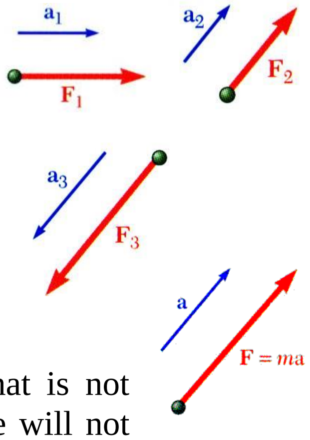
Chapter Two

Kinetics of Particles

2-1 Newton's Second Law of Motion



If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of resultant and in the direction of the resultant.

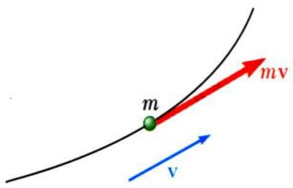
$$\vec{F} = m\vec{a}$$

If particle is subjected to several forces: $\sum \vec{F} = m\vec{a}$

We must use a Newtonian frame of reference, i.e., one that is not accelerating or rotating. If no force acts on particle, particle will not accelerate, i.e., it will remain stationary or continue on a straight line at constant velocity.

2-2 Linear Momentum of a Particle

The principle of conservation of linear momentum is:



$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt}$$

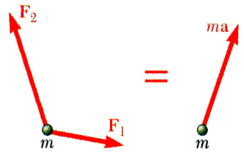
$$= \frac{d}{dt}(m\vec{v}) = \frac{d}{dt}(\vec{L})$$

Where: $\vec{L} = m\vec{v}$ = Linear momentum

Sum of forces = rate of change of linear momentum $\sum \vec{F} = \dot{\vec{L}}$

If $\sum \vec{F} = 0$ then linear momentum is constant

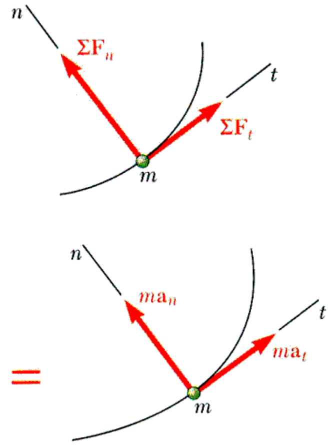
2-3 Equations of Motion



- Newton's second law $\sum \vec{F} = m\vec{a}$

- Convenient to resolve into components:

$$\sum\left(F_x\vec{i} + F_y\vec{j} + F_z\vec{k}\right) = m\left(a_x\vec{i} + a_y\vec{j} + a_z\vec{k}\right)$$
$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$
$$\sum F_x = m\ddot{x} \quad \sum F_y = m\ddot{y} \quad \sum F_z = m\ddot{z}$$



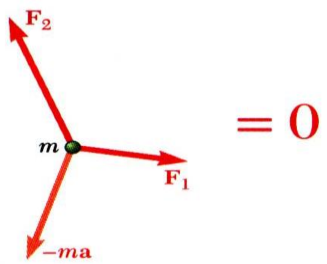
- For tangential and normal components:

$$\sum F_t = ma_t \qquad \sum F_n = ma_n$$
$$\sum F_t = m\frac{dv}{dt} \qquad \sum F_n = m\frac{v^2}{\rho}$$

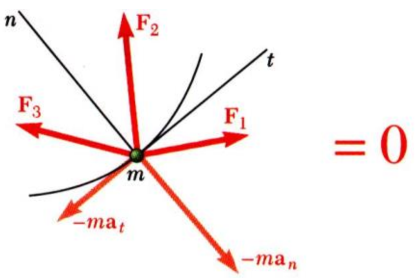
2-4 Dynamic Equilibrium

Alternate expression of Newton's law:: $\sum \vec{F} - m\vec{a} = 0$

Where: $-m\vec{a}$ = inertia vector



If we include inertia vector, the system of forces acting on particle is equivalent to zero. The particle is said to be in dynamic equilibrium.
Inertia vectors are often called *inertia forces* as they measure the resistance that particles offer to changes in motion.



2-5 Equation of Motion for a System of Particles

The summation of the internal forces, if carried out, will equal zero, since internal forces between any two particles occur in equal but opposite collinear pairs. Consequently, only the sum of the external forces will remain, and therefore the equation of motion, written for the system of particles, becomes

$$\sum F_i = \sum m_i a_i$$

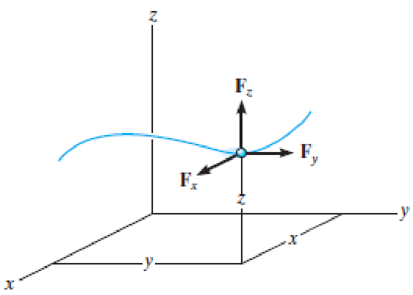
- The equation of motion is based on experimental evidence and is valid only when applied within an inertial frame of reference.
- The equation of motion states that the unbalanced force on a particle causes it to accelerate.
- An inertial frame of reference does not rotate, rather its axes either translate with constant velocity or are at rest.
- Mass is a property of matter that provides a quantitative measure of its resistance to a change in velocity. It is an absolute quantity and so it does not change from one location to another.
- Weight is a force that is caused by the earth's gravitation. It is not absolute; rather it depends on the altitude of the mass from the earth's surface.

2-6 Equations of Motion: Rectangular Coordinates

When a particle moves relative to an inertial *x, y, z* frame of reference, the forces acting on the particle, as well as its acceleration, can be expressed in terms of their **i**, **j**, **k** components. Applying the equation of motion, we have

$$\Sigma \mathbf{F} = m\mathbf{a}; \qquad \Sigma F_x \mathbf{i} + \Sigma F_y \mathbf{j} + \Sigma F_z \mathbf{k} = m(a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k})$$

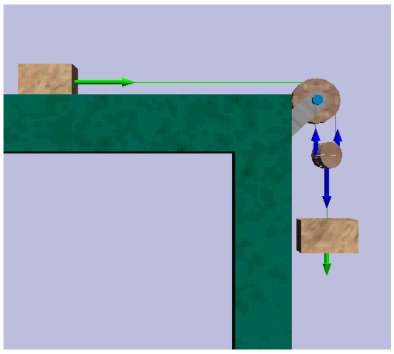
For this equation to be satisfied, the respective i, j, k components on the left side must equal the corresponding components on the right side. Consequently, we may write the following three scalar equations:



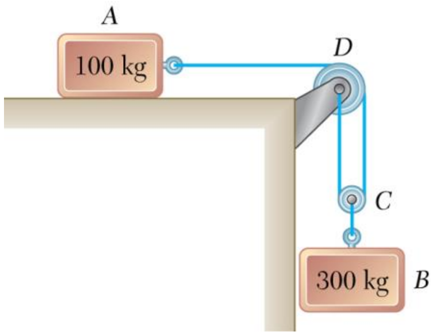
$$\Sigma F_x = ma_x$$
$$\Sigma F_y = ma_y$$
$$\Sigma F_z = ma_z$$

In particular, if the particle is constrained to move only in the x–y plane, then the first two of these equations are used to specify the motion.

Example 1: The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in the cord.



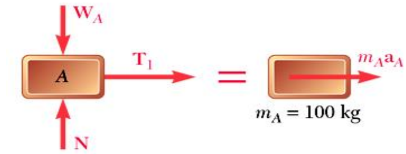
SOLUTION:



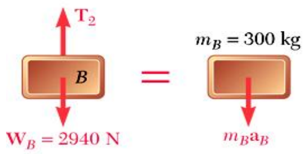
Kinematic relationship: If A moves x_A to the right, B moves down 0.5 x_A:

$$x_B = \tfrac{1}{2} x_A \qquad a_B = \tfrac{1}{2} a_A$$

Draw free body diagrams & apply Newton's law:



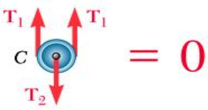
$$\sum F_x = m_A a_A \implies T_1 = (100) a_A$$



$$\sum F_y = m_B a_B \implies m_B g - T_2 = m_B a_B$$

$$300 \times 9.81 - T_2 = (300) a_B$$

$$T_2 = 2940\text{-}(300) a_B$$



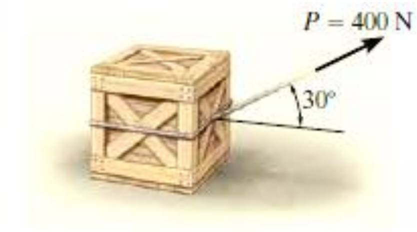
$$\sum F_y = m_C a_C \implies T_2 - 2T_1 = 0$$

$$2940\text{-}(300) a_B - 2T_1 = 0 \qquad 2940\text{-}(300) a_B - 200 a_A = 0$$

$$2940\text{-}(300) a_B - 2 \times 200 a_B = 0$$

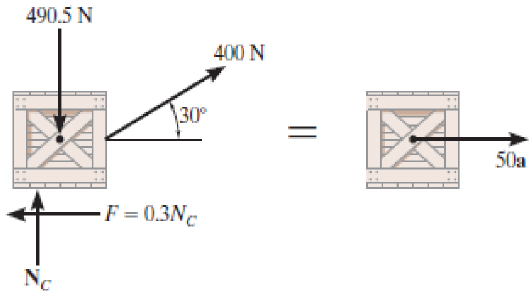
$$a_B = 4.2\, m/s^2 \qquad a_A = 8.4\, m/s^2 \qquad T_1 = 840\ N \qquad T_2 = 1680\ N$$

Example 2: The 50-kg crate shown in Fig. rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400-N towing force as shown, determine the velocity of the crate in 3 s starting from rest.



SOLUTION:

$W = mg = 50$ kg (9.81 m/s^2) = 490.5 N.



$$\overset{+}{\rightarrow} \Sigma F_x = ma_x; \qquad 400 \cos 30° - 0.3N_C = 50a$$
$$+\uparrow \Sigma F_y = ma_y; \qquad N_C - 490.5 + 400 \sin 30° = 0$$

$$N_C = 290.5 \text{ N}$$
$$a = 5.185 \text{ m/s}^2$$
$$v = v_0 + a_c t = 0 + 5.185(3)$$
$$= 15.6 \text{ m/s} \rightarrow$$

Example 3: A 10-kg projectile is fired vertically upward from the ground, with an initial velocity of 50 m/s. Determine the maximum height to which it will travel if (*a*) atmospheric resistance is neglected; and (*b*) atmospheric resistance is measured as $F_D = (0.01v^2)$ N, where v is the speed of the projectile at any instant, measured in m/s.

SOLUTION:

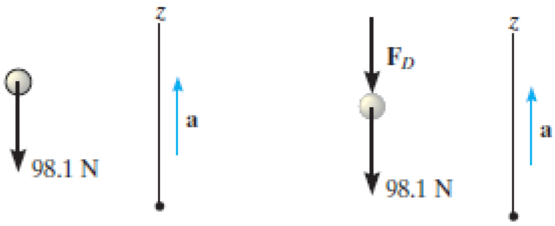
$W = mg = 10(9.81) = 98.1$ N

$$+\uparrow \Sigma F_z = ma_z; \qquad -98.1 = 10a, \qquad a = -9.81 \text{ m/s}^2$$

Kinematics. Initially, $z_0 = 0$ and $v_0 = 50$ m/s, and at the maximum height $z = h$, $v = 0$. Since the acceleration is *constant*, then

$(+\uparrow)$ $$v^2 = v_0^2 + 2a_c(z - z_0)$$
$$0 = (50)^2 + 2(-9.81)(h - 0)$$
$$h = 127 \text{ m}$$ *Ans.*

$$F_D = (0.01v^2)$$



$+\uparrow\Sigma F_z = ma_z$; $\quad -0.01v^2 - 98.1 = 10a, \quad a = -(0.001v^2 + 9.81)$

Kinematics. Here the acceleration is *not constant* since F_D depends on the velocity. Since $a = f(v)$, we can relate a to position using

$(+\uparrow)\ a\,dz = v\,dv; \qquad -(0.001v^2 + 9.81)\,dz = v\,dv$

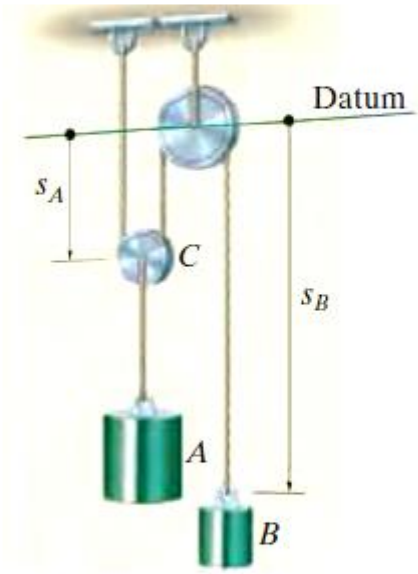
Separating the variables and integrating, realizing that initially $z_0 = 0$, $v_0 = 50$ m/s (positive upward), and at $z = h$, $v = 0$, we have

$$\int_0^h dz = -\int_{50\text{ m/s}}^0 \frac{v\,dv}{0.001v^2 + 9.81} = -500\ln(v^2 + 9810)\Big|_{50\text{ m/s}}^0$$

$$h = 114 \text{ m}$$ *Ans.*

NOTE: The answer indicates a lower elevation than that obtained in part (*a*) due to atmospheric resistance or drag.

Example 4: The 100-kg block *A* shown in Fig. is released from rest. If the masses of the pulleys and the cord are neglected, determine the velocity of the 20-kg block *B* in 2 s.

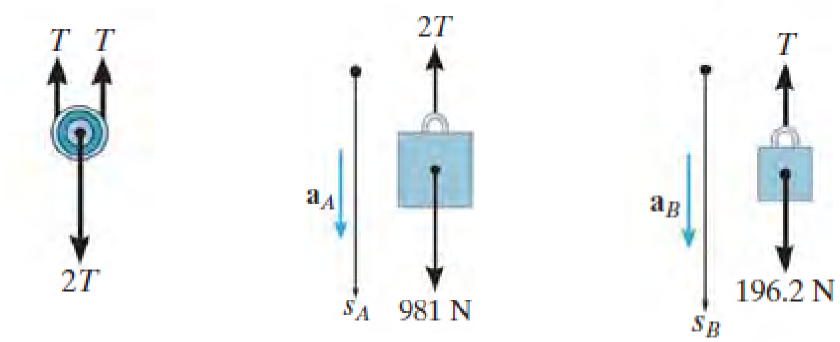


SOLUTION:

Notice from free body diagram that for *A* to remain stationary:
$T = 1/2 * 9.81 *100= 490.5$ N,

whereas for *B* to remain static:

$T = 9.81 *20= 196.2$ N.



Equations of Motion. Block *A*,

$+\downarrow \Sigma F_y = ma_y;$ $\quad 981 - 2T = 100a_A$

Block *B*,

$+\downarrow \Sigma F_y = ma_y;$ $\quad 196.2 - T = 20a_B$

$$2s_A + s_B = l$$

where l is constant and represents the total vertical length of cord. Differentiating this expression twice with respect to time yields

$$2a_A = -a_B$$

$$T = 327.0 \text{ N}$$
$$a_A = 3.27 \text{ m/s}^2$$
$$a_B = -6.54 \text{ m/s}^2$$

Hence when block A accelerates *downward*, block B accelerates *upward* as expected. Since a_B is constant, the velocity of block B in 2 s is thus

$(+\downarrow)$
$$v = v_0 + a_B t$$
$$= 0 + (-6.54)(2)$$
$$= -13.1 \text{ m/s} \qquad \textit{Ans.}$$

The negative sign indicates that block B is moving upward.