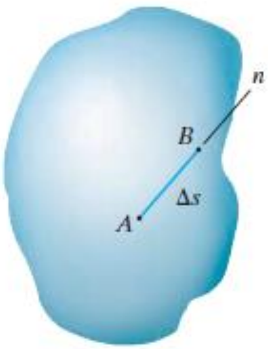
Chapter 3

Strains

Normal Strain. If we define the normal strain as the change in length of a line per unit length, then we will not have to specify the *actual length* of any particular line segment.

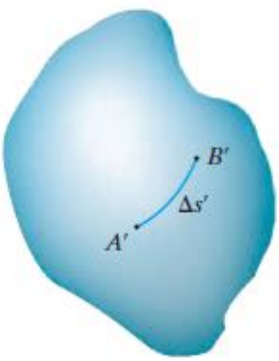


Undeformed body

(a)

$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$

$$\epsilon = \lim_{B \to A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$



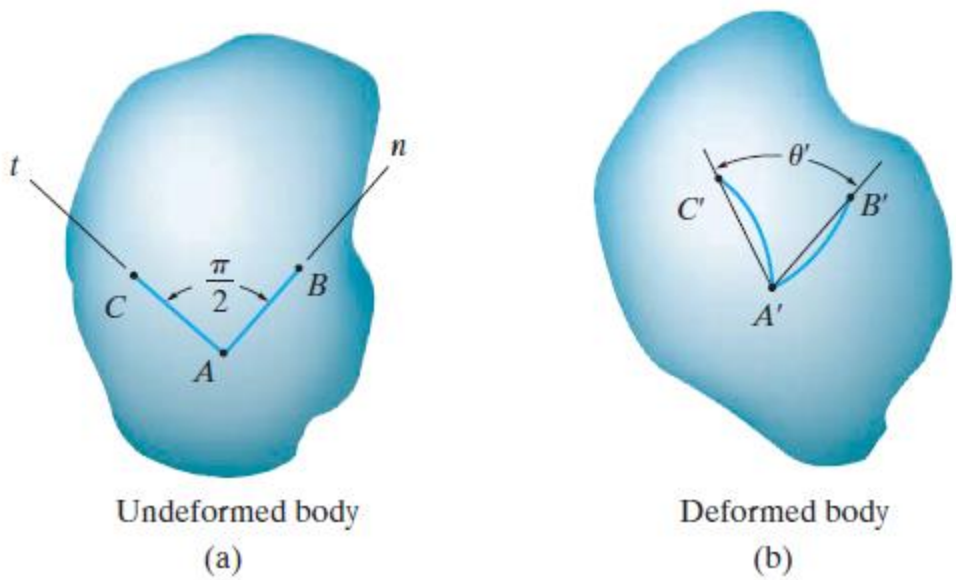
Deformed body

(b)

Units: In SI units: m/m or cm/cm or mm/mm or $\frac{\mu m}{m} = 10^{-6} m/m$ and in Inch-pound in/in.

for experimental work, strain is expressed as a percent, e.g., 0.001 m/m = 0.1%. As an example, a normal strain of $480(10^{-6})$ can be reported as $480(10^{-6})$ in./in., 480 μm/m, or 0.0480%. Also, one can state this answer as simply 480 μ (480 "micros").

Shear Strain. Deformations not only cause line segments to elongate or contract, but they also cause them to change direction. If we select two line segments that are originally perpendicular to one another, then the change in angle that occurs between these two line segments is referred to as ***shear strain***.

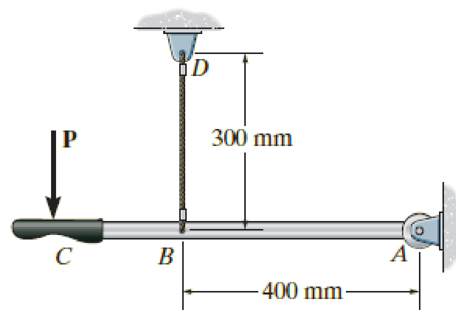


$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

Examples:

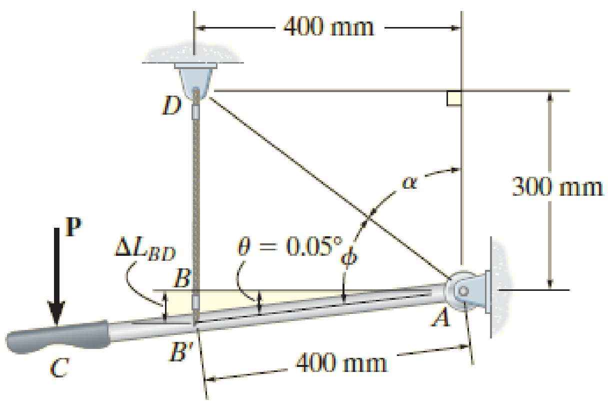
When force **P** is applied to the rigid lever arm ABC in Fig. 2–5a, the arm rotates counterclockwise about pin A through an angle of 0.05°. Determine the normal strain developed in wire BD.

SOLUTION I



Elongation in rod L_{BD}

$\Delta L_{BD} = \sin(0.05) * 400mm = 0.3491\ mm$



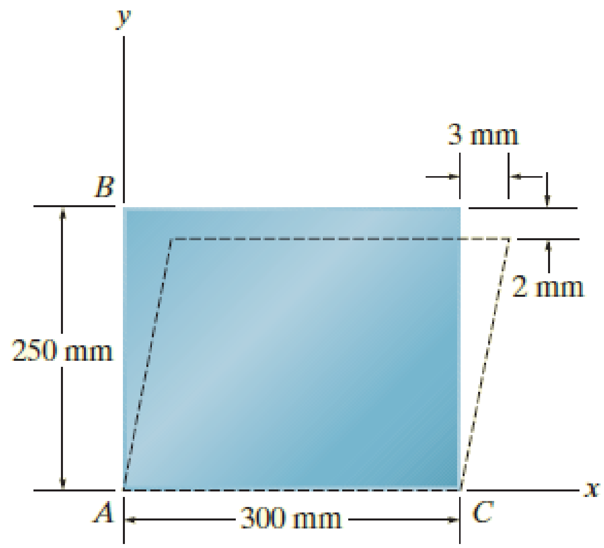
(b)

Fig. 2–5

Strain in rod L_{BD} is

$$\epsilon_{BD} = \frac{\Delta L_{BD}}{L_{BD}} = \frac{0.3491\ \text{mm}}{300\ \text{mm}} = 0.00116\ \text{mm/mm} \qquad \textit{Ans.}$$

Due to a loading, the plate is deformed into the dashed shape shown in Fig. 2–6*a*. Determine (a) the average normal strain along the side AB, and (b) the average shear strain in the plate at A relative to the x and y axes.

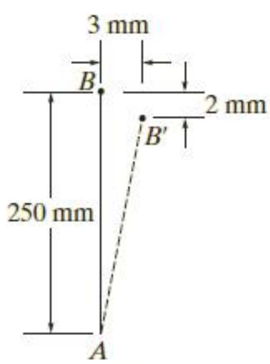


(a)

Fig. 2–6

SOLUTION

Part (a). Line AB, coincident with the y axis, becomes line AB' after deformation, as shown in Fig. 2–6*b*. The length of AB' is



(b)

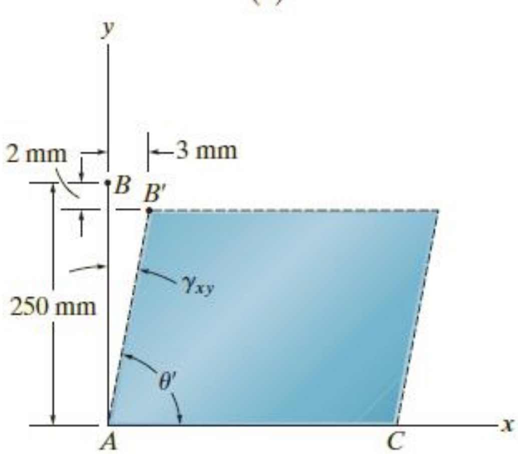
$$AB' = \sqrt{(250 \text{ mm} - 2 \text{ mm})^2 + (3 \text{ mm})^2} = 248.018 \text{ mm}$$

The average normal strain for AB is therefore

$$(\epsilon_{AB})_{\text{avg}} = \frac{AB' - AB}{AB} = \frac{248.018 \text{ mm} - 250 \text{ mm}}{250 \text{ mm}}$$

$$= -7.93(10^{-3}) \text{ mm/mm} \qquad \textit{Ans.}$$

The negative sign indicates the strain causes a contraction of AB.



Part (b). As noted in Fig. 2–6*c*, the once 90° angle BAC between the sides of the plate at A changes to θ' due to the displacement of B to B'. Since $\gamma_{xy} = \pi/2 - \theta'$, then γ_{xy} is the angle shown in the figure. Thus,

$$\gamma_{xy} = \tan^{-1}\left(\frac{3 \text{ mm}}{250 \text{ mm} - 2 \text{ mm}}\right) = 0.0121 \text{ rad} \qquad \textit{Ans.}$$