Chapter 5

Axial Loading

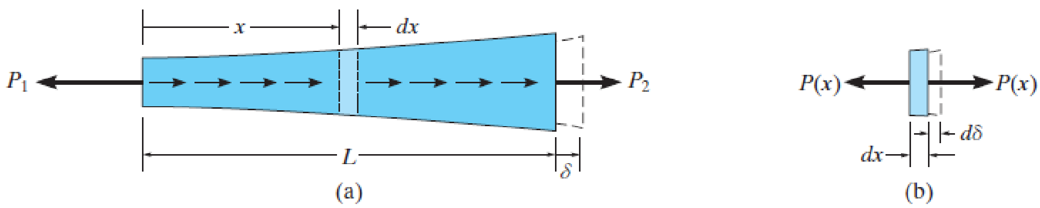
$$\sigma = \frac{P(x)}{A(x)} \quad \text{and} \quad \epsilon = \frac{d\delta}{dx}$$

Provided the stress does not exceed the proportional limit, we can apply Hooke's law; i.e.,

$$\sigma = E\epsilon$$

$$\frac{P(x)}{A(x)} = E\left(\frac{d\delta}{dx}\right)$$

$$d\delta = \frac{P(x)\,dx}{A(x)E}$$



For the entire length L of the bar, we must integrate this expression to find δ. This yields

$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)E}$$

IF the P, E and A are constants then

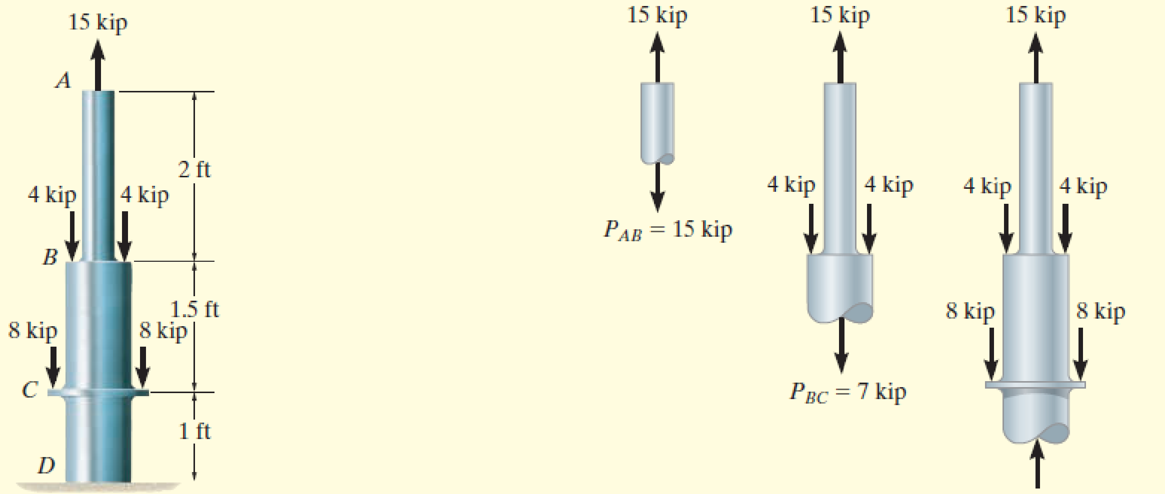
$$\delta = \frac{PL}{AE}$$

If many members have same P, E and A then

$$\delta = \sum \frac{PL}{AE}$$

Examples

The A-36 steel bar shown in Fig. 4–6*a* is made from two segments having cross-sectional areas of $A_{AB} = 1\text{ in}^2$ and $A_{BD} = 2\text{ in}^2$. Determine the vertical displacement of end *A* and the displacement of *B* relative to *C*.



(a) (b)

$$\delta_A = \sum \frac{PL}{AE} = \frac{[+15\text{ kip}](2\text{ ft})(12\text{ in./ft})}{(1\text{ in}^2)[29(10^3)\text{ kip/in}^2]} + \frac{[+7\text{ kip}](1.5\text{ ft})(12\text{ in./ft})}{(2\text{ in}^2)[29(10^3)\text{ kip/in}^2]}$$
$$+ \frac{[-9\text{ kip}](1\text{ ft})(12\text{ in./ft})}{(2\text{ in}^2)[29(10^3)\text{ kip/in}^2]}$$
$$= +0.0127\text{ in.} \qquad \textit{Ans.}$$

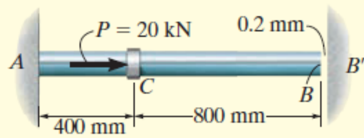
$$\delta_{B/C} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{[+7\text{ kip}](1.5\text{ ft})(12\text{ in./ft})}{(2\text{ in}^2)[29(10^3)\text{ kip/in}^2]} = +0.00217\text{ in.} \qquad \textit{Ans.}$$

Positive displacements mean elongation.

Statically Indeterminate Axially Loaded Member

If the equilibrium equations are not enough to solve the problem, use the compatibility equations with the equilibrium equations to solve the problem.

The steel rod shown in Fig. 4–12*a* has a diameter of 10 mm. It is fixed to the wall at A, and before it is loaded, there is a gap of 0.2 mm between the wall at B' and the rod. Determine the reactions at A and B' if the rod is subjected to an axial force of $P = 20$ kN as shown. Neglect the size of the collar at C. Take $E_{st} = 200$ GPa.



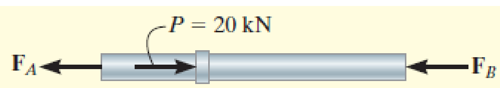
(a)

Let's check if the load is enough to cause the end B to contact the wall.

$\delta_B = \frac{20000*400}{AE} = 0.509\ mm > 0.2mm$ then B will contact the wall.

Equilibrium

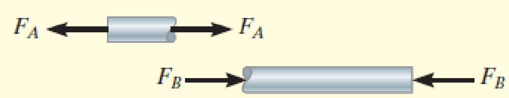
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -F_A - F_B + 20(10^3)\text{ N} = 0 \qquad (1)$$



(b)

Two unknown and one equation then indeterminate.

Compatibility



(c)

$$\delta_{B/A} = 0.0002\text{ m} = \frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE}$$

$$0.0002\text{ m} = \frac{F_A(0.4\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]} - \frac{F_B(0.8\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]}$$

or

$$F_A(0.4\text{ m}) - F_B(0.8\text{ m}) = 3141.59\text{ N}\cdot\text{m} \qquad (2)$$

Solving Eqs. 1 and 2 yields

$$F_A = 16.0\text{ kN} \qquad F_B = 4.05\text{ kN} \qquad \textit{Ans.}$$

Another solution (Called flexibility or force method)

SOLUTION

Compatibility. Here we will consider the support at B' as redundant. Using the principle of superposition, Fig. 4–17*b*, we have

$$(\overset{+}{\rightarrow}) \qquad 0.0002 \text{ m} = \delta_P - \delta_B \qquad (1)$$

The deflections δ_P and δ_B are determined from Eq. 4–2.

$$\delta_P = \frac{PL_{AC}}{AE} = \frac{[20(10^3)\text{ N}](0.4\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]} = 0.5093(10^{-3})\text{ m}$$

$$\delta_B = \frac{F_B L_{AB}}{AE} = \frac{F_B(1.20\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]} = 76.3944(10^{-9})F_B$$

Substituting into Eq. 1, we get

$$0.0002\text{ m} = 0.5093(10^{-3})\text{ m} - 76.3944(10^{-9})F_B$$

$$F_B = 4.05(10^3)\text{ N} = 4.05\text{ kN} \qquad \textit{Ans.}$$

Equilibrium. From the free-body diagram, Fig. 4–17*c*,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_A + 20\text{ kN} - 4.05\text{ kN} = 0 \quad F_A = 16.0\text{ kN}$ *Ans.*