2-11 Power and Efficiency

The term "power" provides a useful basis for choosing the type of motor or machine which is required to do a certain amount of work in a given time. For example, two pumps may each be able to empty a reservoir if given enough time; however, the pump having the larger power will complete the job sooner. The *power* generated by a machine or engine that performs an amount of work *dU* within the time interval *dt* is therefore

$$P = \frac{dU}{dt}$$

$$P = \frac{dU}{dt} = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$

The basic units of power used in the SI and FPS systems are the watt (W) and horsepower (hp), respectively. These units are defined as

1 W = 1 J/s = 1 N . m/s

1 hp = 550 ft . lb/s

For conversion between the two systems of units, 1 hp = 746 W.

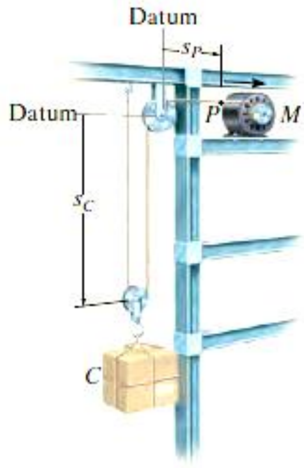
The *mechanical efficiency* of a machine is defined as the ratio of the output of useful power produced by the machine to the input of power supplied to the machine. Hence,

$$\varepsilon = \frac{\text{power output}}{\text{power input}}$$

If energy supplied to the machine occurs during the *same time interval* at which it is drawn, then the efficiency may also be expressed in terms of the ratio. Since machines consist of a series of moving parts, frictional forces will always be developed within the machine, and as a result, extra energy or power is needed to overcome these forces. Consequently, power output will be less than power input and so *the efficiency of a machine is always less than 1*. The procedure for analysis is as follow:

- First determine the external force **F** acting on the body which causes the motion. This force is usually developed by a machine or engine placed either within or external to the body.
- If the body is accelerating, it may be necessary to draw its free-body diagram and apply the equation of motion ($\Sigma \mathbf{F} = m\mathbf{a}$) to determine **F**.
- Once **F** and the velocity **v** of the particle where **F** is applied have been found, the power is determined by multiplying the force magnitude with the component of velocity acting in the direction of **F**, (i.e., $P = \mathbf{F} \cdot \mathbf{v} = Fv \cos\theta$).
- In some problems the power may be found by calculating the work done by **F** per unit of time ($P_{avg} = \Delta U / \Delta t$,).

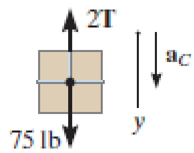
Example 11: The motor of the hoist shown in Fig. lifts the 75-lb crate C so that the acceleration of point P is 4 ft/s^2. Determine the power that must be supplied to the motor at the instant P has a velocity of 2 ft/s. Neglect the mass of the pulley and cable and take e = 0.85.



SOLUTION:

$$+\downarrow \quad \Sigma F_y = ma_y; \qquad -2T + 75\text{ lb} = \frac{75\text{ lb}}{32.2\text{ ft/s}^2}a_c \qquad (1)$$

$$2a_C = -a_P \qquad (2)$$



Since $a_P = +4\text{ ft/s}^2$, then $a_C = -(4\text{ ft/s}^2)/2 = -2\text{ ft/s}^2$. What does the negative sign indicate? Substituting this result into Eq. 1 and *retaining* the negative sign since the acceleration in *both* Eq. 1 and Eq. 2 was considered positive downward, we have

$$-2T + 75\text{ lb} = \left(\frac{75\text{ lb}}{32.2\text{ ft/s}^2}\right)(-2\text{ ft/s}^2)$$

$$T = 39.83\text{ lb}$$

The power output, measured in units of horsepower, required to draw the cable in at a rate of 2 ft/s is therefore

$$P = \mathbf{T}\cdot\mathbf{v} = (39.83\text{ lb})(2\text{ ft/s})[1\text{ hp}/(550\text{ ft}\cdot\text{lb/s})]$$
$$= 0.1448\text{ hp}$$

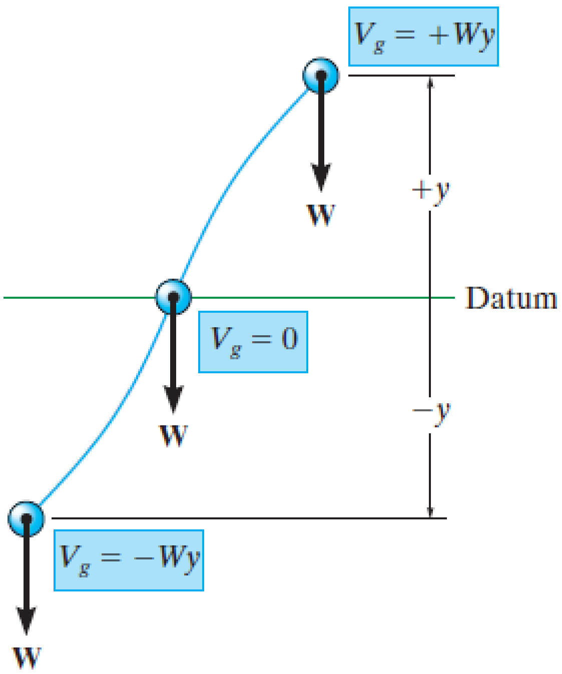
This *power output* requires that the motor provide a *power input* of

$$\text{power input} = \frac{1}{\varepsilon}(\text{power output})$$
$$= \frac{1}{0.85}(0.1448\text{ hp}) = 0.170\text{ hp} \qquad \textit{Ans.}$$

NOTE: Since the velocity of the crate is constantly changing, the power requirement is *instantaneous*.

2-12 Conservative Forces and Potential Energy

If the work of a force is *independent of the path* and depends only on the force's initial and final positions on the path, then we can classify this force as a *conservative force*. Examples of conservative forces are the weight of a particle and the force developed by a spring. The work done by the weight depends *only* on the *vertical displacement* of the weight, and the work done by a spring force depends *only* on the spring's *elongation* or *compression*.



Energy is defined as the capacity for doing work. For example, if a particle is originally at rest, then the principle of work and energy states that $\sum U_{1\text{-}2} = T_2$. In other words, the kinetic energy is equal to the work that must be done on the particle to bring it from a state of rest to a speed v. Thus, the *kinetic energy* is a measure of the particle's *capacity to do work*, which is associated with the *motion* of the particle. When energy comes from the *position* of the particle, measured from a fixed datum or reference plane, it is called potential energy. Thus, *potential energy* is a measure of the amount of work a conservative force will do when it moves from a given position to the datum. In mechanics, the potential energy created by gravity (weight) and an elastic spring is important.

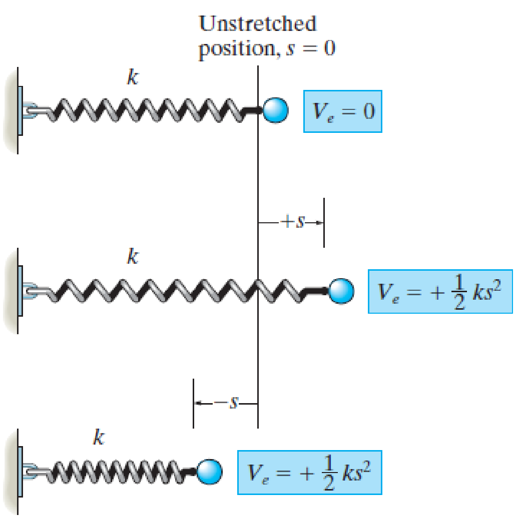
In general, if y is *positive upward*, the gravitational potential energy of the particle of weight W is

$$V_g = Wy$$

When an elastic spring is elongated or compressed a distance s from its unstretched position, elastic potential energy V_e can be stored in the spring. This energy is

$$V_e = +\tfrac{1}{2}ks^2$$

Here V_e is *always positive* since, in the deformed position, the force of the spring has the *capacity* or "potential" for always doing positive work on the particle when the spring is returned to its unstretched position.



Elastic potential energy

In the general case, if a particle is subjected to both gravitational and elastic forces, the particle's potential energy can be expressed as a *potential function*, which is the algebraic sum

$$V = V_g + V_e$$

The work done by a conservative force in moving the particle from one point to another point is measured by the *difference* of this function, i.e.,

$$U_{1-2} = V_1 - V_2$$

For example, the potential function for a particle of weight W suspended from a spring can be expressed in terms of its position, s, measured from a datum located at the unstretched length of the spring, We have

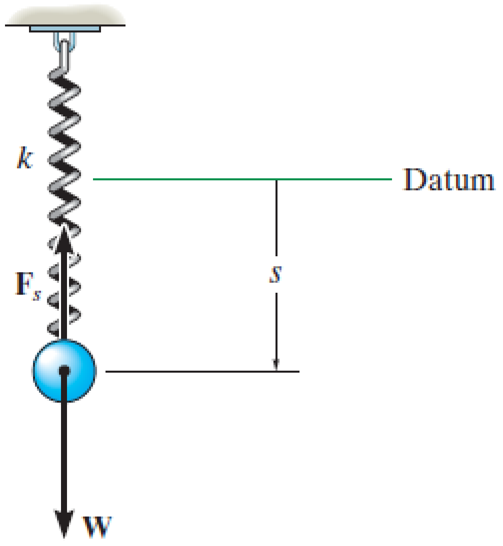
$$V = V_g + V_e$$

$$= -Ws + \tfrac{1}{2}ks^2$$

If the particle moves from $s1$ to a lower position s_2 , it can be seen that the work of **W** and $\mathbf{F}_s$ is

$$U_{1-2} = V_1 - V_2 = \left(-Ws_1 + \tfrac{1}{2}ks_1^2\right) - \left(-Ws_2 + \tfrac{1}{2}ks_2^2\right)$$

$$= W(s_2 - s_1) - \left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)$$



2-13 Conservation of Energy

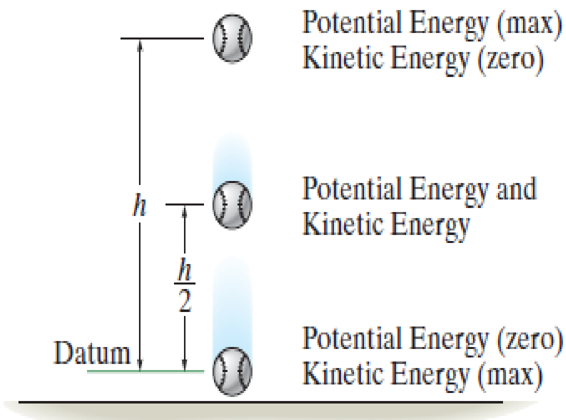
When a particle is acted upon by a system of *both* conservative and nonconservative forces, the portion of the work done by the *conservative forces* can be written in terms of the difference in their potential energies , i.e., $(\sum U_{1\text{-}2})_{cons.} = V_1 - V_2$. As a result, the principle of work and energy can be written as

$$T_1 + V_1 + (\Sigma U_{1-2})_{\text{noncons.}} = T_2 + V_2$$

Here $(\sum U_{1\text{-}2})_{noncons.}$ represents the work of the nonconservative forces acting on the particle. If *only conservative forces* do work then we have

$$T_1 + V_1 = T_2 + V_2$$

This equation is referred to as the *conservation of mechanical energy* or simply the *conservation of energy*. It states that during the motion the sum of the particle's kinetic and potential energies remains *constant*. For this to occur, kinetic energy must be transformed into potential energy, and vice versa. For example, if a ball of weight **W** is dropped from a height *h* above the ground (datum), the potential energy of the ball is maximum before it is dropped, at which time its kinetic energy is zero. The total mechanical energy of the ball in its initial position is thus



$$E = T_1 + V_1 = 0 + Wh = Wh$$

When the ball has fallen a distance *h*>2, its speed can be determined by using

$$v^2 = v_0^2 + 2a_c(y - y_0)$$

which yields

$$v = \sqrt{2g(h/2)} = \sqrt{gh}.$$

The energy of the ball at the mid-height position is therefore

$$E = T_2 + V_2 = \frac{1}{2}\frac{W}{g}(\sqrt{gh})^2 + W\left(\frac{h}{2}\right) = Wh$$

Just before the ball strikes the ground, its potential energy is zero and its speed is

$$v = \sqrt{2gh}$$

Here, again, the total energy of the ball is

$$E = T_3 + V_3 = \frac{1}{2}\frac{W}{g}(\sqrt{2gh})^2 + 0 = Wh$$

Note that when the ball comes in contact with the ground, it deforms somewhat, and provided the ground is hard enough, the ball will rebound off the surface, reaching a new height *h*', which will be *less* than the height *h* from which it was first released. Neglecting air friction, the difference in height accounts for an energy loss,

$$El = W(h - h')$$

Which occurs during the collision. Portions of this loss produce noise, localized deformation of the ball and ground, and heat.

If a system of particles is *subjected only to conservative forces*, then an equation can be written for the particles. Applying the ideas of the preceding discussion, ($\sum T_1 + \sum U_{1\text{-}2} = \sum T_2$) becomes

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

Here, the sum of the system's initial kinetic and potential energies is equal to the sum of the system's final kinetic and potential energies. In other words, $\sum T + \sum V = \text{const.}$ The conservation of energy equation can be used to solve problems involving *velocity*, *displacement*, and *conservative force systems*. It is generally *easier to apply* than the principle of work and energy because this equation requires specifying the particle's kinetic and potential energies at only *two points* along the path, rather than determining the work when the particle moves through a *displacement*. For application it is suggested that the following procedure be used.

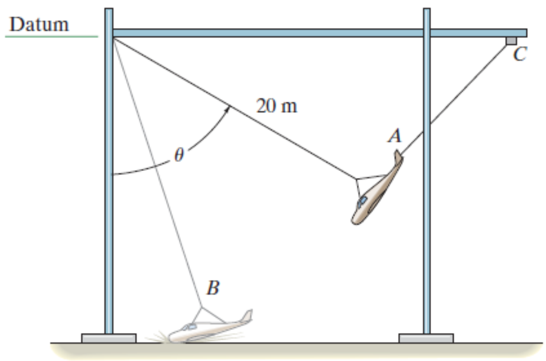
Potential Energy.

- Draw two diagrams showing the particle located at its initial and final points along the path.
- If the particle is subjected to a vertical displacement, establish the fixed horizontal datum from which to measure the particle's gravitational potential energy Vg .
- Data pertaining to the elevation y of the particle from the datum and the stretch or compression s of any connecting springs can be determined from the geometry associated with the two diagrams.
- Recall $Vg = Wy$, where y is positive upward from the datum and negative downward from the datum; also for a spring, Ve = 12 ks2, which is *always positive*.

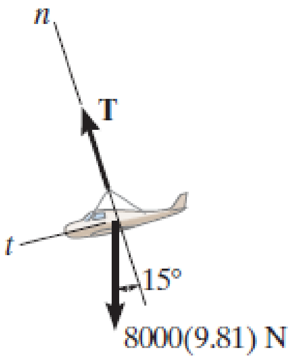
Conservation of Energy.

- Apply the equation $T1 + V1 = T2 + V2$.
- When determining the kinetic energy, T = 12 mv2, remember that the particle's speed v must be measured from an inertial reference frame.

Example 12: The gantry structure in the photo is used to test the response of an airplane during a crash. As shown in Fig. the plane, having a mass of 8 Mg, is hoisted back until $\theta = 60^o$, and then the pull-back cable AC is released when the plane is at rest. Determine the speed of the plane just before it crashes into the ground, $\theta = 15^o$. Also, what is the maximum tension developed in the supporting cable during the motion? Neglect the size of the airplane and the effect of lift caused by the wings during the motion.



SOLUTION:



$$T_A + V_A = T_B + V_B$$

$$0 - 8000\text{ kg }(9.81\text{ m/s}^2)(20\cos 60°\text{ m}) =$$

$$\tfrac{1}{2}(8000\text{ kg})v_B^2 - 8000\text{ kg }(9.81\text{ m/s}^2)(20\cos 15°\text{ m})$$

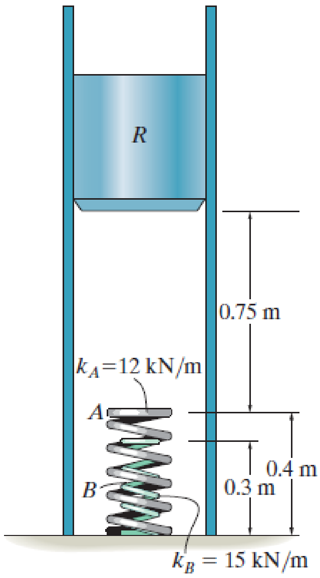
$$v_B = 13.52\text{ m/s} = 13.5\text{ m/s} \qquad \textit{Ans.}$$

$+\nwarrow \quad \Sigma F_n = ma_n;$

$$T - (8000(9.81)\text{ N})\cos 15° = (8000\text{ kg})\frac{(13.52\text{ m/s})^2}{20\text{ m}}$$

$$T = 149\text{ kN} \qquad \textit{Ans.}$$

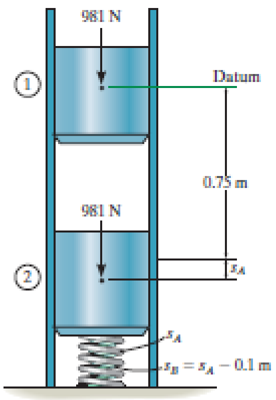
Example 13: The ram *R* shown in Fig. has a mass of 100 kg and is released from rest 0.75 m from the top of a spring, *A*, that has a stiffness k_A = 12 kN/m. If a second spring *B*, having a stiffness k_B = 15 kN/m, is "nested" in *A*, determine the maximum displacement of *A* needed to stop the downward motion of the ram. The unstretched length of each spring is indicated in the figure. Neglect the mass of the springs.



SOLUTION:

Potential Energy. We will *assume* that the ram compresses *both* springs at the instant it comes to rest. The datum is located through the center of gravity of the ram at its initial position, Fig. When the kinetic energy is reduced to zero ($v_2 = 0$), *A* is compressed a distance s_A and s_B compresses $s_B = s_A - 0.1$ m.

Conservation of Energy.



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = 0 + \left\{\tfrac{1}{2}k_A s_A^2 + \tfrac{1}{2}k_B(s_A - 0.1)^2 - Wh\right\}$$

$$0 + 0 = 0 + \left\{\tfrac{1}{2}(12\,000\text{ N/m})s_A^2 + \tfrac{1}{2}(15\,000\text{ N/m})(s_A - 0.1\text{ m})^2 - 981\text{ N }(0.75\text{ m} + s_A)\right\}$$

Rearranging the terms,

$$13\,500 s_A^2 - 2481 s_A - 660.75 = 0$$

Using the quadratic formula and solving for the positive root, we have

$$s_A = 0.331 \text{ m} \qquad \textit{Ans.}$$

Since $s_B = 0.331 \text{ m} - 0.1 \text{ m} = 0.231 \text{ m}$, which is positive, the assumption that *both* springs are compressed by the ram is correct.

NOTE: The second root, $s_A = -0.148 \text{ m}$, does not represent the physical situation. Since positive s is measured downward, the negative sign indicates that spring A would have to be "extended" by an amount of 0.148 m to stop the ram.