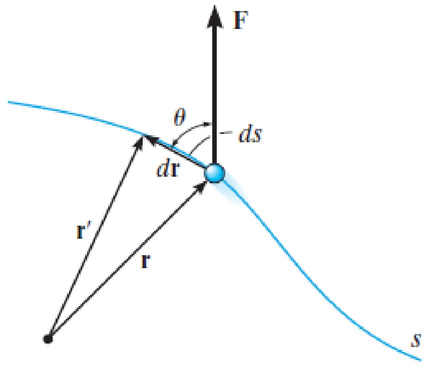
Chapter TWO
Kinetics of Particles: Work And Energy

2-9 The Work of a Force

A force F will do work on a particle only when the particle undergoes a displacement in the direction of the force. For example, if the force F in Fig. causes the particle to move along the path s from position r to a new position r', the displacement is then $dr = r' - r$. The magnitude of dr is ds, the length of the differential segment along the path. If the angle between the tails of dr and F is θ, then the work done by F is a scalar quantity, defined by



$$dU = F\, ds \cos\theta$$

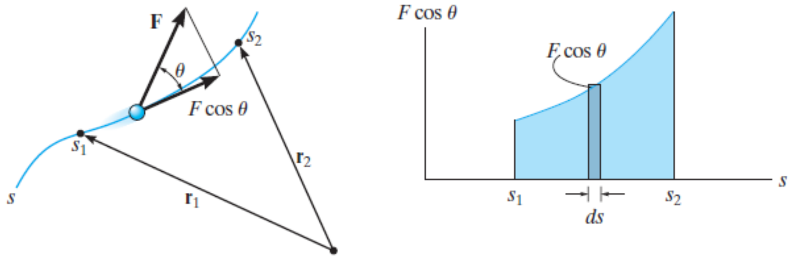
By definition of the dot product this equation can also be written as

$$dU = F \cdot dr$$

2-9-1 Work of a Variable Force

If the particle acted upon by the force F undergoes a finite displacement along its path from r_1 to r_2 or s_1 to s_2 , the work of force F is determined by integration. Provided F and θ can be expressed as a function of position, then

$$U_{1-2} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} \mathbf{F} \cdot d\mathbf{r} = \int_{s_1}^{s_2} F \cos\theta \, ds$$



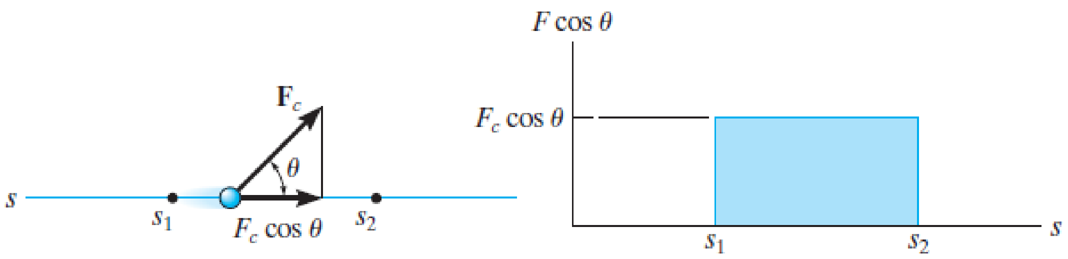
2-9-2 Work of a Constant Force Moving Along a Straight Line

If the force F_c has a constant magnitude and acts at a constant angle θ from its straight-line path, then the component of F_c in the direction of displacement is always $F_c \cos \theta$. The work done by F_c when the particle is displaced from s_1 to s_2 is determined from, in which case

$$U_{1-2} = F_c \cos \theta \int_{s_1}^{s_2} ds$$

$$U_{1-2} = F_c \cos \theta (s_2 - s_1)$$

Here the work of F_c represents the *area of the rectangle* as in Figure below:



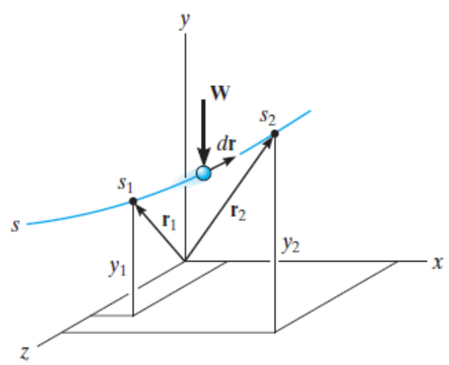
2-9-3 Work of a Weight

Consider a particle of weight W, which moves up along the path s shown in Fig. from position s_1 to position s_2.

$$U_{1-2} = \int \mathbf{F} \cdot d\mathbf{r} = \int_{\mathbf{r}_1}^{\mathbf{r}_2} (-W\mathbf{j}) \cdot (dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$= \int_{y_1}^{y_2} -W\, dy = -W(y_2 - y_1)$$

$$U_{1-2} = -W\,\Delta y$$

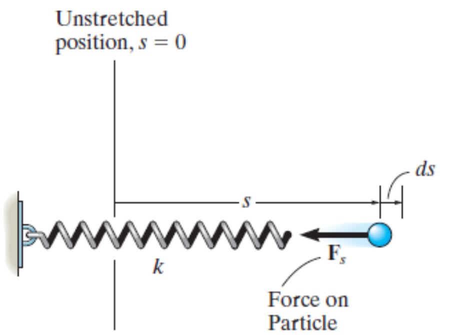


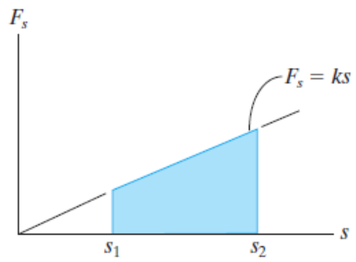
2-9-4 Work of a Spring Force

If an elastic spring is elongated a distance *ds*, then the work done by the force that acts on the attached particle is $dU = -F_s ds = -ks\, ds$. The work is *negative* since $\mathbf{F}_s$ acts in the opposite sense to *ds*. If the particle displaces from s_1 to s_2, the work of $\mathbf{F}_s$ is then

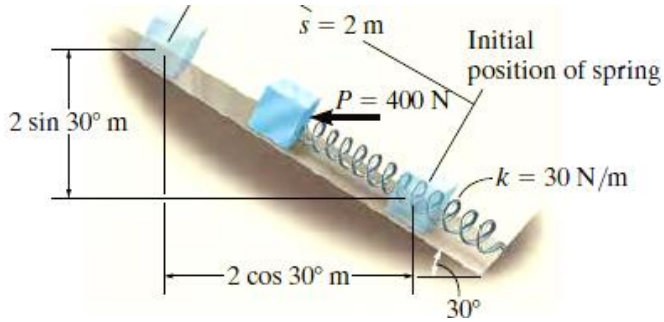
$$U_{1-2} = \int_{s_1}^{s_2} F_s\, ds = \int_{s_1}^{s_2} -ks\, ds$$

$$U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$



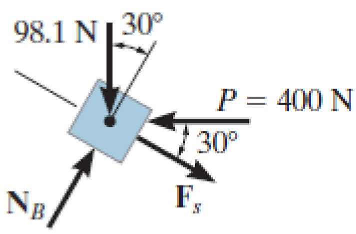


Example 9: The 10-kg block shown in Fig. rests on the smooth incline. If the spring is originally stretched 0.5 m, determine the total work done by all the forces acting on the block when a horizontal force P = 400 N pushes the block up the plane s = 2 m.



SOLUTION:

$$U_P = 400\text{ N}\,(2\text{ m}\cos 30°) = 692.8\text{ J}$$



Spring Force F_s. In the initial position the spring is stretched $s_1 = 0.5$ m and in the final position it is stretched $s_2 = 0.5 \text{ m} + 2 \text{ m} = 2.5$ m. We require the work to be negative since the force and displacement are opposite to each other. The work of $\mathbf{F}_s$ is thus

$$U_s = -\left[\tfrac{1}{2}(30\text{ N/m})(2.5\text{ m})^2 - \tfrac{1}{2}(30\text{ N/m})(0.5\text{ m})^2\right] = -90\text{ J}$$

Weight W. Since the weight acts in the opposite sense to its vertical displacement, the work is negative; i.e.,

$$U_W = -(98.1\text{ N})\,(2\text{ m}\sin 30^\circ) = -98.1\text{ J}$$

Normal Force N_B. This force does *no work* since it is *always* perpendicular to the displacement.

Total Work. The work of all the forces when the block is displaced 2 m is therefore

$$U_T = 692.8\text{ J} - 90\text{ J} - 98.1\text{ J} = 505\text{ J} \qquad \textit{Ans.}$$

2-10 The Principle of Work and Energy

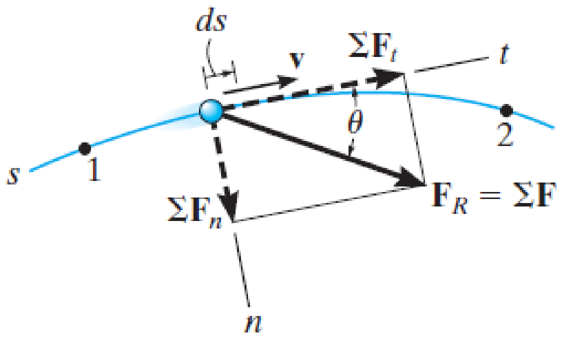
Consider the particle in Fig. which is located on the path defined relative to an inertial coordinate system. If the particle has a mass *m* and is subjected to a system of external forces represented by the resultant $\mathbf{F}_R = \sum\mathbf{F}$, then the equation of motion for the particle in the tangential direction is $\sum F_t = ma_t$. Applying the kinematic equation $a_t = v\,dv>ds$ and integrating both sides, assuming initially that the particle has a position $s = s_1$ and a speed $v = v_1$, and later at $s = s_2$, $v = v_2$, we have

$$\sum\int_{s_1}^{s_2} F_t\,ds = \int_{v_1}^{v_2} mv\,dv$$

$$\sum\int_{s_1}^{s_2} F_t\,ds = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$\sum U_{1-2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

$$T_1 + \sum U_{1-2} = T_2$$



Where:

$$\boldsymbol{T = kinetic\ energy\ \frac{1}{2}mv^2}$$

Principle of Work and Energy.

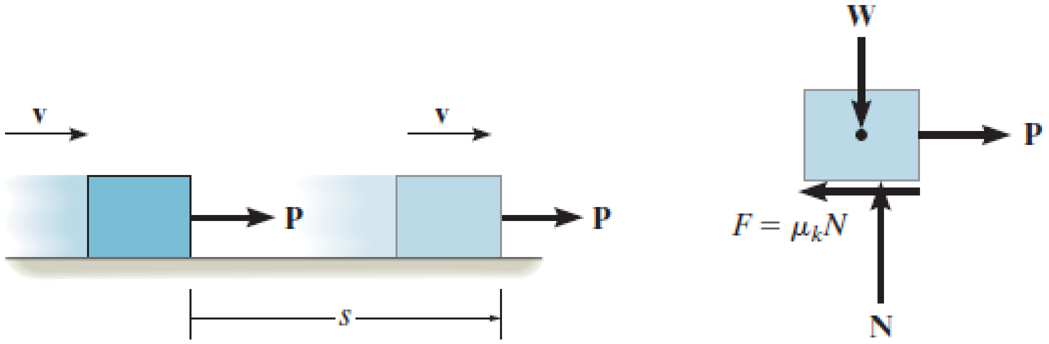
- Apply the principle of work and energy, $T_1 + \Sigma U_{1-2} = T_2$.
- The kinetic energy at the initial and final points is *always positive*, since it involves the speed squared $\left(T = \frac{1}{2}mv^2\right)$.
- A force does work when it moves through a displacement in the direction of the force.
- Work is *positive* when the force component is in the *same sense of direction* as its displacement, otherwise it is negative.
- Forces that are functions of displacement must be integrated to obtain the work. Graphically, the work is equal to the area under the force-displacement curve.
- The work of a weight is the product of the weight magnitude and the vertical displacement, $U_W = \pm Wy$. It is positive when the weight moves downwards.
- The work of a spring is of the form $U_s = \frac{1}{2}ks^2$, where k is the spring stiffness and s is the stretch or compression of the spring.

If we apply the principle of work and energy to this and each of the other particles in the system, then since work and energy are scalar quantities, the equations can be summed algebraically, which gives

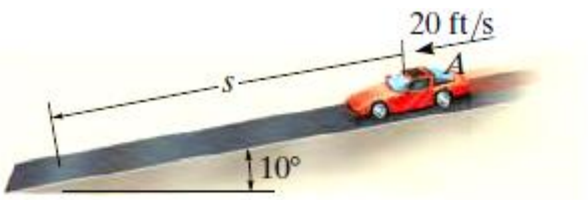
$$\Sigma T_1 + \Sigma U_{1-2} = \Sigma T_2$$

Problems involve cases where a body slides over the surface of another body in the presence of friction considers as special class of problems which requires a careful application. Consider, for example, a block which is translating a distance s over a rough surface as shown in Fig. If the applied force ***P*** just balances the *resultant* frictional force $\mu_k N$.

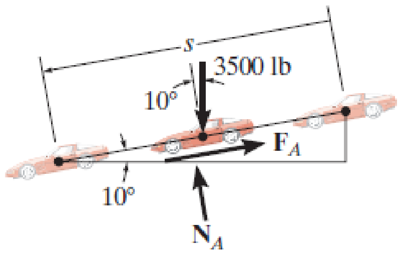
$$\frac{1}{2}mv^2 + Ps - \mu_k Ns = \frac{1}{2}mv^2$$



Example 10: The 3500-lb automobile shown in Fig. travels down the 10° inclined road at a speed of 20 ft/s. If the driver jams on the brakes, causing his wheels to lock, determine how far *s* the tires skid on the road. The coefficient of kinetic friction between the wheels and the road is $\mu_k = 0.5$.



SOLUTION:



Applying the equation of equilibrium normal to the road, we have

$$+\nwarrow \Sigma F_n = 0; \qquad N_A - 3500 \cos 10^\circ \text{ lb} = 0 \qquad N_A = 3446.8 \text{ lb}$$

Thus,

$$F_A = \mu_k N_A = 0.5\,(3446.8 \text{ lb}) = 1723.4 \text{ lb}$$

Principle of Work and Energy.

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\frac{1}{2}\left(\frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(20 \text{ ft/s})^2 + 3500 \text{ lb}(s \sin 10^\circ) - (1723.4 \text{ lb})s = 0$$

Solving for *s* yields

$$s = 19.5 \text{ ft} \qquad \textit{Ans.}$$

NOTE: If this problem is solved by using the equation of motion, *two steps* are involved. First, from the free-body diagram, Fig. , the equation of motion is applied along the incline. This yields

$$+\swarrow \Sigma F_s = ma_s; \qquad 3500 \sin 10^\circ \text{ lb} - 1723.4 \text{ lb} = \frac{3500 \text{ lb}}{32.2 \text{ ft/s}^2}a$$

$$a = -10.3 \text{ ft/s}^2$$

Then, since *a* is constant, we have

$$(+\swarrow) \quad v^2 = v_0^2 + 2a_c(s - s_0);$$

$$(0)^2 = (20 \text{ ft/s})^2 + 2(-10.3 \text{ ft/s}^2)(s - 0)$$

$$s = 19.5 \text{ ft} \qquad \textit{Ans.}$$