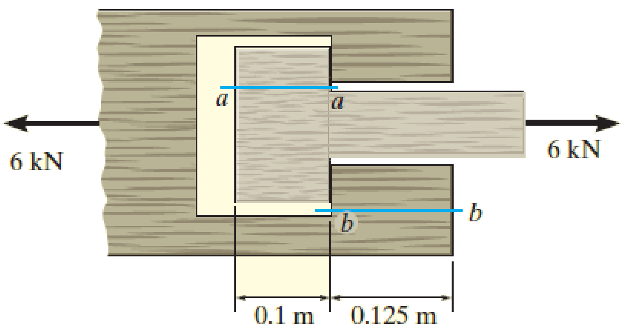
Fig. 1–20*b*, indicates that the shear force $V = F/2$ must be applied at each section to hold the segment in equilibrium. The ***average shear stress*** distributed over each sectioned area that develops this shear force is defined by

$$\tau_{avg} = \frac{V}{A} \tag{1–7}$$

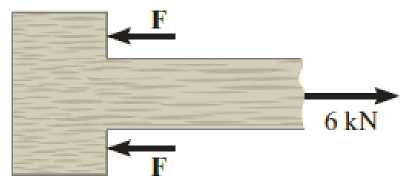
The loading case discussed here is an example of ***simple or direct shear***, since the shear is caused by the *direct action* of the applied load **F**.

Examples

If the wood joint in Fig. 1–23*a* has a width of 150 mm, determine the average shear stress developed along shear planes *a–a* and *b–b*. For each plane, represent the state of stress on an element of the material.



(a)



(b)

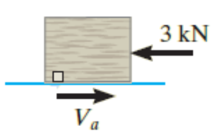
Fig. 1–23

SOLUTION

Internal Loadings. Referring to the free-body diagram of the member, Fig. 1–23*b*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 6 \text{ kN} - F - F = 0 \qquad F = 3 \text{ kN}$$

Now consider the equilibrium of segments cut across shear planes *a–a* and *b–b*, shown in Figs. 1–23*c* and 1–23*d*.



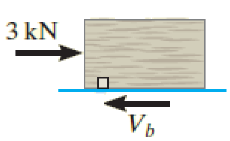
(c)

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad V_a - 3 \text{ kN} = 0 \qquad V_a = 3 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3 \text{ kN} - V_b = 0 \qquad V_b = 3 \text{ kN}$$

Average Shear Stress.

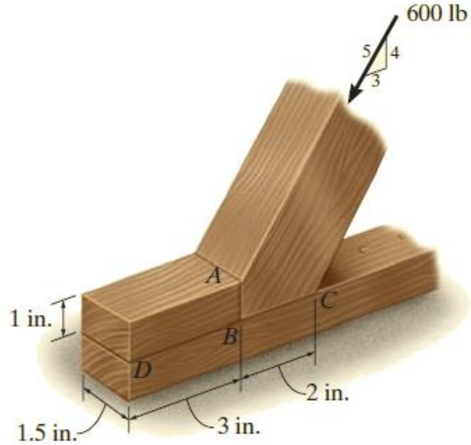
$$(\tau_a)_{avg} = \frac{V_a}{A_a} = \frac{3(10^3) \text{ N}}{(0.1 \text{ m})(0.15 \text{ m})} = 200 \text{ kPa} \qquad \textit{Ans.}$$



(d)

$$(\tau_b)_{avg} = \frac{V_b}{A_b} = \frac{3(10^3) \text{ N}}{(0.125 \text{ m})(0.15 \text{ m})} = 160 \text{ kPa} \qquad \textit{Ans.}$$

The inclined member in Fig. 1–24a is subjected to a compressive force of 600 lb. Determine the average compressive stress along the smooth areas of contact defined by AB and BC, and the average shear stress along the horizontal plane defined by DB.

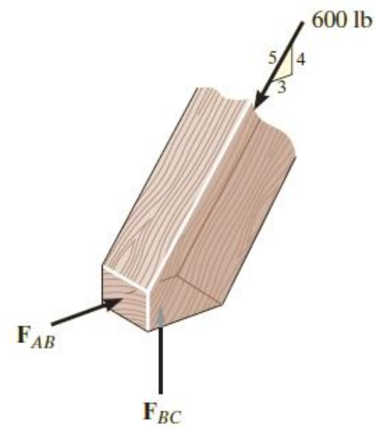


(a)

Fig. 1–24

SOLUTION

Internal Loadings. The free-body diagram of the inclined member is shown in Fig. 1–24b. The compressive forces acting on the areas of contact are

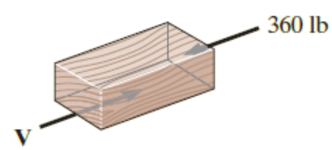


(b)

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{AB} - 600\text{ lb}\left(\tfrac{3}{5}\right) = 0 \qquad F_{AB} = 360\text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} - 600\text{ lb}\left(\tfrac{4}{5}\right) = 0 \qquad F_{BC} = 480\text{ lb}$$

Also, from the free-body diagram of the top segment ABD of the bottom member, Fig. 1–24c, the shear force acting on the sectioned horizontal plane DB is



(c)

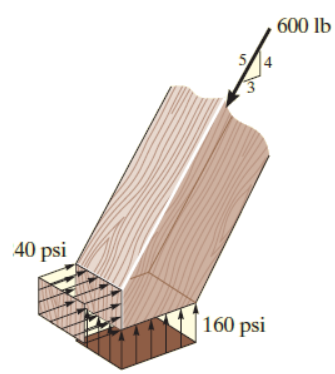
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad V = 360\text{ lb}$$

Average Stress. The average compressive stresses along the horizontal and vertical planes of the inclined member are

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{360\text{ lb}}{(1\text{ in.})(1.5\text{ in.})} = 240\text{ psi} \qquad \textit{Ans.}$$

$$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{480\text{ lb}}{(2\text{ in.})(1.5\text{ in.})} = 160\text{ psi} \qquad \textit{Ans.}$$

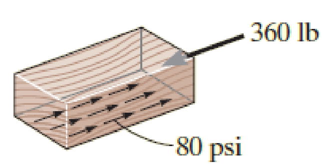
These stress distributions are shown in Fig. 1–24d.



(d)

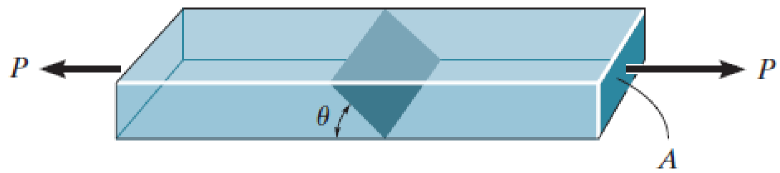
The average shear stress acting on the horizontal plane defined by DB is

$$\tau_{\text{avg}} = \frac{360\text{ lb}}{(3\text{ in.})(1.5\text{ in.})} = 80\text{ psi} \qquad \textit{Ans.}$$



(e)

Show that the average normal and shear stresses on the shade section in the figure below are given as



$$V = P\cos\theta,\ N = P\sin\theta,$$

$$\sigma = \frac{P}{A}\sin^2\theta,\ \tau_{avg} = \frac{P}{2A}\sin 2\theta$$

The exercise below is on the normal stress (DIY)

The plate has a width of 0.5 m. If the stress distribution at the support varies as shown, determine the force **P** applied to the plate and the distance *d* to where it is applied.