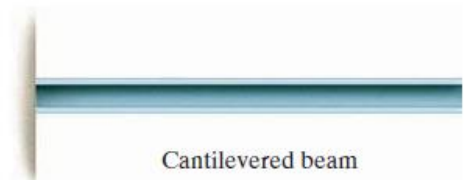
Chapter 7

Shear and Bending diagrams

Members that are slender and support loadings that are applied perpendicular to their longitudinal axis are called **beams** and classified as how they are supported as shown below.

Simply supported beam

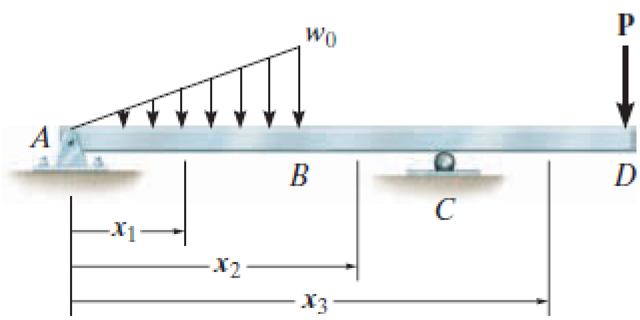


Cantilevered beam

Overhanging beam

To design beams, we need to know the shear and bending moment diagrams and find the maximum values.

Because shear and moment are discontinuous functions, we divide their diagrams into regions as shown below.



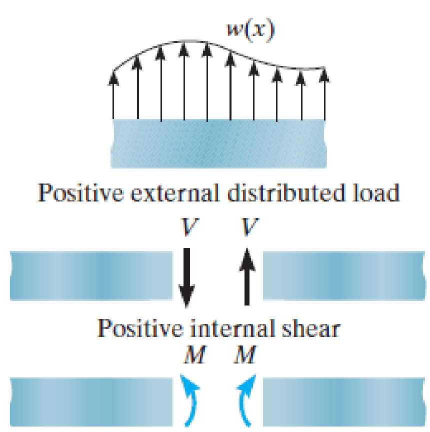
Consider the origin to the left. Then x1 will be used for region AB, x2 for region BC and x3 for region CD.

Beams sign convention

It is arbitrary but the one often used in engineering practice is as shown below.

OR,

When starting from left side of the beam, upward forces cause positive shear and bending moment and downward forces cause negative shear and bending moment.



Positive external distributed load

Positive internal shear

Positive internal moment

Beam sign convention

EXAMPLE 6.1

Draw the shear and moment diagrams for the beam shown in Fig. 6–4*a*.

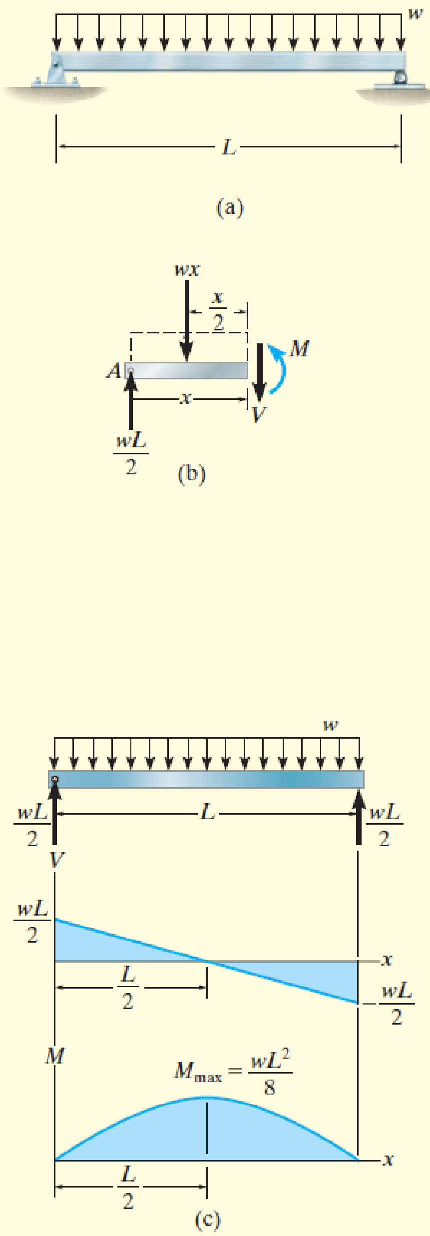


Fig. 6–4

SOLUTION

Support Reactions. The support reactions are shown in Fig. 6–4*c*.

Shear and Moment Functions. A free-body diagram of the left segment of the beam is shown in Fig. 6–4*b*. The distributed loading on this segment, wx, is represented by its resultant force only *after* the segment is isolated as a free-body diagram. This force acts through the centroid of the area comprising the distributed loading, a distance of $x/2$ from the right end. Applying the two equations of equilibrium yields

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \tag{1}$$

$$\circlearrowleft + \Sigma M = 0; \qquad -\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w}{2}(Lx - x^2) \tag{2}$$

Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 6–4*c* are obtained by plotting Eqs. 1 and 2. The point of *zero shear* can be found from Eq. 1:

$$V = w\left(\frac{L}{2} - x\right) = 0$$

$$x = \frac{L}{2}$$

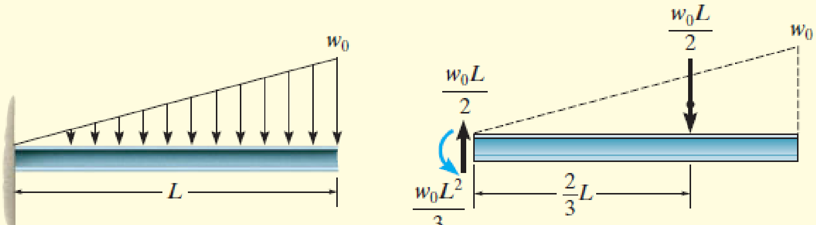
NOTE: From the moment diagram, this value of x represents the point on the beam where the *maximum moment* occurs, since by Eq. 6–2 (see Sec. 6.2) the *slope* $V = dM/dx = 0$. From Eq. 2, we have

$$M_{max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{wL^2}{8}$$

EXAMPLE 6.2

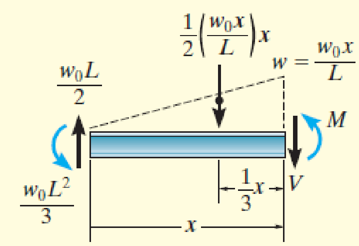
Draw the shear and moment diagrams for the beam shown in Fig. 6–5*a*.



(a) (b)

SOLUTION

Support Reactions. The distributed load is replaced by its resultant force and the reactions have been determined as shown in Fig. 6–5*b*.



(c)

Shear and Moment Functions. A free-body diagram of a beam segment of length x is shown in Fig. 6–5*c*. Note that the intensity of the triangular load at the section is found by proportion, that is, $w/x = w_0/L$ or $w = w_0 x/L$. With the load intensity known, the resultant of the distributed loading is determined from the area under the diagram. Thus,

$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{2} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)x - V = 0$$

$$V = \frac{w_0}{2L}(L^2 - x^2) \qquad (1)$$

$$\circlearrowright + \Sigma M = 0; \qquad \frac{w_0 L^2}{3} - \frac{w_0 L}{2}(x) + \frac{1}{2}\left(\frac{w_0 x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0$$

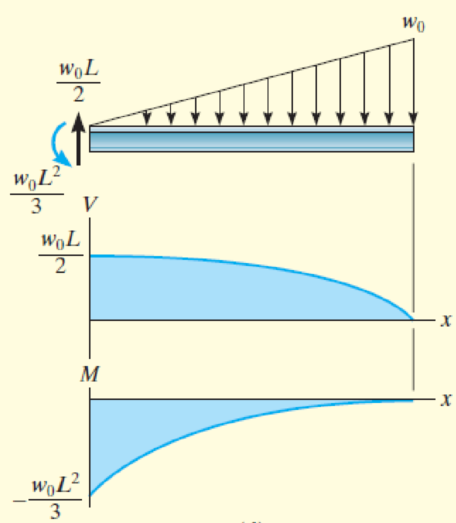
$$M = \frac{w_0}{6L}(-2L^3 + 3L^2 x - x^3) \qquad (2)$$

These results can be checked by applying Eqs. 6–1 and 6–2 of Sec. 6.2, that is,

$$w = \frac{dV}{dx} = \frac{w_0}{2L}(0 - 2x) = -\frac{w_0 x}{L} \qquad \text{OK}$$

$$V = \frac{dM}{dx} = \frac{w_0}{6L}(0 + 3L^2 - 3x^2) = \frac{w_0}{2L}(L^2 - x^2) \qquad \text{OK}$$

Shear and Moment Diagrams. The graphs of Eqs. 1 and 2 are shown in Fig. 6–5*d*.

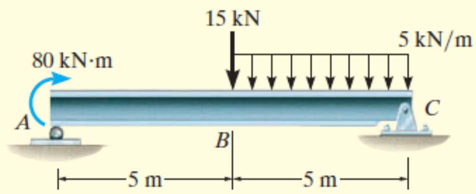


(d)

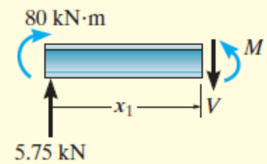
Fig. 6–5

EXAMPLE 6.4

Draw the shear and moment diagrams for the beam shown in Fig. 6–7*a*.



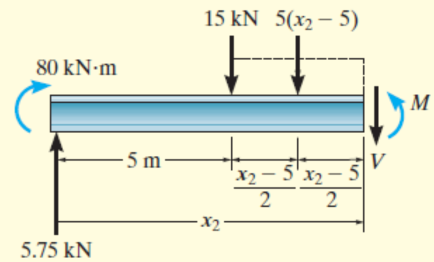
(a)



(b)

SOLUTION

Support Reactions. The reactions at the supports have been determined and are shown on the free-body diagram of the beam, Fig. 6–7*d*.



(c)

Shear and Moment Functions. Since there is a discontinuity of distributed load and also a concentrated load at the beam's center, two regions of x must be considered in order to describe the shear and moment functions for the entire beam.

$0 \le x_1 < 5$ m, Fig. 6–7*b*:

$$+\uparrow \Sigma F_y = 0; \qquad 5.75 \text{ kN} - V = 0$$

$$V = 5.75 \text{ kN} \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_1 + M = 0$$

$$M = (5.75x_1 + 80) \text{ kN}\cdot\text{m} \qquad (2)$$

$5 \text{ m} < x_2 \le 10$ m, Fig. 6–7*c*:

$$+\uparrow \Sigma F_y = 0; \quad 5.75 \text{ kN} - 15 \text{ kN} - 5 \text{ kN/m}(x_2 - 5 \text{ m}) - V = 0$$

$$V = (15.75 - 5x_2) \text{ kN} \qquad (3)$$

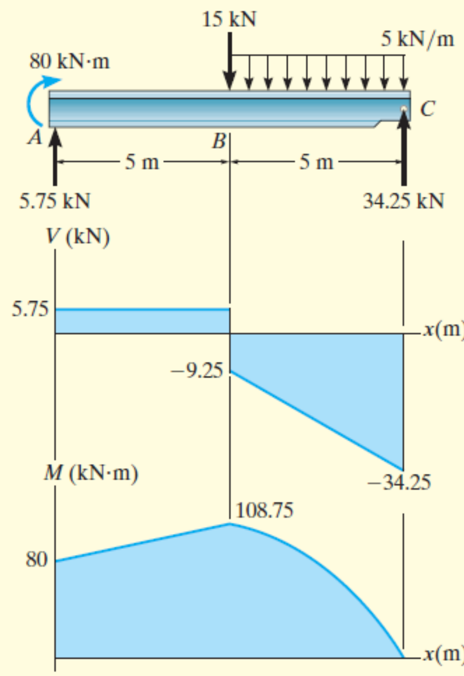
$$\circlearrowleft + \Sigma M = 0; \quad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_2 + 15 \text{ kN}(x_2 - 5 \text{ m})$$

$$+ 5 \text{ kN/m}(x_2 - 5 \text{ m})\left(\frac{x_2 - 5 \text{ m}}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5) \text{ kN}\cdot\text{m} \qquad (4)$$

These results can be checked in part by noting that $w = dV/dx$ and $V = dM/dx$. Also, when $x_1 = 0$, Eqs. 1 and 2 give $V = 5.75$ kN and $M = 80$ kN·m; when $x_2 = 10$ m, Eqs. 3 and 4 give $V = -34.25$ kN and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. 6–7*d*.

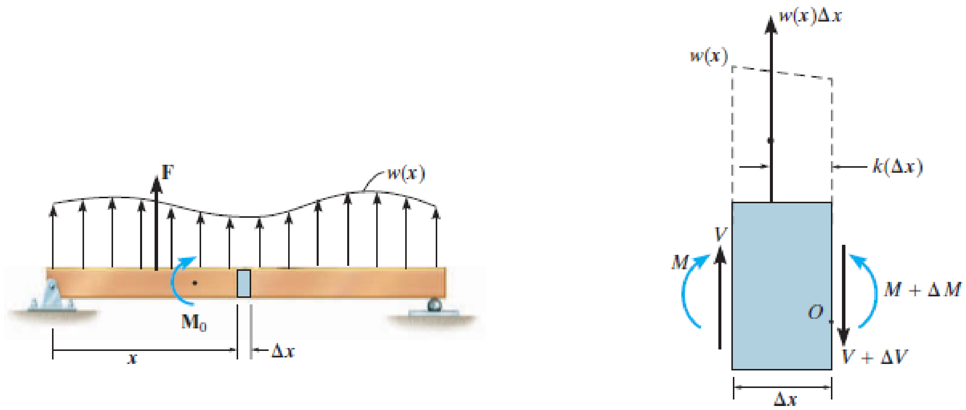
Shear and Moment Diagrams. Equations 1 through 4 are plotted in Fig. 6–7*d*.



(d)

Fig. 6–7

Graphical Method for Constructing Shear and Moment Diagrams



(a)

Free-body diagram of segment Δx

(b)

$$+\uparrow \Sigma F_y = 0; \qquad V + w(x)\,\Delta x - (V + \Delta V) = 0$$
$$\Delta V = w(x)\,\Delta x$$

$$\circlearrowleft + \Sigma M_O = 0; \quad -V\,\Delta x - M - w(x)\,\Delta x[k(\Delta x)] + (M + \Delta M) = 0$$
$$\Delta M = V\,\Delta x + w(x)\,k(\Delta x)^2$$

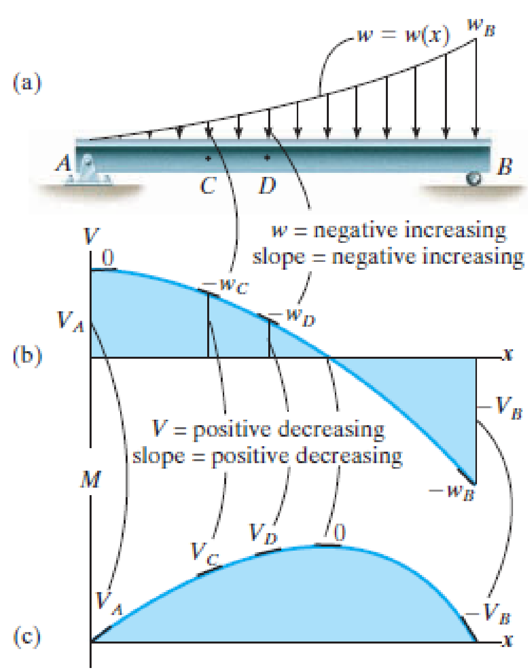
Dividing by Δx and taking the limit as $\Delta x \to 0$, the above two equations become

$$\frac{dV}{dx} = w(x) \qquad (6\text{–}1)$$

slope of shear diagram at each point = distributed load intensity at each point

$$\frac{dM}{dx} = V \qquad (6\text{–}2)$$

slope of moment diagram at each point = shear at each point



Integrating equations 6-1 and 6-2,

$$\Delta V = \int w(x)\,dx \qquad (6\text{–}3)$$

change in shear = area under distributed loading

$$\Delta M = \int V(x)\,dx \qquad (6\text{–}4)$$

change in moment = area under shear diagram