Biostatistics
Lecture 9

T test

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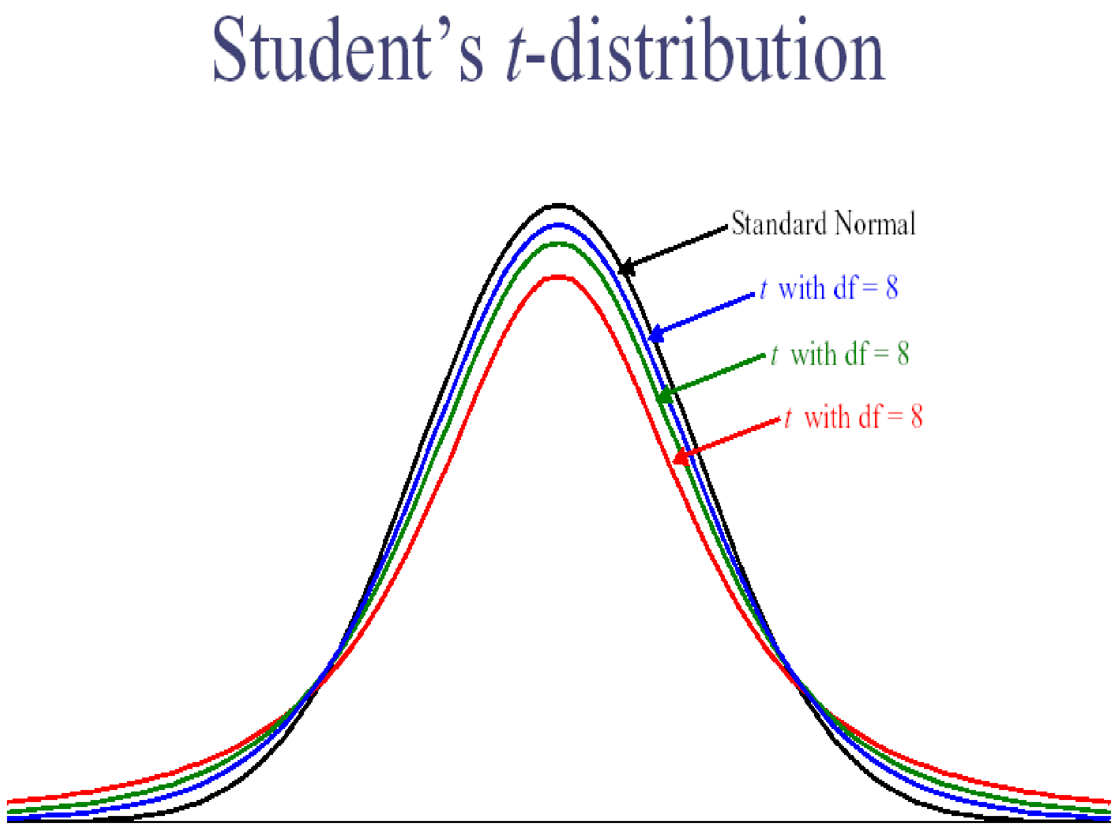
(T) test (student test):-

- **It's used to study & compare: 1- Between the mean of sample & mean of population.**

2- Compare between means of 2 independent samples. 3- Compare between the mean of sample before & after treatment, that if has sample size < 60 can use t test but if equal or below 30 should use t test (= or<30).

- t-distribution has one parameter ((*degree of freedom(df)*

- DF=n-1
- The t-distribution is similar to the normal distribution:
  - Symmetric about 0
  - Bell shaped
- The main differences between the t-distribution and the normal distribution is in tails:
- T-distribution has larger tails than the normal
- Larger DF means smaller tails, the larger the DF, the closer to the normal distribution, small DF means larger tails



T Test

- **<u>One mean</u>:** $$t = \frac{x^{-} - \mu}{\frac{sd}{\sqrt{n}}}$$

where x^{-} = mean of sample μ = mean of standard population

SD = S.D n = sample size (≤ 30)

Example: -

26 patients after surgery, has standard mean of temp 99F, S.D =1. standard temp (normal temp of people)= 98Fr.

- Find if there is statistically deference between temp of patient &normal people

<u>Steps of</u>:

1. **Data :X^{-}= 99Fr^{0}, S.D =1 , μ= 98F^{0} , n=26**
2. **Ho : $x^{-} = \mu$, HA: $x^{-} \neq \mu$.**
3. **Assume T distribution.**
4. **Calculate df = n – 1 = 26-1=25**

T test

- 6. Compare t-calculate with t- tabulate: If t-calculate is > t- tabulate, reject the Ho, if t-calculate is < t- tabulate, do not reject Ho.
- $T\ test = \frac{99-98}{1\,|\,\sqrt{26}} = 2.55$ (calculated T)

- Degree of freedom = n-1 = 26-1=25 - Tabulated T = 2.06
- Calculated T (2.55) > Tabulated T = (2.06)
- Reject Ho , - p. value < 0.05, - there it is statistically significant difference between temp of patients & temp of people after 48 hours post-surgery.

Probability p. value				
degree of freedom	0.5	0.1	0.05	0.01
1	1.000	6.31	12.71	63.66
5	0.727	2.02	2.57	4.03
10	0.700	1.71	2.23	3.17
20	0.687	1.71	2.06	2.84
25	0.674	1.64	2.06	2.79

2- T test of 2means of 2 independent samples with equal variance ;

- if has sample size < 60 can use t test but if equal or below 30 should use student samples.
- T test = (X1”-X”2)
- SP (S1,2)× $\sqrt{\frac{1}{n1} + \frac{1}{n2}}$
- Where SP (S1,2) = $\sqrt{\frac{(n1-1)\ x\ S^2 1 + (n2-1)\ x\ S^2 2}{n1 + n2 - 2}}$ (find first)

Example; The following data represents weight in Kg for 10 males and 12 females. p=0.01

- Males: 80 75 95 55 60 70 75 72 80 65
- Females: 60 70 50 85 45 60 80 65 70 62 77 82
- Note; should find Mean & Variance

STEPS

1. Data: Mean1=72.7 , N1= 10 , Mean2=67.17 , N2 =12

- Variance1 =128.46 Variance2=157.78

2. Ho: mean1 = mean2 , HA: mean1 ≠ mean2.

3. Assume T distribution, p=0.01

4. Calculate df = n1+n2 – 2 = 20

5. Test statistics: t-test to calculate. T test =

$$\frac{(X1'' - X2'')}{SP \times \sqrt{\frac{1}{n1} + \frac{1}{n2}}}$$

6. Compare t-calculate with t-tabulate: If t-calculate is > t- tabulate, reject the Ho, If t-calculate is < t-tabulate, do not reject Ho.

- $$SP = \sqrt{\frac{((\ (n1-1\ )x\ S^2 1\ +\ (n2-1)x\ S^2 2}{n1+n2-2}}$$

- $$S1,2 = \sqrt{\frac{((\ 10-1\ )x\ 128.6\ +\ (12-1)x\ 157.78}{10+12-2}}$$ $$S1,2 = \sqrt{\frac{(1156.14\ +\ 1735.58}{20}}$$

$$= \sqrt{144.586} = (12.024)$$

T TEST

$$T\ test = \frac{72.7 - 67.17}{(12.024 \times \sqrt{(0.1 + 0.083)})} = \frac{5.53}{12.024 \times \sqrt{(0.183)}}$$

T test = 1.075, p=0.01

conclusion

1- The tabulated t, for alpha 0.01 is 2.53

2- the calculated t (1.075) < tabulated t (2.53) , - p>0.01

3- accept Ho and reject HA,

4- there is no significant difference between the 2 means, this difference may be due to chance.

3 . Compare after & before treatment of one sample with 2 occasions):

$$Td = \frac{\bar{d}\,mean}{\frac{S\bar{d}}{\sqrt{n}}} \qquad S\bar{d} = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}}$$

df = n – 1 , d‾ = mean , Sd‾ = SD of deference between after & before

- **Steps of test :**

1. Data : d‾ mean = mean , $\sum d^2$, $(\sum d)^2$

2. testing hypothesis : Ho: there is no significant difference between readings before and after treatment. HA: there is significant difference between readings before and after treatment

- 3. Assume T distribution
- 4. Calculate df = n – 1 , p=0.05
- 5. Test statistics: Td = d‾ mean / Sd‾/√ (n)

6. Conclusion: compare t-calculate with t-tabulated: If t-calculate is > tabulated reject the Ho, if t-calculate is < t- tabulate, do not reject Ho.

Example:

- Systolic Blood pressure of 8 patients, before & after treatment

Data : d¯ mean = 58.125 , d ² = 29175, (∑d)²= (465) ². n = 8

2. testing hypothesis : Ho: there is no significant difference

between readings before and after treatment

HA: there is significant difference between readings before and after treatment

3. Assume T distribution

4. Calculate df = n – 1, Level of significance = 0.05

5. Test statistics: Td = d¯mean / Sd¯/√ (n)

6. Conclusion: If t-calculated is >t- tabulated then reject the Ho,

if t-calculated is < t- tabulated, do not reject Ho.

$$Td = \frac{d^{-}mean}{\frac{Sd^{-}}{\sqrt{n}}}$$

BP before	BP after	d	d ²
180	140	40	1600
200	145	55	3025
230	150	80	6400
240	155	85	7225
170	120	50	2500
190	130	60	3600
200	140	60	3600
165	130	35	1225
Mean= d¯ mean = 465 / 8 = 58.125		∑d =465	29175 ∑d ² 29175

T TEST

- $Sd^{-} = \sqrt{\left(\frac{\sum d^2 - (\sum d)^2/(n)}{n-1} \right)}$
- $Sd^{-} = \sqrt{(29175 - ((465)^2/ 8)/ 7)} = \sqrt{(29175 - 27028.125 / 7)}$
- $= \sqrt{(2146.875 / 7)} = \sqrt{306.606}$
- $Sd^{-} = 17.510$
- $Td = \frac{d^{-}mean}{\frac{Sd^{-}}{\sqrt{n}}}$
- $= 58.125 / 17.510 / \sqrt{8} = 58.125 / 17.510 / 2.83$
- $= 58.125 / 6.19 = 9.39$
- -Tabulated t (df 7), with level of significance 0.05 = 1.895 (from table)
- - Calculated t > Tabulated t, - P value < 0.05
- -We reject Ho and accept HA, - there is significant difference between BP
- readings before and after treatment at level P <0.05.

Confidence intervals for (T) test

To estimate the population parameter: at 95%, 99%, 90% C. Level

1. **one sample of one mean:**

   $\bar{X} \pm (t_{df\ n-1}) \times (SD/(\sqrt{n})$

   (($t_{df\ n-1} = t_{1-\alpha}$ = t from table))

2. **Two samples of two means :**

$$(\bar{X}_1 - \bar{X}_2) \pm (t_{df\ n1+n2-2}) \times Sp\ (S1, 2) \times \sqrt{\left(\frac{1}{n1} + \frac{1}{n2}\right)}$$

3. **After & before :**

   $\bar{d}$ mean $\pm$ $t_{df\ n-1} \times (S\bar{d}/(\sqrt{n})$

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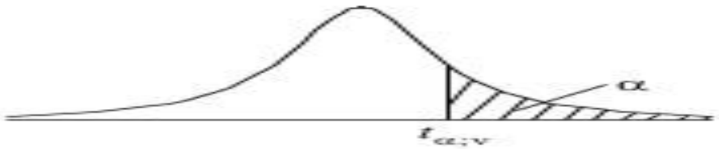
Table of the Student's *t*-distribution

The table gives the values of $t_{\alpha;\nu}$ where
$\Pr(T_\nu > t_{\alpha;\nu}) = \alpha$, with ν degrees of freedom



ν \ α	0.1	0.05	0.025	0.01	0.005	0.001	0.0005
1	3.078	6.314	12.076	31.821	63.657	318.310	636.620
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291