Chapter 7

Thermal Stress

A change in temperature can cause a body to change its dimensions. Generally, if the temperature increases, the body will expand, whereas if the temperature decreases, it will contract. Ordinarily this expansion or contraction is linearly related to the temperature increase or decrease that occurs. If this is the case, and the material is homogeneous and isotropic, it has been found from experiment that the displacement of a member having a length L can be calculated using the formula.

$$\delta_T = \alpha \, \Delta T L \qquad (4\text{–}4)$$

where

α = a property of the material, referred to as the ***linear coefficient of thermal expansion***. The units measure strain per degree of temperature. They are 1/°F (Fahrenheit) in the FPS system, and 1/°C (Celsius) or 1/K (Kelvin) in the SI system. Typical values are given on the inside back cover

ΔT = the algebraic change in temperature of the member

L = the original length of the member

δ_T = the algebraic change in the length of the member

EXAMPLE 4.10

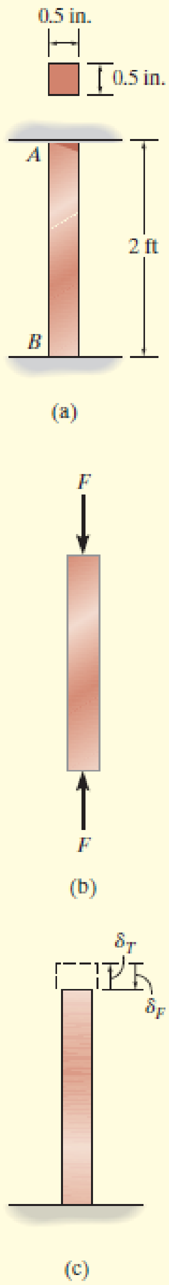

Fig. 4–18

The A-36 steel bar shown in Fig. 4–18*a* is constrained to just fit between two fixed supports when $T_1 = 60°\text{F}$. If the temperature is raised to $T_2 = 120°\text{F}$, determine the average normal thermal stress developed in the bar.

SOLUTION

Equilibrium. The free-body diagram of the bar is shown in Fig. 4–18*b*. Since there is no external load, the force at *A* is equal but opposite to the force at *B*; that is,

$$+\uparrow \Sigma F_y = 0; \qquad F_A = F_B = F$$

The problem is statically indeterminate since this force cannot be determined from equilibrium.

Compatibility. Since $\delta_{A/B} = 0$, the thermal displacement δ_T at *A* that occurs, Fig. 4–18*c*, is counteracted by the force **F** that is required to push the bar δ_F back to its original position. The compatibility condition at *A* becomes

$$(+\uparrow) \qquad \delta_{A/B} = 0 = \delta_T - \delta_F$$

Applying the thermal and load–displacement relationships, we have

$$0 = \alpha \Delta T L - \frac{FL}{AE}$$

Thus, from the data on the inside back cover,

$$\begin{aligned} F &= \alpha \Delta T A E \\ &= [6.60(10^{-6})/°\text{F}](120°\text{F} - 60°\text{F})(0.5\text{ in.})^2[29(10^3)\text{ kip/in}^2] \\ &= 2.871\text{ kip} \end{aligned}$$

Since **F** also represents the internal axial force within the bar, the average normal compressive stress is thus

$$\sigma = \frac{F}{A} = \frac{2.871\text{ kip}}{(0.5\text{ in.})^2} = 11.5\text{ ksi} \qquad \textit{Ans.}$$

NOTE: From the magnitude of **F**, it should be apparent that changes in temperature can cause large reaction forces in statically indeterminate members.

EXAMPLE 4.11

The rigid beam shown in Fig. 4–19*a* is fixed to the top of the three posts made of A-36 steel and 2014-T6 aluminum. The posts each have a length of 250 mm when no load is applied to the beam, and the temperature is $T_1 = 20°\text{C}$. Determine the force supported by each post if the bar is subjected to a uniform distributed load of 150 kN/m and the temperature is raised to $T_2 = 80°\text{C}$.

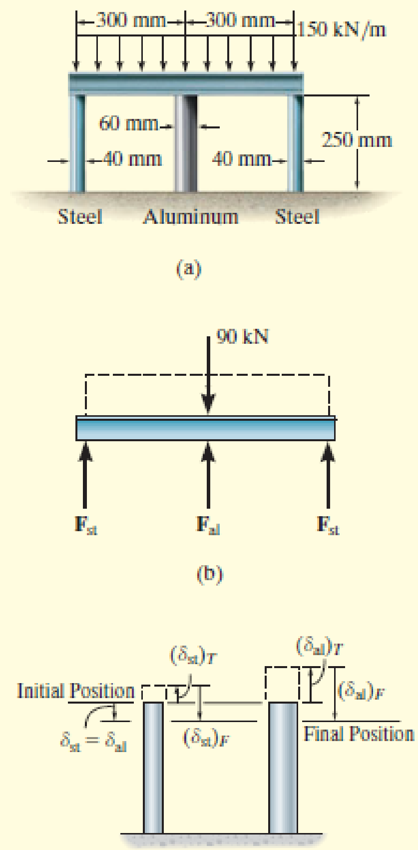

Fig. 4–19

SOLUTION

Equilibrium. The free-body diagram of the beam is shown in Fig. 4–19*b*. Moment equilibrium about the beam's center requires the forces in the steel posts to be equal. Summing forces on the free-body diagram, we have

$$+\uparrow \Sigma F_y = 0; \qquad 2F_{st} + F_{al} - 90(10^3)\text{ N} = 0 \qquad (1)$$

Compatibility. Due to load, geometry, and material symmetry, the top of each post is displaced by an equal amount. Hence,

$$(+\downarrow) \qquad \delta_{st} = \delta_{al} \qquad (2)$$

The final position of the top of each post is equal to its displacement caused by the temperature increase, plus its displacement caused by the internal axial compressive force, Fig. 4–19*c*. Thus, for the steel and aluminum post, we have

$$(+\downarrow) \qquad \delta_{st} = -(\delta_{st})_T + (\delta_{st})_F$$

$$(+\downarrow) \qquad \delta_{al} = -(\delta_{al})_T + (\delta_{al})_F$$

Applying Eq. 2 gives

$$-(\delta_{st})_T + (\delta_{st})_F = -(\delta_{al})_T + (\delta_{al})_F$$

Using Eqs. 4–2 and 4–4 and the material properties on the inside back cover, we get

$$-[12(10^{-6})/°\text{C}](80°\text{C} - 20°\text{C})(0.250\text{ m}) + \frac{F_{st}(0.250\text{ m})}{\pi(0.020\text{ m})^2[200(10^9)\text{ N/m}^2]}$$

$$= -[23(10^{-6})/°\text{C}](80°\text{C} - 20°\text{C})(0.250\text{ m}) + \frac{F_{al}(0.250\text{ m})}{\pi(0.030\text{ m})^2[73.1(10^9)\text{ N/m}^2]}$$

$$F_{st} = 1.216F_{al} - 165.9(10^3) \qquad (3)$$

To be *consistent*, all numerical data has been expressed in terms of newtons, meters, and degrees Celsius. Solving Eqs. 1 and 3 simultaneously yields

$$F_{st} = -16.4\text{ kN} \quad F_{al} = 123\text{ kN} \qquad \textit{Ans.}$$

The negative value for F_{st} indicates that this force acts opposite to that shown in Fig. 4–19*b*. In other words, the steel posts are in tension and the aluminum post is in compression.