Advances in Concrete Mechanics

Course code

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Advances in Concrete Mechanics

Syllabus:

1. Basic concepts

2. Stress-strain relationships

3. Elastic behaviour

4. Viscoelasticity

- Rheological models
- Superposition principle and integral representation
- Prediction of creep and shrinkage

Temperature distribution in mass concrete

- Heat transfer analysis
- Finite element formulation

Fracture mechanics

- Linear elastic fracture mechanics
- Nonlinear fracture mechanics for concrete
- Fracture process zone

References to extend your knowledge:

1) P. K. MEHTA and P. J. M. MONTEIRO, “Microstructure, Properties, and Materials,” McGraw-Hil, 2006, 684 pp.

2) Wai-Fah CHEN, “Plasticity in Reinforced Concrete,” J. Ross Publishing, 2007, 475 pp.

3) REDDY J.N. (2004). “Mechanics of Laminated Composite Plates - Theory and Analysis,” CRC Press, USA, 2nd Edition, 831 pp.

INTRODUCTION

This course can be seen as a mathematical approach to understand reinforced concrete mechanics.

Concrete strength both in tension and compression in addition to the constitutive models of stress-strain relationships is a key subject for modern analysis using any commercially available solver. Mastering finite element method and fracture analysis is practically not possible without a good insight into these key concepts regardless how sophisticated your model is (linear-elastic, nonlinear-elastic, viscoelastic, or plastic) and for all types of loading and boundary conditions (static/dynamic).

The theory of composite materials has been extensively used to model the elastic behaviour of advanced ceramics, rocks, soils, and so forth. In this course, the theory will be applied to estimate the elastic moduli of concrete, when the elastic moduli and the volume fractions of cement paste and aggregate are known. Concrete is a porous material with a number of microcracks even before load is applied. The *differential scheme* and *Mori-Tanaka method* allow the computation of the effect of these imperfections on the elastic moduli of concrete. The concept of upper and lower bounds for elastic moduli is discussed, and the *Hashin-Shtrikman bounds* are presented.

In this course, a number of rheological models that aid understanding of the underlying mechanisms of creep and stress relaxation are presented. Changes in properties of concrete, with time, makes the task of mathematical modelling more complex. Methods for incorporating the age of concrete into mathematical models are discussed. Some viscoelastic formulations, such as the principle of superposition, are also discussed. Finally, *methods of estimating the creep and shrinkage* are illustrated by the use of CEB-FIP and ACI codes as well as the *Bazant-Panula model*.

The finite element method for computing the *temperature distribution in mass concrete* is introduced. Examples of application of this method to a number of practical situations in concrete are also given.

Fracture mechanics of concrete has become a powerful method for studying the behaviour of plain and reinforced concrete members in tension. The traditional concept of linear fracture mechanics and its limitations when applied to concrete are discussed in this course. The use of the finite element method for cracks in structures is demonstrated. Also presented are nonlinear fracture mechanics models, such as the *fictitious crack model, and the smeared crack model*,

1. BASIC CONCEPTS

1.1 The Characteristic of Reinforced Concrete Behaviour

The characteristics stages of reinforced concrete behaviour can be illustrated by a typical load-displacement relationship, as shown in Figure 1.1. This relationship can be the result of a beam test, for example. The highly nonlinear relationship can be roughly divided into three intervals: the *uncracked elastic stage*, *crack propagation*, and the *plastic stage*. The nonlinear response is caused by two major material effects, *cracking* of the concrete and *plasticity* of the reinforcement and of the compression concrete. Other time-independent nonlinearities arise from the nonlinear action of the individual constituents of reinforced concrete, e.g., bond-slip between steel and concrete, aggregate interlock of a cracked concrete (Figure 1.2), and dowel action of reinforcing steel (Figure 1.3). The time-dependent effects such as creep, shrinkage, and temperature change also contribute to the nonlinear response.

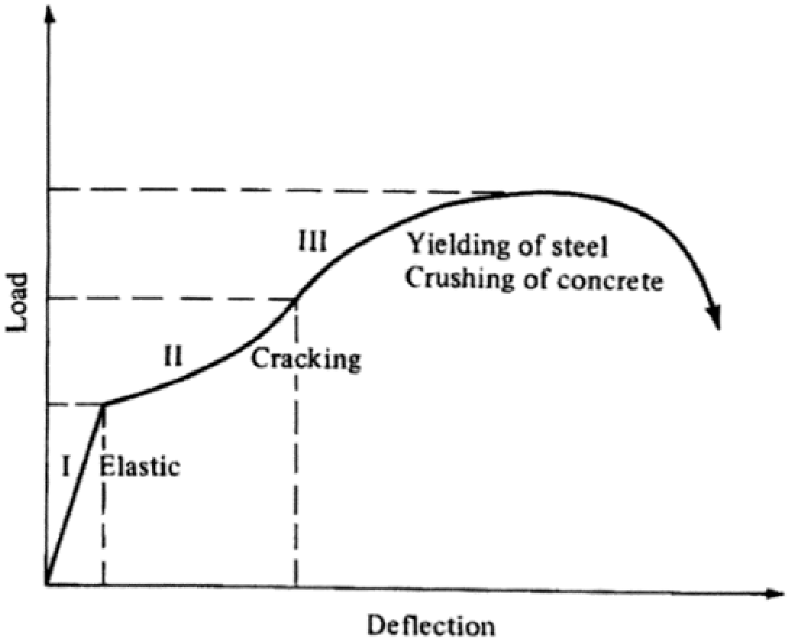
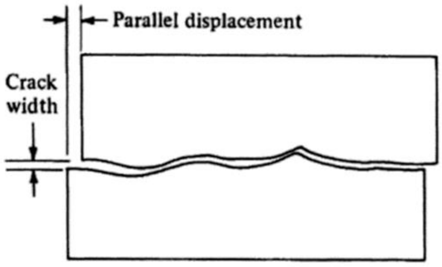


Fig. 1.1: Typical load-displacement relationship for a reinforced concrete element.



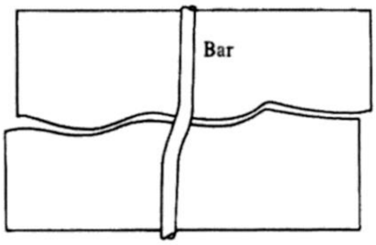


Fig. 1.2: Aggregate interlock. **Fig. 1.3: Dowel action of a reinforcing bar.**

1.2 Progressive-Failure Analysis

A complete progressive-failure analysis of reinforced concrete structures under static and dynamic conditions requires the consideration of loading input, generalized material behaviour, and analytical procedure.

Loading refers to the specific forces and motions that should be considered in design and analysis of reinforced concrete structures. This is beyond the scope of this course.

Generalized material behaviour refers to multidimensional stress-strain relations which adequately describe the basis characteristics of reinforced concrete materials subjection to monotonic and cyclic loading. These *constitutive equations* are the most fundamental relations required for any analysis of reinforced concrete structures. To discuss the mathematical modelling of nonlinear reinforced concrete behaviour, three areas must be examined: the behaviour of concrete, the response of steel reinforcement, and the bond-slip phenomenon between steel and concrete.

Since steel reinforcement is comparatively thin, it is generally assumed to be capable of transmitting axial force only; thus, a uniaxial stress-strain relationship is sufficient for general use. For concrete, however, a knowledge of multiaxial stress-strain behaviour is required. Once the stress-strain relation of each material is available and a bond-slip relation is assumed, steel reinforcements can be placed in the proper positions in concrete elements and constitutive equations for the composite response of reinforced concrete elements can readily be formulated. The mechanics of the bond-slip phenomenon between steel and concrete will not be considered. In most practical applications, a perfect bond is generally assumed.

Analytical procedure refers to the mathematical and numerical aspects of calculation used to obtain solutions. In recent years, there has been a growing interest in the application of finite-element procedure to the analysis of reinforced concrete problems.

1.3 Constitutive Modelling of Reinforced Concrete

The several approaches for defining this complicated stress-strain behaviour of reinforced concrete under various stress states can be divided in four main groups: (1) representation of given stress-strain curves by using curve-fitting methods, interpolation, or mathematical functions; (2) linear and nonlinear-elasticity theories; (3) perfect- and work-hardening-plasticity theories; (4) the endochronic theory of plasticity. Current analysis procedures for two-dimensional reinforced concrete structures such as plates and shells are essentially one-dimensional. The common approach uses an equivalent uniaxial stress-strain relation for the biaxial stress-strain behaviour of concrete. Thus, various empirical stress-strain equations expressed in terms of their respective principal stress and strain values have been established by curve fitting many biaxial-test data. This one-dimensional approach is appealing because of its broad data base and its simplicity. Multidimensional analyses are usually made by taking the concrete to be incrementally elastic with variable moduli.

Some of the main constitutive models used in the numerical analysis of reinforced concrete structures are described briefly below.

1.3.1 Linear elasticity

In spite of its shortcomings, the linear-elasticity theory is the most commonly used material model for concrete in the pre- and post-failure range.

1.3.2 Non-linear elasticity

The linear-elastic models can be significantly improved by assuming nonlinear-elastic stress-strain relationship in secant-modulus form. The most prominent models of this class are the hyperelastic type of formulation, which approximates a path-independent reversible process with no memory. In contrast, differential or incremental material descriptions of the hypoelastic type do not exhibit these shortcomings. The hypoelastic model approximates a path-dependent, irreversible process with limited memory and no pronounced reference state.

Incremental stress-strain relations provide a natural mathematical extension for materials with some memory and with no pronounced reference state. The hypoelastic model with variable moduli of tangential material stiffness describes the instantaneous behaviour directly in terms of the rates of stress and strain tensors. Thus, the hypoelastic formulation offers a more general description of materials with limited memory and reduces to the hyperelastic formulation as its limiting case

1.3.3 Failure criteria

The elastic models must of course be combined with criteria defining failure of the concrete material. Various mathematical models with one to five parameters are developed for the description of initial concrete failure under triaxial stress conditions. These failure surfaces, as shown schematically in Figure (1.4) in principal-stress space, can be combined with elasticity-based constitutive models for the overload and ultimate load analysis of three-dimensional reinforced concrete structures. Alternatively, the failure surfaces can be applied to a working stress design using relevant safety philosophies. The failure surfaces are used to construct initial yield surfaces (Figure 1.4) and subsequent loading surfaces, from which the incremental stress-strain relations of concrete based on the flow or incremental theory of plasticity can be constructed.

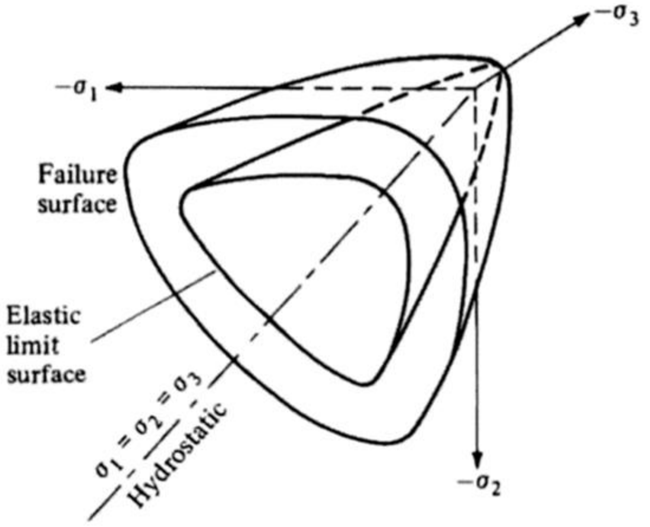


Fig. 1.4: Schematic failure surface of concrete in three-dimensional stress space.

1.3.4 Perfect plasticity

It is know that under triaxial compression concrete can flow like a ductile material on the yield or failure surface before reaching its crushing strains. The assumption that concrete crushes completely once the fracture surface is reached is rather rough but a fair first approximation. To account for this limited plastic-flow ability of concrete before crushing, a *perfectly plastic model* can be introduced, see Figure 1.5.

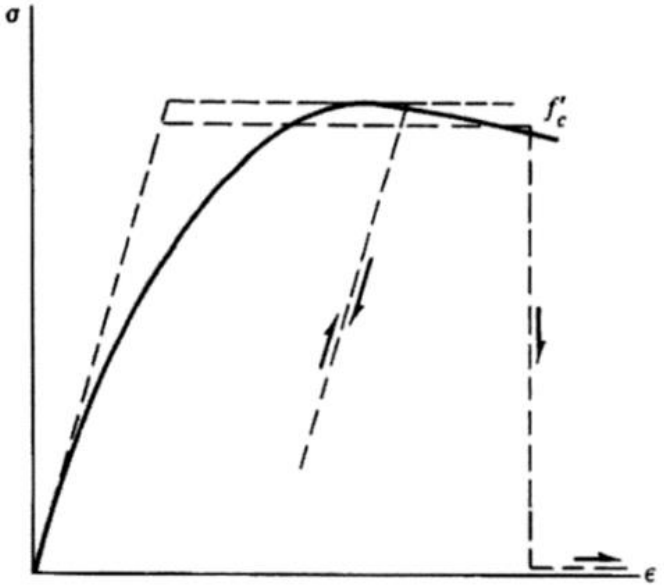


Fig. 1.5: Idealized stress-strain curve for concrete (*perfectly plastic-brittle-fracture model*).

The complete stress-strain relationships for a perfectly plastic-brittle-fracture model are developed in three parts: (1) before yield, (2) during plastic flow, and (3) after fracture. The linear-elastics stress-strain relationships before yield and after fracture have frequently been used in these regimes. Only, the plastic stress-strain relationship during the plastic flow needs to be added here. To achieve this, one must first define the condition of yield and the strain criterion for fracture. With these boundaries determined, the plastic stress-strain relationship in an incremental form can be established.

The failure or fracture criterion described in terms of stress invariants can be taken as the perfectly plastic yield surface. A considerable amount of numerical work has been done using (1) the von Mises criterion, (2) the extended von Mises criterion (or Drucker-Prager criterion), and (3) the Coulomb or modified Coulomb criterion. Except for the von Mises criterion, which is used widely in metal plasticity, all yield criteria developed for concrete incorporate a dependence of yield-point stress on the mean normal stress in addition to the dependence on invariant of the "average" maximum shear stress.

1.3.5 Work-hardening plasticity

One of the latest steps in development of concrete constitutive models is the use of *strain- or work-hardening theory* of elasticity. A yield surface called *loading surface*, which combines both perfect plasticity and strain hardening, is postulated, and an associated flow rule is used for the plastic concrete before fracture. The model is shown schematically in Figure (1.6).

According to this approach, the stresses in a structure under operating conditions are expected to be in the initial yield range such that the concrete behaviour can be characterized as liner elastic and formation of microcracks can be minimized; i.e, one can expect to avoid fatigue. This initial yield surface is the limiting surface for elastic behaviour. Figure (1.7) shows a trace of this initial yield surface in comparison to the failure or fracture surface and the subsequent loading surfaces. When the state of stress lies within the initial yield surface, the material is assumed to be linear and the linear-elastic constitutive equations can be applied.

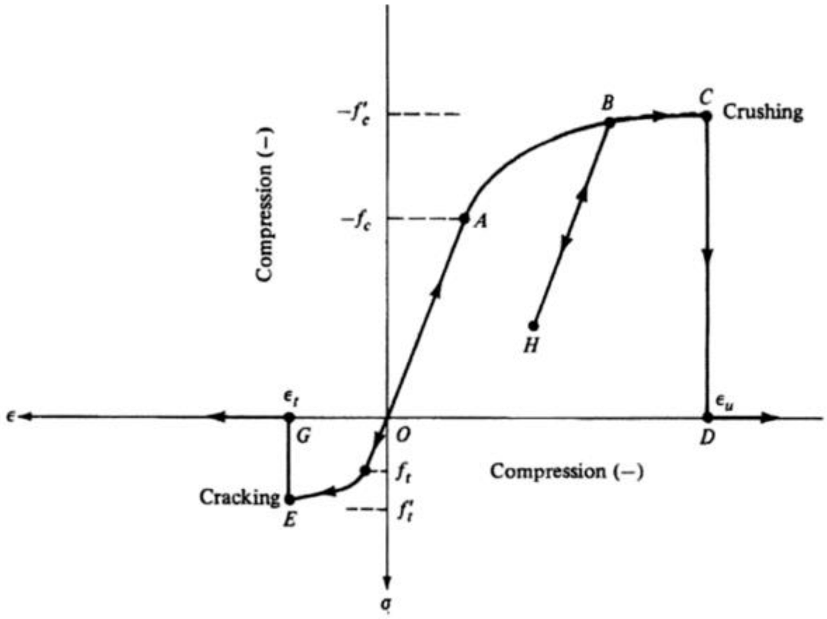


Fig. 1.6: Idealized uniaxial stress-strain curve for concrete.

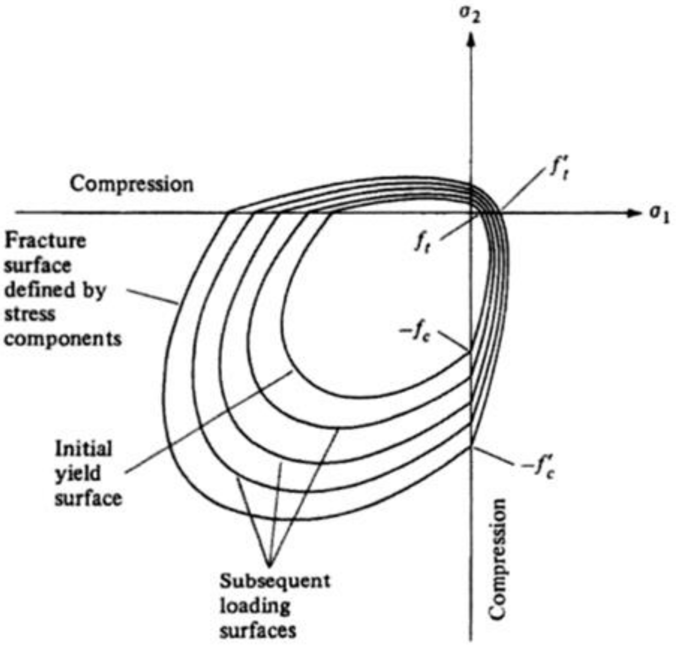


Fig. 1.7: Loading surfaces of concrete in the biaxial stress plane for a work-hardening-plasticity model.

When the material is stressed beyond the initial-yield or elastic-limit surface, a subsequent new yield surface called the *loading surface* is developed. The new surface replaces the initial yield surface. If the material in unloaded from, and reloaded within, this subsequent loading surface, no additional irrecoverable deformation will occur until this new surface is reached. If straining is continued beyond this surface, further discontinuity and additional irrecoverable deformation results. Beyond the elastic limit, the normality condition or the so-called associated flow rule is assumed to govern the post-yielding stress-strain relations for concrete. Once the loading surface has been defined, constitutive equations based on the concept of flow rule can be derived.

A crush and crack surface, called a strain-fracture surface, in terms of strain must also be postulated to define the complete collapse for the yielded concrete. Once the stress-fracture surface has been reached, the concrete begins to flow under constant stress. Finally, the concrete is assumed to crack or crush, depending upon the nature of the stress states, when the strain-fracture surface is reached. The stress-strain relationship developed previously for a fracture concrete can now be applied.

An important characteristic of concrete that cannot be adequately treated by the classical work- or strain-hardening theory of plasticity is the full stress-strain behaviour beyond the peak stress called *stress-softening* effects. Since the question of strain softening is highly controversial, a simple engineering approach to the material-degradation problem is to assume the collapse of the yield surface after the maximum value has been reached (*remember that concrete is a highly non-homogenous material and that locally contained instabilities may contribute to the reduction of the overall strength, however, the extensive microcracking necessary for increasing deformations certainly absorbs much energy and may thus stabilize the material*) . The collapsed failure surface corresponding to the Mohr-Coulomb surface describes a frictional material behaviour without cohesion.

1.3.6 Endochronic plasticity

In the preceding sections, classical incremental-flow theory of plasticity ahs been used as the basis for develping constitutieve equations for conrete. Fundamentally, the incremental-flow theory assumes the existence of a yield criterion coupled with a hardening rule to define the subsequent yild surfaces. Thus, the elastoplastic model can be viewed as a discontinuous-material model which separates the material responses into several stages. Loading, unloading, and relaoding processes can then be considered as differnet steps in the sturctural analysis and desing, and thus the analytical proscedures are considerabely simplified. However, real mateial behaviour is usually continuous and include many complicated cross effects. Furthermore, with the advent of numerical analysi, the necessity of separating compliated problems into several stage is questionable, and frequantly such discontinuous models, instead of simplifying the problem, are the source of numerical difficulities and inefficiencies. Valanis (1971) showed that by employing a pseudo time scale, the *intrinsic time*, a constitutive equation in integral of differential form can successfully be used to describe metal behaviour, including strain hardening, unloading and reloading, cross hardening, and continued cyclic straining. The theory does not require specific definitions of yieldin and hardening. Using Valanis' concept, Bazant and his coworkers (1976-1978) have extended the theory to describe the behaviour of rock, sand, and plain concrete and reinforced concrete under various conditions. They have shown that the new constitutive formulations can correctly prdict the nonlinear effect, creep behviour, and concrte responses to cyclic loadings. Although some fundamental questions of the theory seem to need further study, it appears to have remarkable potential for practical applications (*needs further research*).

1.4 Finite-Element Modelling of Reinforced Concrete Structures

Modern computational techniques like the *finite-element method* have been used in nonlinear analysis of reinforced concrete structures since (Ngo and Scordelis, 1967). Reinforced concrete has a very complex behaviour, involving such phenomena as inelasticity, cracking, and interactive effects between concrete and reinforcement. The complexities involved in developing an accurate finite-element analytical solution for problems in reinforced concrete can be illustrated by considering the simple beam shown in Figure (1.8), subjected to a simple two-point load.

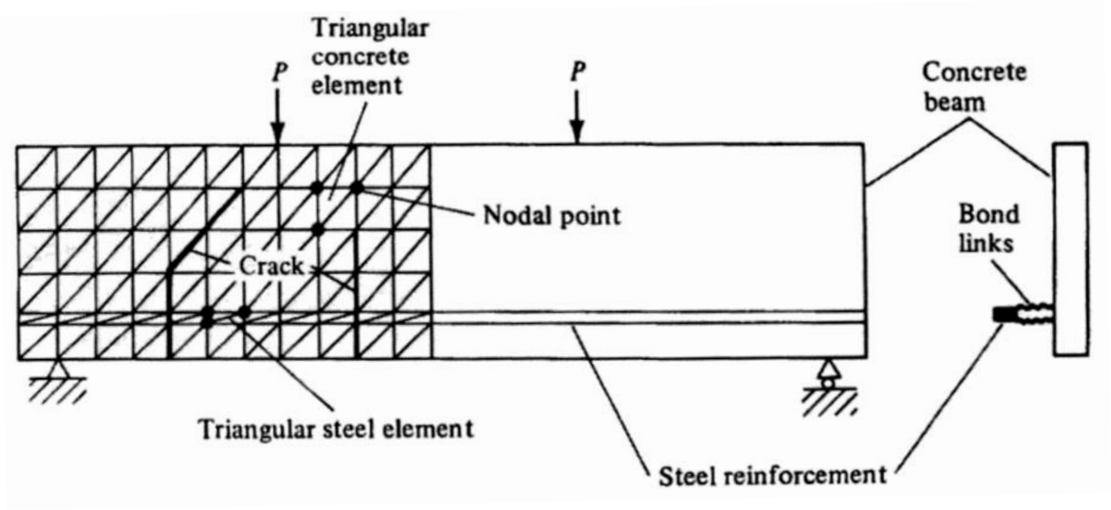


Fig. 1.8: Original analytical model, (*Ngo and Scordelis, 1967*).

The finite-element meshes for concrete and reinforcement are represented by triangular elements, and special bond links are used to connect the steel to the concrete. Under increasing load, the following sequence of events takes place:

1. at low loads the beam acts essentially as an uncracked elastic member,
2. vertical flexural cracks then occur at midspan, resulting in a redistribution of stress and causing increased steel stresses, bond stresses, and some bond slip,
3. under additional load these flexural cracks spread and increase, and if shear and diagonal tension are not critical, the beam eventually fails by yielding of the longitudinal tensile steel reinforcement or by crushing of the concrete in the compressive zone,
4. if shear and diagonal tension are critical (as in Figure 1.8), a much more complex situation evolves due to the formation of a significant diagonal-tension crack,

5. this crack activates resistance to vertical shear by dowel action in the main longitudinal reinforcement, aggregate interlock along the diagonal crack, stresses in vertical stirrups (if any), and resistance in the uncracked concrete above the head of the crack,
6. a sudden increase in the longitudinal steel stress at the base of the diagonal-tension crack also occurs,
7. under increasing load, the diagonal crack propagates toward the loading point, causing an increase in the dowel shear,
8. final failure may occur when the head of the diagonal crack has decreased the uncracked compression block of concrete to a critical point, when shear-compression failure occurs under a combined state of stress. In some beams without stirrups, failure occurs by splitting along the longitudinal reinforcement cause by the heavy dowel shear in the main reinforcement.

Since the first paper by Ngo and Scordelis (1967) on the application of finite-element method to reinforced concrete beam problem, reinforced concrete structures modelled successively as plane-stress, plane-strain, plate-bending, thin-shell, axisymmetric-solid, or three-dimensional solid systems have been analyzed as nonlinear systems using various assumptions for cracking, constitutive relationships, failure criteria, bond, dowel action, and aggregate interlock.

Most finite-element studies consider concrete to act like an elastoplastic solid in compression and like an elastic brittle material in tension. Various elasticity- and plasticity-based constitutive models have been proposed for uncracked concrete. For cracking concrete, two different approaches have been employed for modelling. The most popular procedure is to treat the cracking as *distributed* cracks on the *continuum* level; i.e., the cracks are smeared out in a continuous fashion. An alternative to the continuous model is the introduction of discrete *cracks,* which are traced individually as they progressively alter the topology of the structure, as shown in the original model by Ngo and Scordelis (Figure 1.8). In this approach, the stress concentrations at the crack tip, normally disregarded, should be dealt with according to the concepts of fracture mechanics to predict crack extension (*to extend your knowledge in this area, look for the recent extended finite element approach, xFEM*).

In general, a full bond is assumed between the reinforcement and the concrete components, implying compatible deformation. As a result of this assumption, the material stiffness of the composite element is obtained by superimposing material stiffnesses of the individual material components, concrete and reinforcement. In special cases, differential movements have been modelled, link-type elements simulating bond slip between reinforcement and concrete, as shown in Figure (1.8).

The material properties of steel reinforcement bars are easily established in uniaxial testing. The material properties are normally defined in using classical plasticity formulations. The reinforcement bars are usually incorporated in the computations model using *discrete bar elements* or by introducing *equivalent* layers. The bending rigidity of the bars is normally neglected.

In modelling interaction between concrete and reinforcement, two important mechanisms have been identified. In the first type the reinforcement and the concrete are both subjected to tension, so that large cracks form. The shear forces at the contact surface feed tension stresses into the concrete between cracks. The concrete hangs on the bar and contributes to the overall stiffness of the system. This stiffness effect, often called *tension stiffening*, may be quite significant for concrete beams under normal working loads (Figure 1.8). It can be accounted for in an indirect way by assuming a gradual loss of tension strength in concrete, or by special spring-type material models, or be a more involved model for the interaction.

In the second type of interaction between the reinforcement and the concrete there is a major shear deformation after tension cracking has first occurred. The bars act as dowels under such conditions (Figure 1.3). This dowel effect can be incorporated into a continuum model by using an *equivalent shear stiffness* and shear strength for the cracked concrete. A similar procedure can be applied for the *aggregate interlocking effect* (Figure 1.2).