Chapter 6

Circular Motion and other Applications of Newton's laws

Newntos Second Law Applied to Uniform Circular Motion

$$\mathbf{a_c = v^2/r}$$

Where ($\mathbf{a_c}$): the centierpetal acceleration of a particle moving with uniform speed Consider figure (1), when we apply the newtons second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:

$$\mathbf{\Sigma F = m.a_c = m(v^2/r)}$$

* This force acts toward the center of the circlular path and causes a change in the direction of a velocity vecrot.
* The force causing centripetal acceleration is called a (centripetal force).

Example 1: A small object of mass (m) is suspended from a string of length (L). The object revolves with constant speed (v) in a horizontal circle of radius (r). The system is known as a (conical pendulum). Find the expression of (v)?

Solution:

Because the object does not accelerate in thr vertical direction, $\mathbf{\Sigma F_y = 0}$

$\mathbf{T\cos\theta - m.g = 0 \implies T\cos\theta = m.g}$ **....... (1)**

$\mathbf{\Sigma F_x = ma_c \implies T\sin\theta = m(v^2/r)}$ **....... (2)**

Solve eq (1) and (2) $\mathbf{\implies F\ \tan\theta = v^2/g.r}$ **or** $\mathbf{v = \sqrt{r\,g\,\tan\theta}}$

But $\mathbf{r = L\sin\theta \implies v = \sqrt{Lg\sin\theta\tan\theta}}$

So the speed is independent of the mass of the object.

Example 2: A ball of mass (0.5 kg) is attached to the end of the cord (L = 1.5 m) long. The ball is whirled in a horizontal circle as shown in fig-(3). If the cord can withstand a maximum tention of (5 N). **(a)** What is the max speed at which the ball can be whirled the cord breaks? Assume that the string remains horizontal during motion. **(b)** Suppose that the ball is whirled in a circle of larger radius at the same speed (v), is the cord more likely to break or less likely?

Solution:

(a)

$\mathbf{T = m\,(v^2/r)}$ .... (1)   and $\mathbf{v = \sqrt{T.r/m}}$

This equation shows that (v) increases with (T) and decreases with larger (m).

$\mathbf{v_{max} = \sqrt{T_{max}/m}} = \sqrt{(5\ N)\ (1.5\ m)\ /\ (0.5\ kg)} = 12.2$ m/s

(b) The larger radius means that the change in direction of the velocity vector will be smaller for a given time interval. Thus the acceleration is smaller and the required force from the string is small. As a result, the string is less likely to break when the ball travels in the in a circle of larger radius.

$\mathbf{T_1 = mv^2 / r_1}$ **or** $\mathbf{T_2 = mv^2 / r_2}$

$\mathbf{T_2 / T_1 = (mv^2 / r_2) / (mv^2 / r_1)}$ $\Longrightarrow$ $\mathbf{T_2 / T_1 = r_1 / r_2}$

If we choose $r_2 > r_1$ we see that $T_2 < T_1$

* Less tension required to whirl the ball in the larger circle and the string is less likely to break.

<u>Example 3</u>: A civil engineer wish to design a curved exit ramp for a highway in such away that a car will not have to rely on friction around the curve without skidding. In other words a car moving at the designated speed can negotiate the curve even when the road is covered with ice. The roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be (13.4 m/s) and radius of the curve is (50 m). At what angle should the curve be banked?

Solution: Because the is to be designed so that the force of static friction is zero. Only the component ($\mathbf{n_x = n\,\sin\theta}$) causes the centripetal acceleration.

$\mathbf{\Sigma F_r = n\,\sin\theta = m\,(v^2/r)}$ .... (1)

$\mathbf{\Sigma F_y = 0}$ $\Longrightarrow$ $\mathbf{n\,\cos\theta = m\,g}$ .... (2)

From equations (1) and (2)

$\mathbf{\tan\theta = v^2 / r.g}$ .... (3)

$\mathbf{\theta = \tan^{-1}[(13.4 / (50*9.8)] = 20.1^\circ}$

*** Note**: If a car round the curve at a speed less than (13.4 m/s), friction is needed to kep it from sliding down the bank (to the left). At speed greater than (13.4 m/s) friction required to keep it from sliding up the bank (to the right).

Non Uniform Circular Motion:

* In addition to radial component of acceleration ($\mathbf{a_r}$), there is a tangential component ($\mathbf{a_t}$) having a magnitude of (**dv/dt**).

* The total force exerted on the particle is:

$$\Sigma F = \Sigma F_r + \Sigma F_t$$

*** a_t:** represent the change in the speed of particle with time.

Example 4: A small sphere of mass (m) is attached to the end of a cord of length (R) and set into the motion in a vertical circle about a fixed point (o). (a): Determine the tension in the cord at any instant when the speed of the sphere is (v) and the cord makes an angle (ɵ) with the vertical? (b): If we set the ball in motion of a slower speed, what would the ball have as it passes over the top of the circle if the tension in the goes to zero instantaneously at this point?

Solution:

(a): $\Sigma F_t = m\,g\,\sin\theta = m\,a_t \Longrightarrow a_t = g\,\sin\theta$

$\Sigma F_r = T - m\,g\,\cos\theta = m(v^2/R)$ or $T = m\,(v^2/R + g\,\cos\theta)$

(b): At the top of the path where (ɵ = 180°), we have (cos 180°) = -1, so:

$T_{top} = m\,[(v^2_{top} / R) - g]$

Let us set $T_{top} = 0 \Longrightarrow 0 = m\,[(v^2_{top} / R) - g]$

$V_{top} = \sqrt{g\,R}$