Chapter 4

Mechanical properties

The Stress–Strain Diagram

Conventional Stress–Strain Diagram. We can determine the ***nominal*** or ***engineering stress*** by dividing the applied load P by the specimen's *original* cross-sectional area A_0.

$$\boxed{\sigma = \frac{P}{A_0}}$$

Likewise, the ***nominal*** or ***engineering strain*** is found directly from the strain gauge reading, or by dividing the change in the specimen's gauge length, δ, by the specimen's original gauge length L_0.

$$\boxed{\epsilon = \frac{\delta}{L_0}}$$

Elastic behavior

If the load is removed, the specimen will return back to its original shape.

proportional limit

The stress-strain relationship up to this point is linear elastic.

Elastic limit

After this point the stress-strain curve is not linear but still elastic.

Yielding

A slight increase in stress above the elastic limit will result in a breakdown of the material and cause it to *deform permanently* and becomes ***plastic***.

Strain hardening

Increase in the stress after yielding until it reaches maximum stress σ_u.

Necking

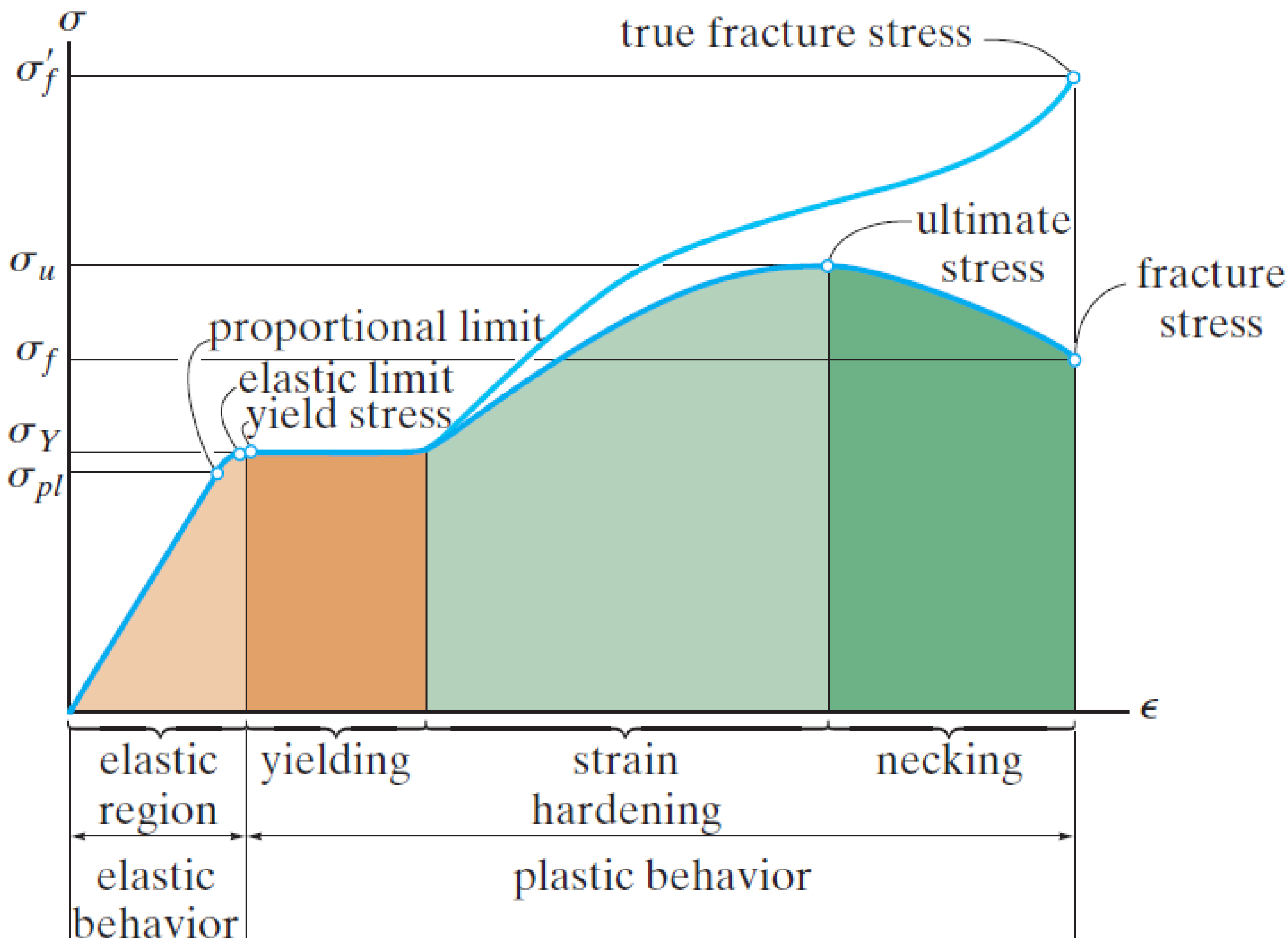

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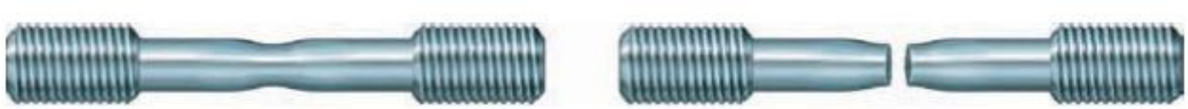
Figure: Stress strain curve for steel.

Necking

A “neck” tends to form due to the elongation in the specimen and finally the specimen fails at the fracture stress σ_f.



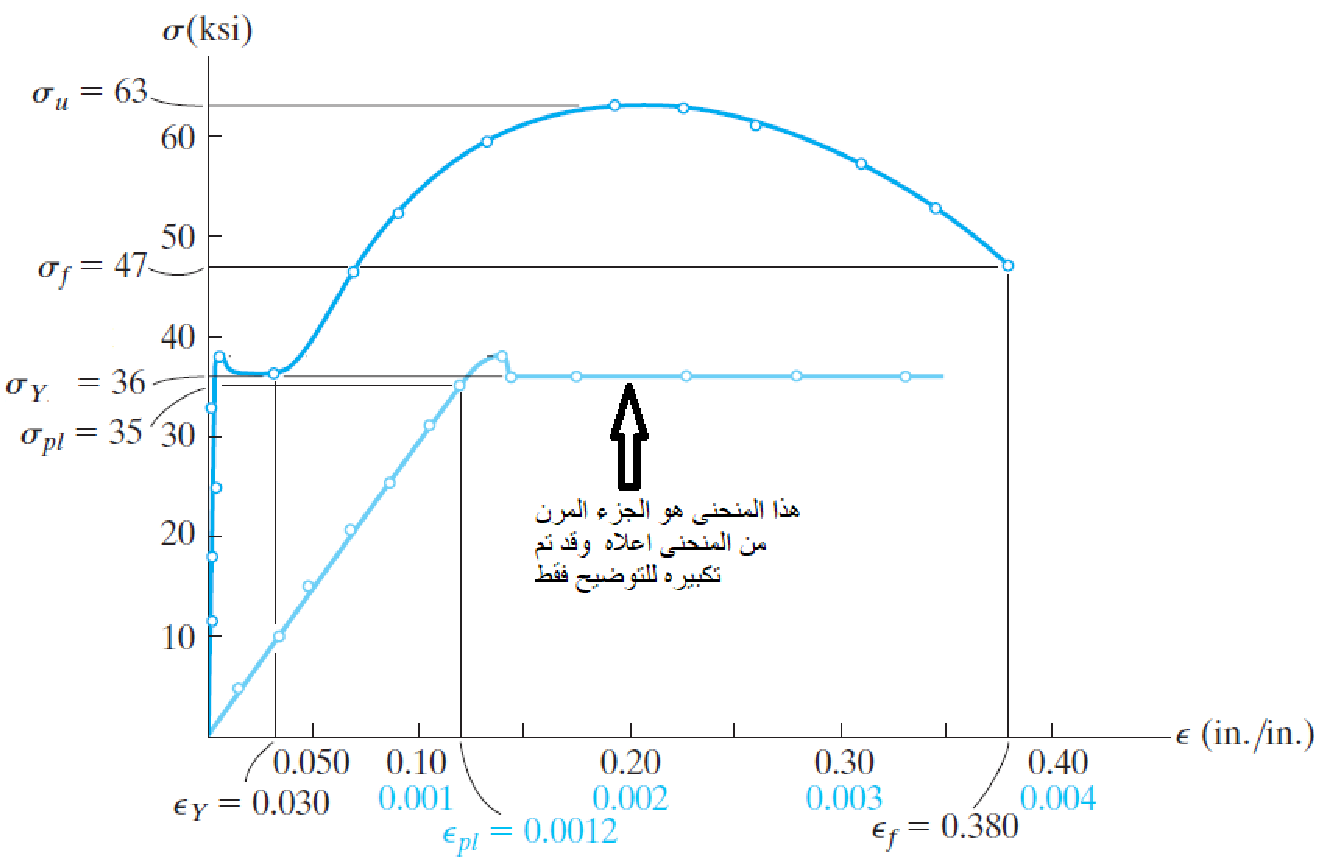
Necking

Failure of a ductile material

True Stress–Strain Diagram.

In reality, due to the elongation of the specimen, the cross-sectional area decreases and the length increases. As a result, the true stress-strain curve is obtained by using the real area A then $\sigma = \frac{P}{A}$ and the true strain is obtained bu using the true length L then $\epsilon = \frac{\delta L}{L}$.

Ductile Materials. Any material that can be subjected to large strains before it fractures is called a ***ductile material*** *(an example is steel).*



Example of stress-strain curve for steel.

From the above figure we see:

Proportional limit stress is $\sigma_{pl} = 35\ ksi$.

Yield stress is $\sigma_Y = 36\ ksi$

Ultimate stress is $\sigma_Y = 63\ ksi$

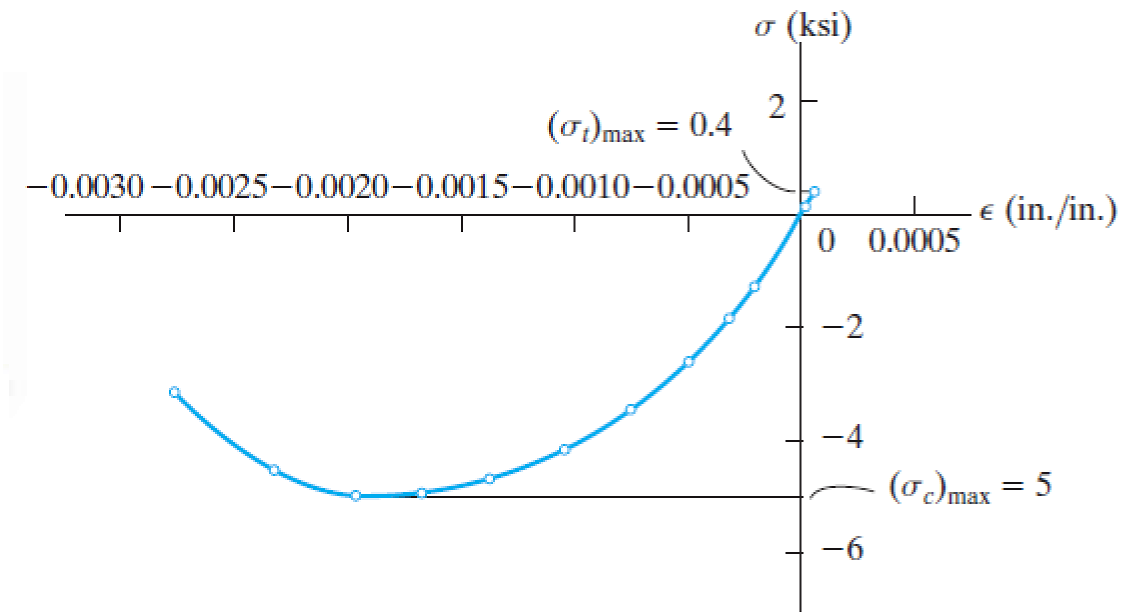
Failure stress is $\sigma_f = 47\ ksi$

Proportional limit strain $\epsilon_{pl} = 0.0012$

Yield strain $\epsilon_Y = 0.03$

Failure strain $\epsilon_f = 0.38$

Brittle Materials. *Materials that exhibit little or no yielding before failure are referred to as* ***brittle materials (****an example is concrete).*



Hook's law (Hook 1676)

$$\boxed{\sigma = E\epsilon}$$

From the above stress-strain curve of the steel

$$E = \frac{\sigma_{pl}}{\epsilon_{pl}} = \frac{35\ \text{ksi}}{0.0012\ \text{in./in.}} = 29(10^3)\ \text{ksi}$$

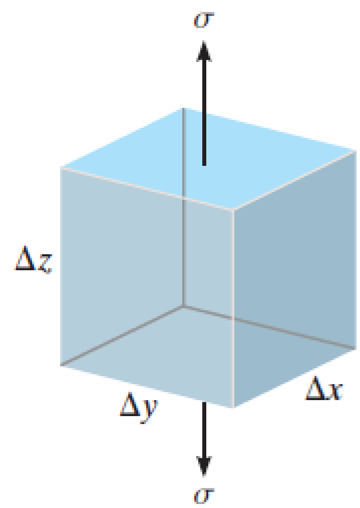
And *E is Young's modulus of elasticity (Young 1807)*

Strain Energy density

Energy is work= Force*distance and strain energy is work due to deformations.

For the specimen in tension in this figure, assume the elongation is δL and the cross-sectional area is A, then

Strain energy density is the strain energy divided by the volume



$$Strain\ energy\ density = \frac{\frac{1}{2}F * \delta L}{\Delta V}$$

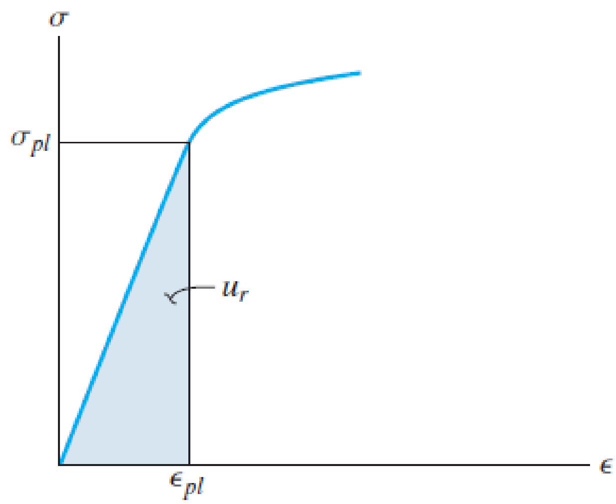
$$= \frac{1}{2\Delta V}(\sigma * A) * (\epsilon * \Delta z) = \frac{1}{2\Delta V}(\sigma\epsilon) * (A * \Delta z) = \frac{1}{2\Delta V}(\sigma\epsilon) * (\Delta V)$$

$$Strain\ energy\ density = \frac{1}{2}(\sigma\epsilon) = \frac{1}{2}(\sigma * \sigma/E) = \frac{1}{2E}\sigma^2$$

Modulus of Resilience. In particular, when the stress σ reaches the proportional limit, the strain-energy density is referred to as the ***modulus of resilience***, i.e.,

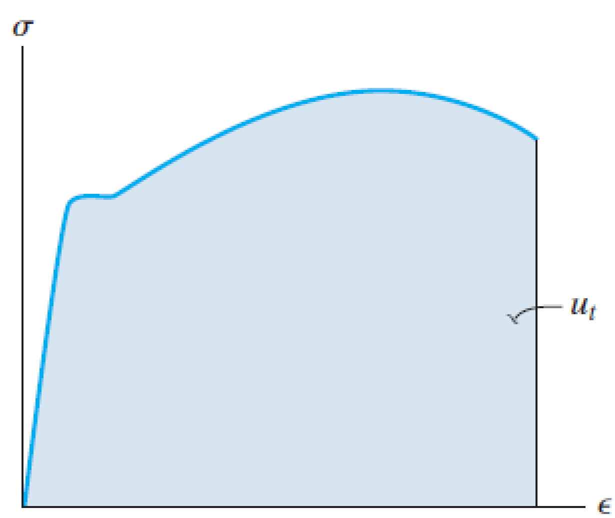
$$\boxed{u_r = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}\frac{\sigma_{pl}^2}{E}}$$

We notice that strain energy density and modulus of resilience are the area under the stress-strain curve.



Modulus of resilience u_r

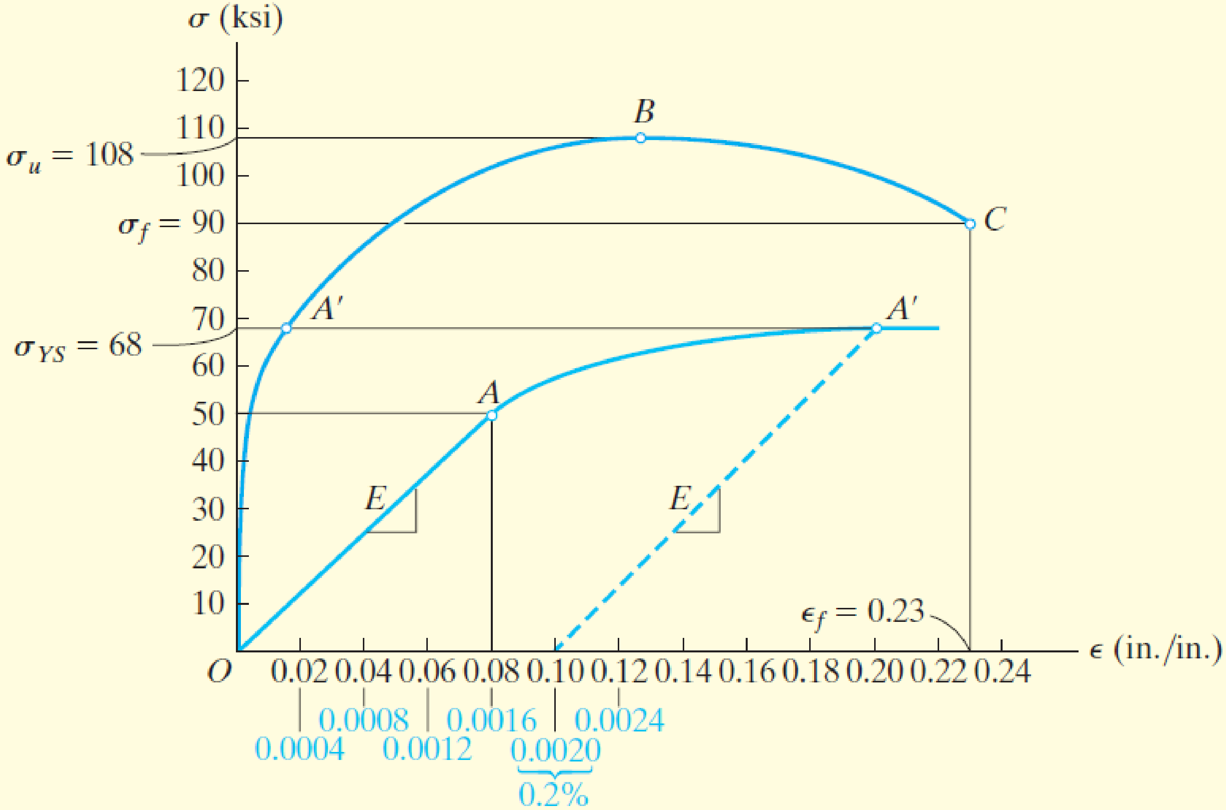
Modulus of Toughness. Another important property of a material is the ***modulus of toughness***, u_t.
entire area under the stress–strain diagram



Modulus of toughness u_t

Examples

A tension test for a steel alloy results in the stress–strain diagram shown in Fig. 3–18. Calculate the modulus of elasticity and the yield strength based on a 0.2% offset. Identify on the graph the ultimate stress and the fracture stress.



Solution

$$E = \frac{50 \text{ ksi}}{0.0016 \text{ in./in.}} = 31.2(10^3) \text{ ksi}$$

Yield Strength. For a 0.2% offset, we begin at a strain of 0.2% or 0.0020 in./in. and graphically extend a (dashed) line parallel to OA until it intersects the $\sigma-\epsilon$ curve at A'. The yield strength is approximately

$$\sigma_{YS} = 68 \text{ ksi} \qquad \textit{Ans.}$$

Ultimate Stress. This is defined by the peak of the σ–ϵ graph, point B in Fig. 3–18.

$$\sigma_u = 108 \text{ ksi} \qquad \textit{Ans.}$$

Fracture Stress. When the specimen is strained to its maximum of $\epsilon_f = 0.23$ in./in., it fractures at point C. Thus,

$$\sigma_f = 90 \text{ ksi} \qquad \textit{Ans.}$$

Example

The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. 3–19. If a specimen of this material is stressed to 600 MPa, determine the permanent strain that remains in the specimen when the load is released. Also, find the modulus of resilience both before and after the load application.

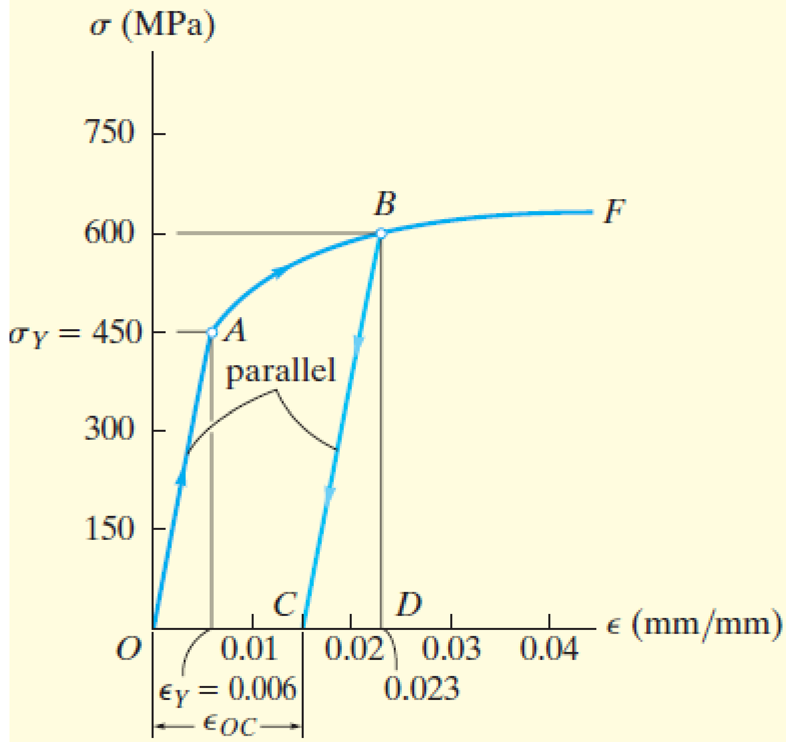


Fig. 3–19

SOLUTION

Permanent Strain. When the specimen is subjected to the load, it strain-hardens until point B is reached on the $\sigma-\epsilon$ diagram. The strain at this point is approximately 0.023 mm/mm. When the load is released, the material behaves by following the straight line BC, which is parallel to line OA.

$$E = \frac{450 \text{ MPa}}{0.006 \text{ mm/mm}} = 75.0 \text{ GPa}$$

From triangle CBD, we require

$$E = \frac{BD}{CD}; \qquad 75.0(10^9) \text{ Pa} = \frac{600(10^6) \text{ Pa}}{CD}$$

$$CD = 0.008 \text{ mm/mm}$$

This strain represents the amount of *recovered elastic strain*. The permanent strain, ϵ_{OC}, is thus

$$\epsilon_{OC} = 0.023 \text{ mm/mm} - 0.008 \text{ mm/mm}$$
$$= 0.0150 \text{ mm/mm}$$

Modulus of Resilience. Area under the curve OA

$$(u_r)_{\text{initial}} = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}(450 \text{ MPa})(0.006 \text{ mm/mm})$$
$$= 1.35 \text{ MJ/m}^3 \qquad \textit{Ans.}$$

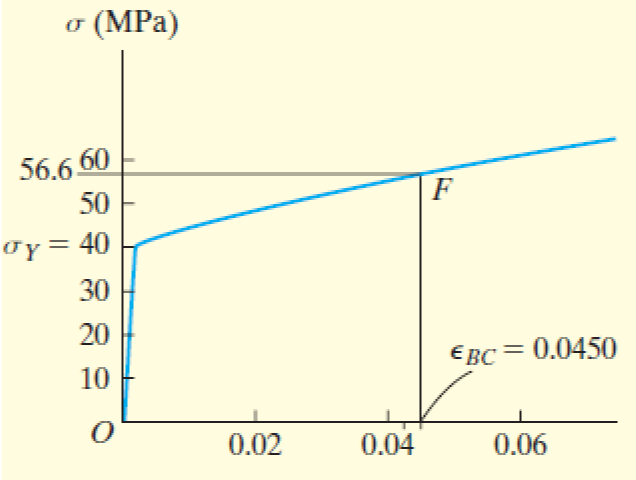
Area under the curve CB

$$(u_r)_{\text{final}} = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}(600 \text{ MPa})(0.008 \text{ mm/mm})$$
$$= 2.40 \text{ MJ/m}^3 \qquad \textit{Ans.}$$

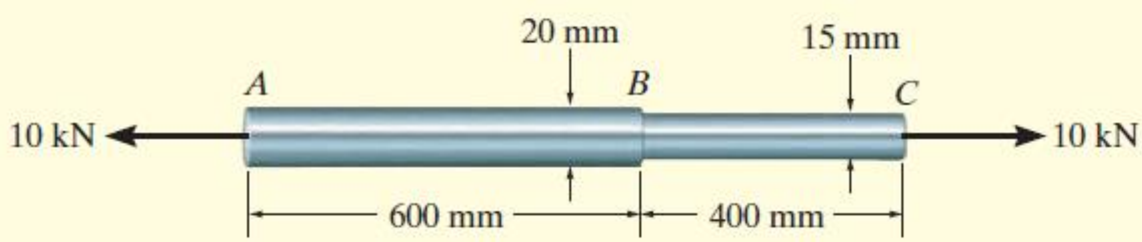
* Work in the SI system of units is measured in joules, where $1 \text{ J} = 1 \text{ N} \cdot \text{m}$.

An aluminum rod shown in Fig. 3–20*a* has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress–strain diagram is shown in Fig. 3–20*b*, determine the approximate elongation of the rod when the load is applied. Take $E_{al} = 70$ GPa.

Example



(b)



(a)

$$\sigma_{AB} = \frac{P}{A} = \frac{10(10^3)\text{ N}}{\pi(0.01\text{ m})^2} = 31.83\text{ MPa}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{10(10^3)\text{ N}}{\pi(0.0075\text{ m})^2} = 56.59\text{ MPa}$$

From the stress–strain diagram, the material in segment AB is strained *elastically* since $\sigma_{AB} < \sigma_Y = 40$ MPa. Using Hooke's law,

$$\epsilon_{AB} = \frac{\sigma_{AB}}{E_{\text{al}}} = \frac{31.83(10^6)\text{ Pa}}{70(10^9)\text{ Pa}} = 0.0004547\text{ mm/mm}$$

The material within segment BC is strained plastically, since $\sigma_{BC} > \sigma_Y = 40$ MPa. From the graph, for $\sigma_{BC} = 56.59$ MPa, $\epsilon_{BC} \approx 0.045$ mm/mm.

The total elongation of the rod is the elongation in AB (which is elastic) and the elongation in BC (which is plastic) as follow

$$\begin{aligned}\delta &= \Sigma \epsilon L = 0.0004547(600\text{ mm}) + 0.0450(400\text{ mm}) \\ &= 18.3\text{ mm} \qquad \textit{Ans.}\end{aligned}$$

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