1. Flexible Pavement..(3 Weeks)

2.1. Analysis of: Stress, Strain and Deflection in Flexible Pavement

2.1.1. One Layer System

2.1.1.1. Point loading

2.1.1.2. Circular Loading

2.1.1.3. Methods of solutions.

2.1.2. Layard Systems

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2. Flexible Pavement

2.1. Analysis of: Stress, Strain and Deflection in Flexible Pavement

A pavement structure is not so easily to accurate structural analysis because the materials forming the flexible pavement layers and soils supporting the pavement are not same, so their exhibiting are not similar and their response under loads are different.

2.1.1. One Layer System

Boussinesq (1885) analysed the stresses in flexible pavement as a single layer due to an applied load based on the assumptions that: the pavement and supporting soils subgrade below form a **homogeneous, isotropic, single elastic layer with the same value of elastic modulus (E).** The first analysis approach represented the load as a point load and then the load was represented as a circular load which is more realistic than the point load.

Note:

- **Isotropic materials are materials whose properties remain the same when tested in different directions. Isotropic materials differ from anisotropic materials, which display varying properties when tested in different directions. Common isotropic materials include glass, plastics, and metals.**
- **A half-space has an infinitely large area and an infinite depth with a top plane on which the loads are applied .**

2.1.1.1. Point loading

The closed-form solution for a point load on an elastic half-space was originally developed by Boussinesq (Fig. 2.1.) as shown in the following forms:

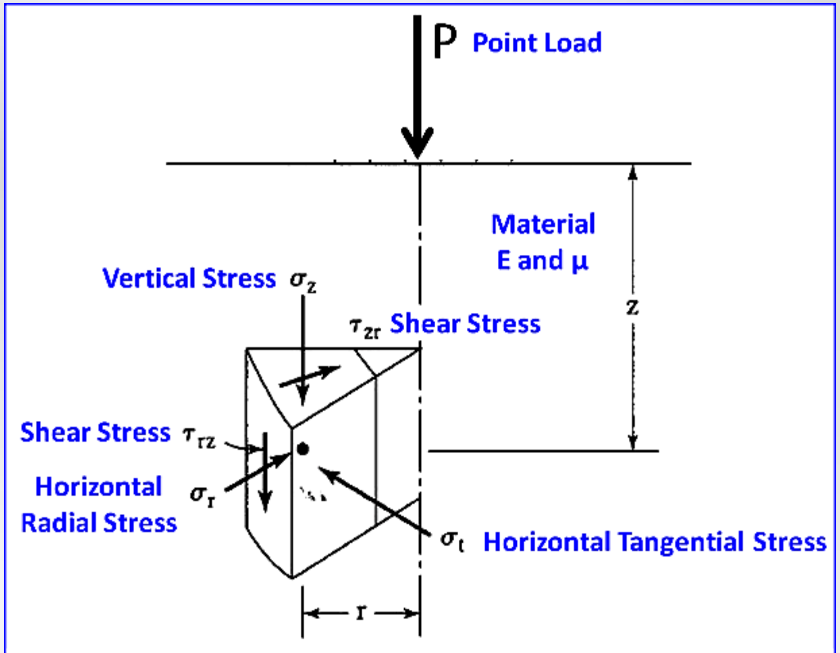


Figure 2.1. Stresses due to point loading

P = Point Load
μ = Poisson's Ratio
σ_z = Vertical normal stress
σ_r = Radial normal stress (Horizontal)
σ_t = Tangential normal stress (Horizontal)
τ_{zr} = Horizontal Shear stress (radial direction)
ϵ_z = Vertical normal strain
ϵ_r = Radial normal strain (Horizontal)
ϵ_t = Tangential normal strain (Horizontal)
γ_{zr} = Horizontal Shear strain (radial direction)
w = Vertical Deflection

$$\sigma_z = \frac{P}{2\pi}\frac{3z^3}{(r^2+z^2)^{5/2}} \quad , \quad \tau_{zr} = \frac{P}{2\pi}\frac{3rz^2}{(r^2+z^2)^{5/2}}$$

$$\sigma_r = \frac{P}{2\pi}\left[\frac{3r^2 z}{(r^2+z^2)^{5/2}} - \frac{1-2\mu}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$$

$$\sigma_t = \frac{P}{2\pi}(1-2\mu)\left[\frac{z}{(r^2+z^2)^{3/2}} - \frac{1}{r^2+z^2+z\sqrt{r^2+z^2}}\right]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_r+\sigma_t)] \quad , \quad \varepsilon_r = \frac{1}{E}[\sigma_r - \mu(\sigma_z+\sigma_t)]$$

$$\varepsilon_t = \frac{1}{E}[\sigma_t - \mu(\sigma_z+\sigma_r)] \quad , \quad \gamma_{zr} = \frac{2\tau_{zr}(1+\mu)}{E} = \frac{\tau_{zr}}{G}$$

$$w = \frac{P}{2\pi E}\left[\frac{(1+\mu)z^2}{(r^2+z^2)^{3/2}} + \frac{2(1-\mu^2)}{\sqrt{(r^2+z^2)}}\right]$$

2.1.1.2. Circular Loading

For pavement analysis, the equivalent circular contact area of a tire on pavement surface is taken. For this purposes a uniformly loaded circular area is considered for calculating the stresses in the soil mass. The equation of vertical stress under point load may be integrated over the circular area as shown in Figure 2.2.

$$\sigma_z = \int_0^{2\pi}\int_0^{a}\left(\frac{3p((rd\theta)dr)}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}\right)$$

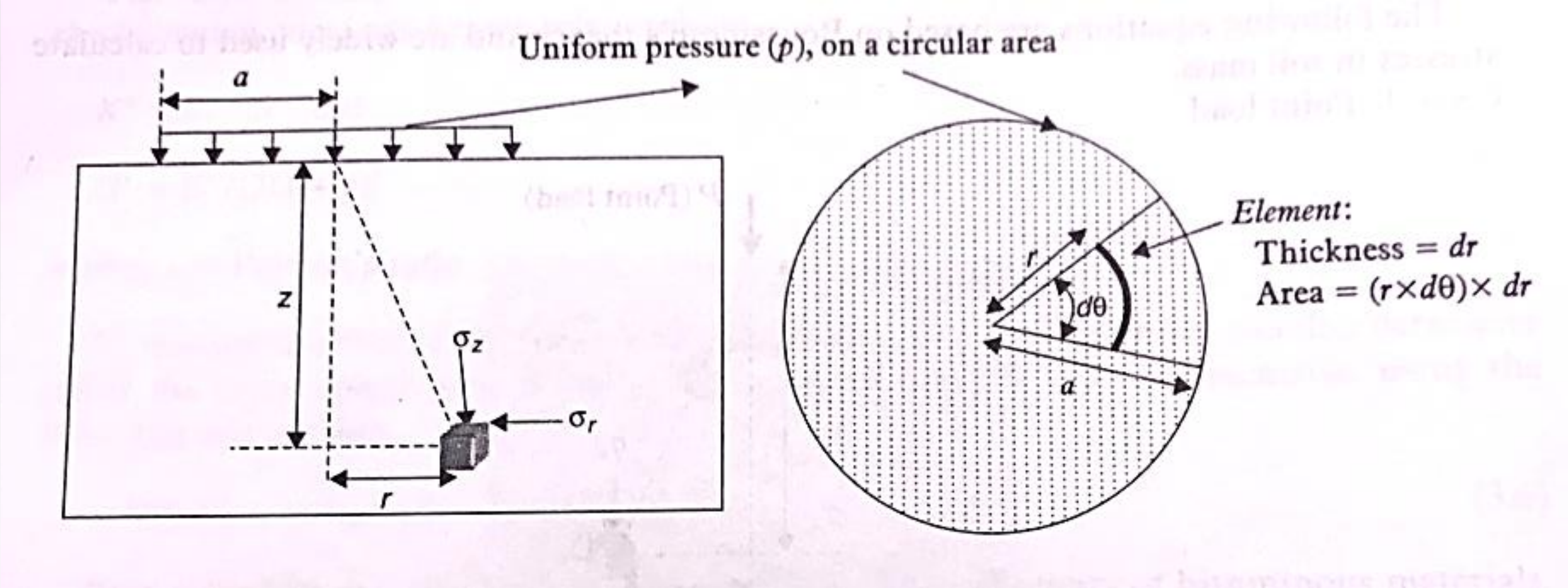


Figure 2.2. Stresses under uniformly circular loading

➢ The response due to a circular load with **(a)** radius and uniform pressure **(q)** on an elastic homogeneous half-space is obtained by integrating the Boussinesq's components due to a concentrated load.

➢ **When the load is applied over a single circular loaded area, the most critical stress, strain, and deflection occur under the center of circular area on the axis of symmetry, where: $\tau_{zr} = 0$ and $\sigma_r = \sigma_t$, so σ_z and σ_r are the principal stresses.** For points on the centerline of the load (i.e., r = 0), these stress components are given by:

$$\varepsilon_z = \frac{(1+\mu)q}{E}\left[(1-2\mu) + \frac{2\mu z}{\sqrt{a^2+z^2}} - \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$

$$\varepsilon_r = \frac{(1+\mu)q}{2E}\left[(1-2\mu) - \frac{2(1-\mu)z}{\sqrt{a^2+z^2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$

and the vertical deflection under the centerline of the load is given by

$$w = \frac{(1+\mu)qa}{E}\left\{\frac{a}{\sqrt{(a^2+z^2)}} + \frac{1-2\mu}{a}\left[\sqrt{a^2+z^2} - z\right]\right\}$$

$$w = \frac{3qa^2}{2E\sqrt{a^2+z^2}} \qquad \text{when } \mu = 0.5$$

On the surface of the half-space (i.e., z = 0)

$$w = qa\left[\frac{2(1-\mu^2)}{E}\right] \qquad \text{when } z = 0$$

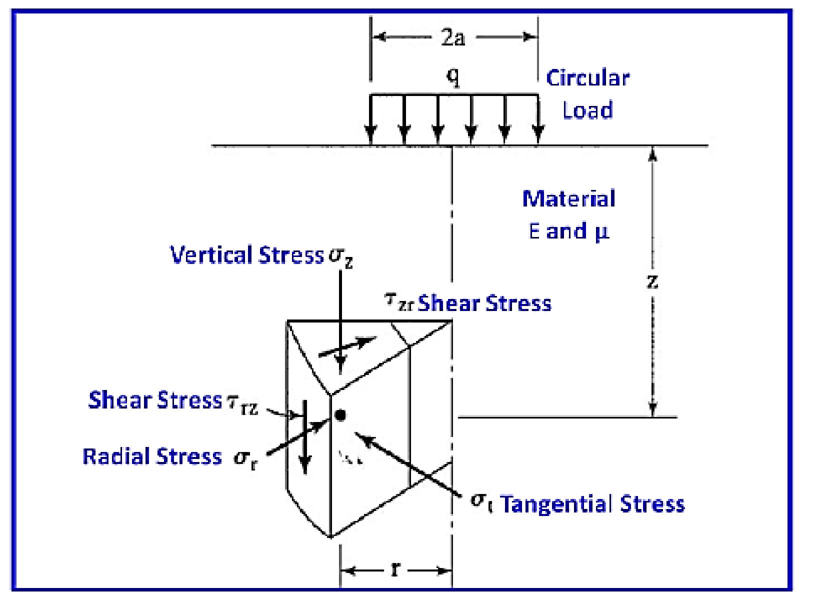


Figure 2.3. Stresses due to circular loading

$$\sigma_z = q\left[1 - \frac{z^3}{(a^2+z^2)^{3/2}}\right], \quad \tau_{zr} = 0$$

$$\sigma_r = \sigma_t = \frac{q}{2}\left[(1+2\mu) - \frac{2(1+\mu)z}{\sqrt{a^2+z^2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$

Flexible and Rigid Plates Loading

Flexible Plate:

The load applied from tire to pavement is similar to a flexible plate with a radius ***(a)*** and a uniform pressure ***(q)***. The deflection beneath the center of the plate can be determined from:

$$w_0 = \frac{2(1-\mu^2)qa}{E}$$

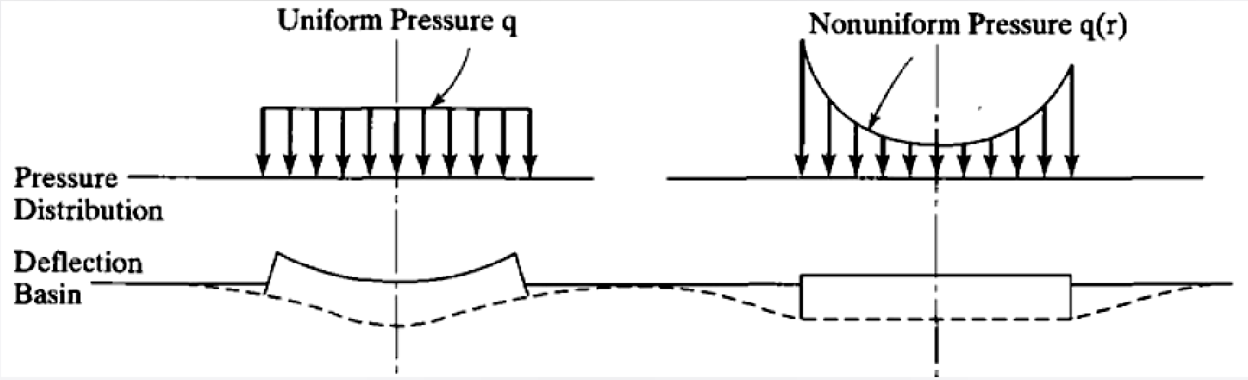
Rigid Plate:

All the above analyses are based on the assumption that the load is applied on a flexible plate, such as a rubber tire . If the load is applied on a rigid plate, such as that used in a plate loading test, the deflection is the same at all points on the plate, but the pressure distribution under the plate is not uniform. The differences between a flexible and a rigid plate are shown in Figure *2.4*.

Flexible plate:
- Uniform Contact Pressure
- Variable Deflection Profile

Rigid Plate plate:
- Non-Uniform Contact Pressure
- Equal Deflection

$$w_0 = \frac{\pi(1-\mu^2)qa}{2E}$$



(a) Flexible plate (b) Rigid plate

Figure *2.4*. Differences between flexible and rigid plates.

❑ A comparison of these two equations indicates that the surface deflection under a rigid plate is only 79% of that under the center of a uniformly distributed load (flexible plate). This is reasonable because the pressure under the rigid plate is smaller near the center of the loaded area but greater near the edge . The pressure near the center has a greater effect on the surface deflection at the center . The same factor, 0 .79, can be applied if the plates are placed on a layer system, as indicated by Yoder and Witczak (1975), as shown in Figure 2.5..

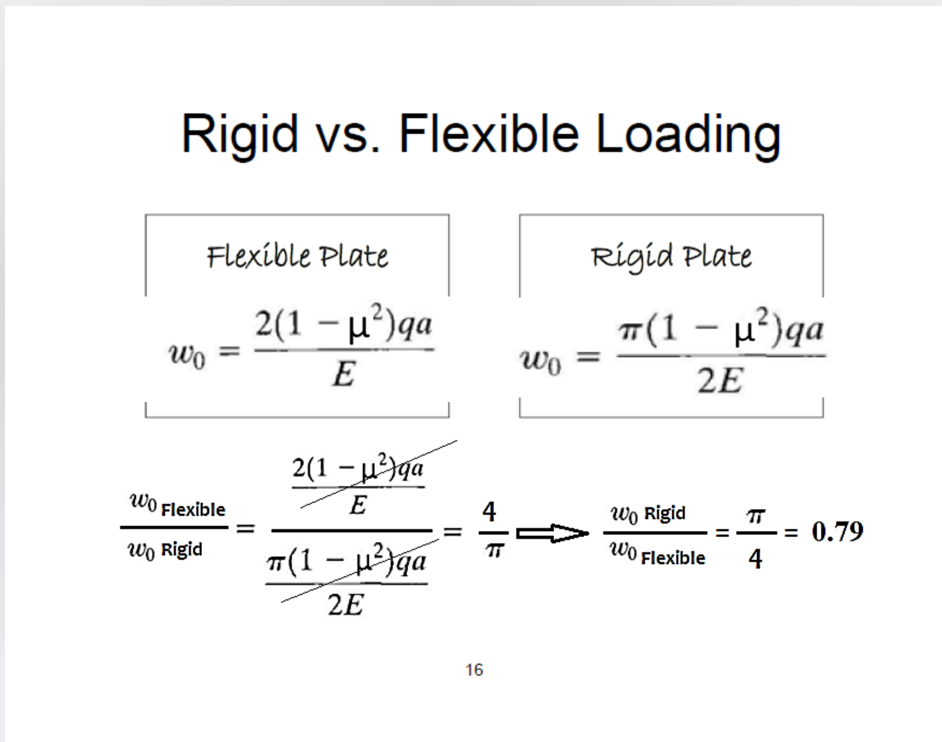


Figure 2.5. Deflection induced by rigid and flexible plate loading.

2.1.1.3. Methods of Solution

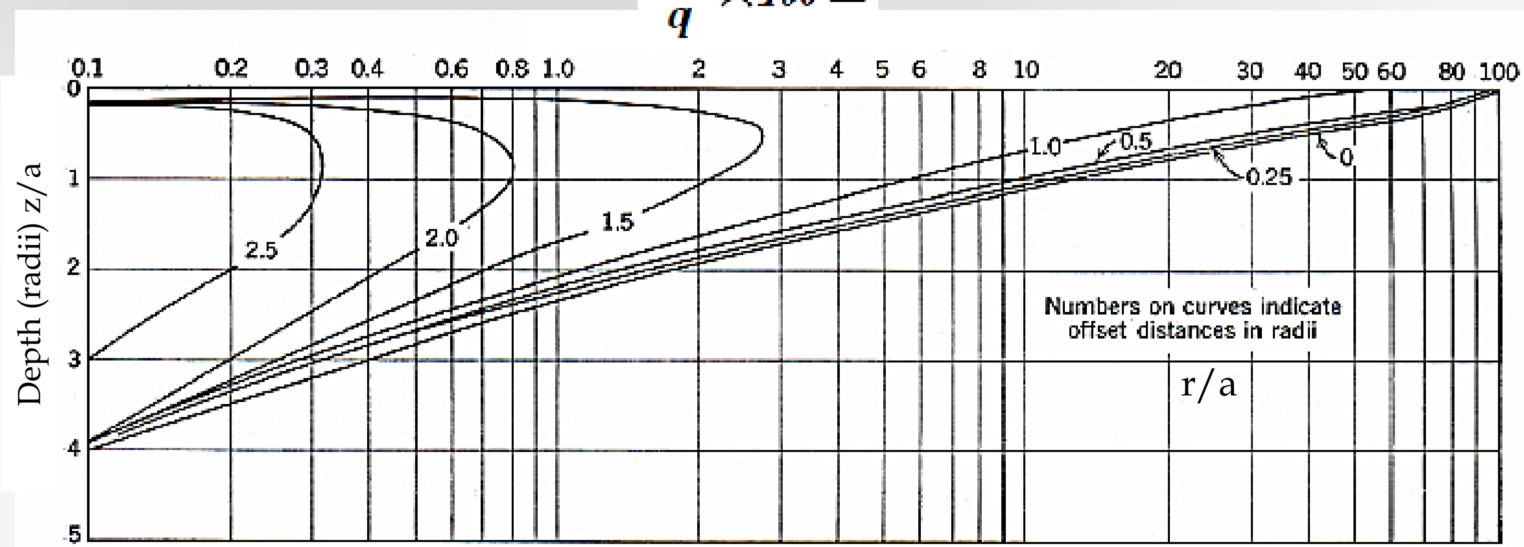
In addition to using the theoretical formulas suggested by Bossinseq's method (circular loading method) , there another two methods as explained in the following articles:

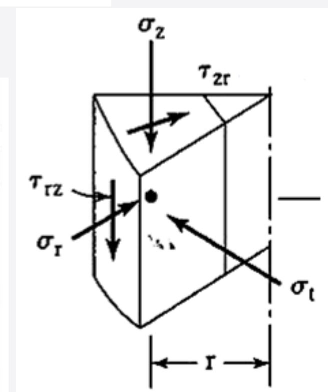
2.1.1.3.1. Foster and Ahlvin Charts (Poisson's ratio is constant = 0.5)

Foster and Ahlvin (1954) presented charts for determining vertical stress $\boldsymbol{\sigma_z}$, tangential stress $\boldsymbol{\sigma_t}$, radial stress $\boldsymbol{\sigma_r}$, shear stress $\boldsymbol{\tau_{zr}}$, and vertical deflection ***w***, as shown in **Figures 2.6** through **2.10**. The load is applied over a circular area with radius ***(a)*** and an load intensity ***(q)***. Because Poisson ratio has relatively small effect on stresses and deflections, Foster and Ahlvin assumed the **Poisson's ratio value 0.5**.

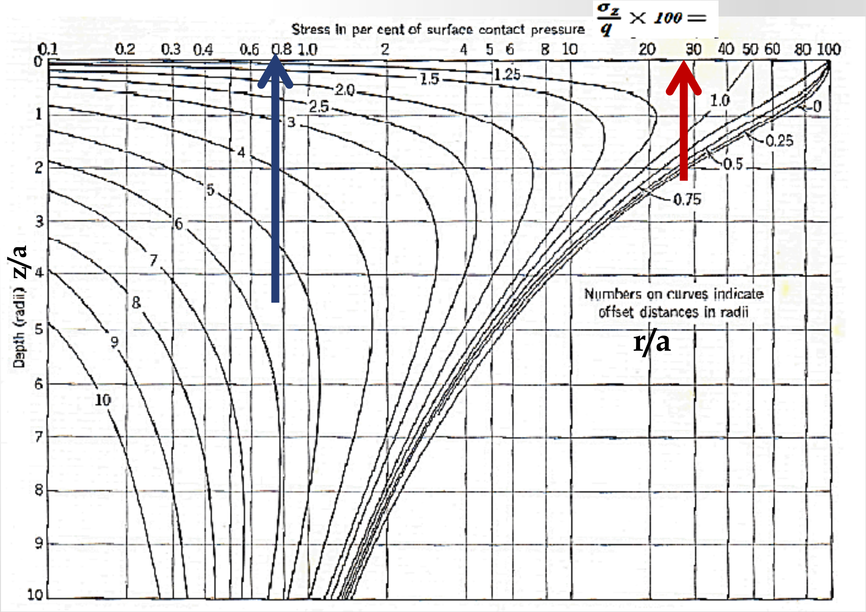
Note : In the charts, $x\text{-}axis \rightarrow \dfrac{stress\ type}{q} \times 100$, $y\text{-}axis \rightarrow \dfrac{z}{a}$ $No.\,on\ the\ Curve \rightarrow \dfrac{r}{a}$

$$\frac{\sigma_t}{q} \times 100 =$$

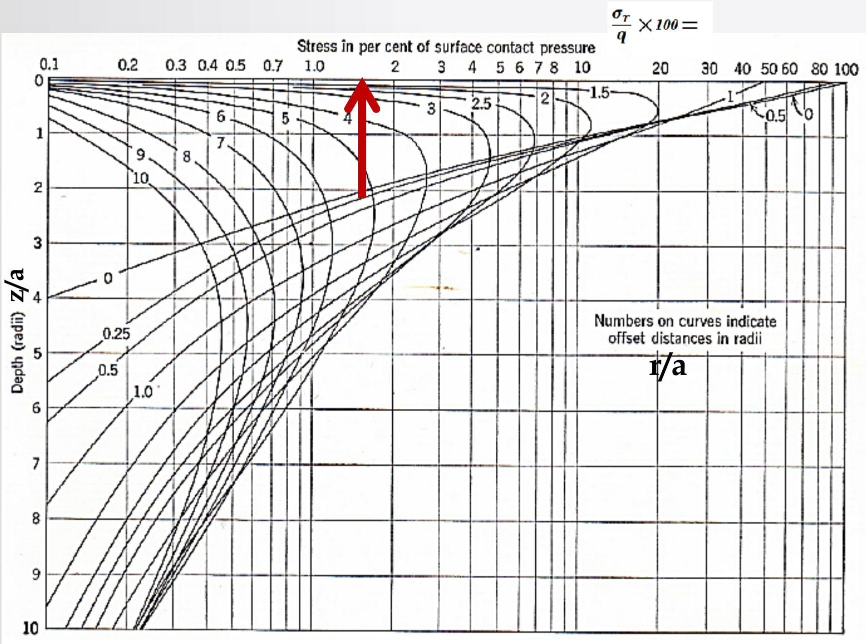




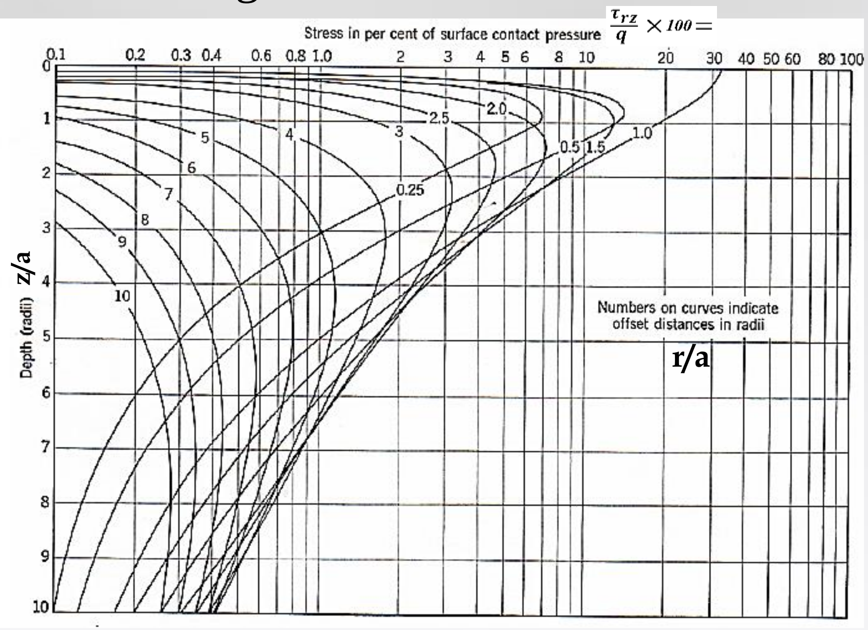
Figures 2.6. Tangential Stresses



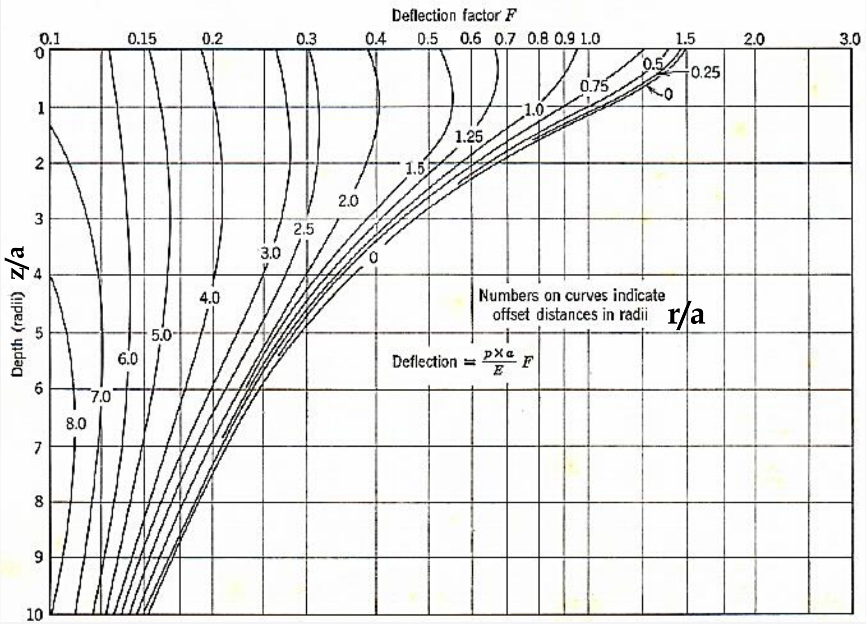
Figures 2.7. Vertical Stresses



Figures 2.8. Radial Stresses



Figures 2.9. Shear Stresses



Figures 2.10. Vertical Deflection *(w)*

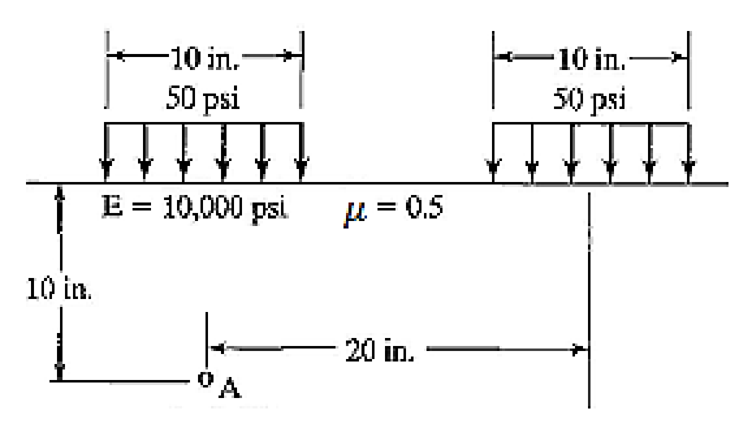
After the stresses are obtained from the charts, the strains can be obtained from

$$\epsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_r + \sigma_t)]$$

$$\epsilon_r = \frac{1}{E}[\sigma_r - \mu(\sigma_t + \sigma_z)]$$

$$\epsilon_t = \frac{1}{E}[\sigma_t - \mu(\sigma_z + \sigma_r)]$$

Example 1: Figure (2.11) shows a homogeneous half-space subjected to two circular loads, each 10 in.(254 mm) in diameter and spaced at 20 in.(508 mm) on centers. The pressure on the circular area is 50 psi (345 kPa (1 psi=6.9 kPa). The half-space has elastic modulus 10,000 psi (69 MPa) and **Poisson's ratio 0.5**. Determine the vertical stress, strain, and deflection at point A, which is located 10 in.(254 mm) below the center of one circle.



Figures 2.11. Example 1.

Solution :

From Figures 2.7, 2.8, and 2.10, the stresses at point ***A:***

Due to the **left load** with r/a = 0 and z/a = 10/5 = 2 are:

$\boldsymbol{\sigma_z}$ = 0.28 × 50 = 14.0 psi (96 .6 kPa), and

$\boldsymbol{\sigma_r}$ = $\boldsymbol{\sigma_t}$ = 0.016 × 50 = 0.8 psi (5 .5 kPa) ;

Due to **the right load** with *r/a* = 20/5 = 4

and z/a = 2 are:

$\boldsymbol{\sigma_z}$ = 0.0076 × 50 = 0.38 psi (2 .6 kPa),

$\boldsymbol{\sigma_r}$=0.026 × 50 =1.3 psi (9.0 kPa), and

$\boldsymbol{\sigma_t}$ = 0. (Out of the right load's range).

By superposition:

$\boldsymbol{\sigma_z}$ = 14.0 + 0 .38 =14 .38 psi (99.2 kPa),

$\boldsymbol{\sigma_r}$= 0.8+1.3 = 2 .10 psi (14 .5 kPa) , and

$\boldsymbol{\sigma_t}$ = 0.8 psi (5.5 kPa).

Strain:

$\boldsymbol{\varepsilon_z}$= [14 .38 - 0 .5(2.10+ 0.8)/10,000 = 0.00129.

From Figure 2.10, the deflection factor at point **A** due to the left load is 0 .68 and, due to the right load is 0.21 .

The total deflection ***w*** = (0 .68 + 0 .21) × 50 × 5/10,000 = 0 .022 in . (0.56 mm) .

The final answer is $\boldsymbol{\sigma_z}$ = 14.38 psi (99 .2 kPa), $\boldsymbol{\varepsilon_z}$ = 0.00129, and ***w*** = 0.022 in . (0.56 mm) $\boldsymbol{\tau_{zr}}$,***w***,

2.1.1.3.2. Ahlvin and Ulery Tables (Any value of Poisson's ratio)

Tables for One-layer Solutions are suggested by Ahlvin and Ulery (1962), to find stresses, strains, and deflection in one layer system for any value of Poisson's ratio, as shown in Figure 2.12 and Tables 2.1. and 2.2.

Figure 2.12. Summary of One-Layer Elastic Equations[a] (after Ahlvin and Ulery)

Parameter	General Case	Special Case ($\mu = 0.5$)
Vertical stress	$\sigma_z = p[A + B]$	(same)
Radial horizontal stress	$\sigma_r = p[2\mu A + C + (1 - 2\mu)F]$	$\sigma_r = p[A + C]$
Tangential horizontal stress	$\sigma_t = p[2\mu A - D + (1 - 2\mu)E]$	$\sigma_t = p[A - D]$
Vertical radial shear stress	$\tau_{rz} = \tau_{zr} = pG$	(same)
Vertical strain	$\epsilon_z = \dfrac{p(1 + \mu)}{E_1} [(1 - 2\mu)A + B]$	$\epsilon_z \quad \dfrac{1.5p}{E_1} B$
Radial horizontal strain	$\epsilon_r = \dfrac{p(1 + \mu)}{E_1} [(1 - 2\mu)F + C]$	$\epsilon_r = \dfrac{1.5p}{E_1} C$
Tangential horizontal strain	$\epsilon_t = \dfrac{p(1 + \mu)}{E_1} [(1 - 2\mu)E - D)]$	$\epsilon_t = -\dfrac{1.5p}{E_1} D$
Vertical deflection	$\Delta_z = \dfrac{p(1 + \mu)a}{E_1} \left[\dfrac{z}{a} A + (1 - \mu)H\right]$	$\Delta_z = \dfrac{1.5pa}{E_1} \left(\dfrac{z}{a} A + \dfrac{H}{2}\right)$
Bulk stress	$\theta = \sigma_z + \sigma_r + \sigma_t$	
Bulk strain	$\epsilon_\theta = \epsilon_z + \epsilon_r + \epsilon_t$	
Vertical tangential shear stress	$\tau_{zt} = \tau_{tz} = 0 \therefore [\sigma_t(\epsilon_t)$ is principal stress (strain)$]$	
Principal stresses	$\sigma_{1,2,3} = \dfrac{(\sigma_z + \sigma_r) \pm \sqrt{(\sigma_z - \sigma_r)^2 + (2\tau_{rz})^2}}{2}$	
Maximum shear stress	$\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2}$	

Table 2.1. Function A.

Function A

Depth (z) in Radii	Offset (r) in Radii: 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	1.0	1.0	1.0	1.0	1.0	.5	0	0	0	0	0	0	0	0	0	0	0
0.1	.90050	.89748	.88679	.86126	.78797	.43015	.09645	.02787	.00856	.00211	.00084	.00042					
0.2	.80388	.79824	.77884	.73483	.63014	.38269	.15433	.05251	.01680	.00419	.00167	.00083	.00048	.00020			
0.3	.71265	.70518	.68316	.62690	.52081	.34375	.17964	.07199	.02440	.00622	.00250						
0.4	.62861	.62015	.59241	.53767	.44329	.31048	.18709	.08593	.03118								
0.5	.55279	.54403	.51622	.46448	.38390	.28156	.18556	.09499	.03701	.01013	.00407	.00209	.00118	.00053	.00025	.00014	.00009
0.6	.48550	.47691	.45078	.40427	.33676	.25588	.17952	.10010									
0.7	.42654	.41874	.39491	.35428	.29833	.21727	.17124	.10228	.04558								
0.8	.37531	.36832	.34729	.31243	.26581	.21297	.16206	.10236									
0.9	.33104	.32492	.30669	.27707	.23832	.19488	.15253	.10094									
1	.29289	.28763	.27005	.24697	.21468	.17868	.14329	.09849	.05185	.01742	.00761	.00393	.00226	.00097	.00050	.00029	.00018
1.2	.23178	.22795	.21662	.19890	.17626	.15101	.12570	.09192	.05260	.01935	.00871	.00459	.00269	.00115			
1.5	.16795	.16552	.15877	.14804	.13436	.11892	.10296	.08048	.05116	.02142	.01013	.00548	.00325	.00141	.00073	.00043	.00027
2	.10557	.10453	.10140	.09647	.09011	.08269	.07471	.06275	.04496	.02221	.01160	.00659	.00399	.00180	.00094	.00056	.00036
2.5	.07152	.07098	.06947	.06698	.06373	.05974	.05555	.04880	.03787	.02143	.01221	.00732	.00463	.00214	.00115	.00068	.00043
3	.05132	.05101	.05022	.04886	.04707	.04487	.04241	.03839	.03150	.01980	.01220	.00770	.00505	.00242	.00132	.00079	.00051
4	.02986	.02976	.02907	.02802	.02832	.02749	.02651	.02490	.02193	.01592	.01109	.00768	.00536	.00282	.00160	.00099	.00065
5	.01942	.01938				.01835			.01573	.01249	.00949	.00708	.00527	.00298	.00179	.00113	.00075
6	.01361					.01307			.01168	.00983	.00795	.00628	.00492	.00299	.00188	.00124	.00084
7	.01005					.00976			.00894	.00784	.00661	.00548	.00445	.00291	.00193	.00130	.00091
8	.00772					.00755			.00703	.00635	.00554	.00472	.00398	.00276	.00189	.00134	.00094
9	.00612					.00600			.00566	.00520	.00466	.00409	.00353	.00256	.00184	.00133	.00096
10								.00477	.00465	.00438	.00397	.00352	.00326	.00241			

Table 2.2. Function B.

Function B

Depth (z) in Radii	Offset (r) in Radii																
	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2	3	4	5	6	8	10	12	14
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1	.09852	.10140	.11138	.13424	.18796	.05388	−.07899	−.02672	−.00845	−.00210	−.00084	−.00042					
0.2	.18857	.19306	.20772	.23524	.25983	.08513	−.07759	−.04448	−.01593	−.00412	−.00166	−.00083	−.00024	−.00010			
0.3	.28362	.26787	.28018	.29483	.27257	.10757	−.04316	−.04999	−.02166	−.00599	−.00245						
0.4	.32016	.32259	.32748	.32273	.26925	.12404	−.00766	−.04535	−.02522								
0.5	.35777	.35752	.35323	.33106	.26236	.13591	.02165	−.03455	−.02651	−.00991	−.00388	−.00199	−.00116	−.00049	−.00025	−.00014	−.00009
0.6	.37831	.37531	.36308	.32822	.25411	.14440	.04457	−.02101									
0.7	.38487	.37962	.36072	.31929	.24638	.14986	.06209	−.00702	−.02329								
0.8	.38091	.37408	.35133	.30699	.23779	.15292	.07530	.00614									
0.9	.36962	.36275	.33734	.29299	.22891	.15404	.08507	.01795									
1	.35355	.34553	.32075	.27819	.21978	.15355	.09210	.02814	−.01005	−.01115	−.00608	−.00344	−.00210	−.00092	−.00048	−.00028	−.00018
1.2	.31485	.30730	.28481	.24836	.20113	.14915	.10002	.04378	.00023	−.00995	−.00632	−.00378	−.00236	−.00107			
1.5	.25602	.25025	.23338	.20694	.17368	.13732	.10193	.05745	.01385	−.00669	−.00600	−.00401	−.00265	−.00126	−.00068	−.00040	−.00026
2	.17889	.18144	.16644	.15198	.13375	.11331	.09254	.06371	.02836	.00028	−.00410	−.00371	−.00278	−.00148	−.00084	−.00050	−.00033
2.5	.12807	.12633	.12126	.11327	.10298	.09130	.07869	.06022	.03429	.00661	−.00130	−.00271	−.00250	−.00156	−.00094	−.00059	−.00039
3	.09487	.09394	.09099	.08635	.08033	.07325	.06551	.05354	.03511	.01112	.00157	−.00134	−.00192	−.00151	−.00099	−.00065	−.00046
4	.05707	.05666	.05562	.05383	.05145	.04773	.04532	.03995	.03066	.01515	.00595	.00155	−.00029	−.00109	−.00094	−.00068	−.00050
5	.03772	.03760				.03384			.02474	.01522	.00810	.00371	.00132	−.00043	−.00070	−.00068	−.00049
6	.02666					.02468			.01968	.01380	.00867	.00496	.00254	.00028	−.00037	−.00047	−.00045
7	.01980					.01868			.01577	.01204	.00842	.00547	.00332	.00093	−.00002	−.00029	.00037
8	.01526					.01459			.01279	.01034	.00779	.00554	.00372	.00141	.00035	−.00008	−.00025
9	.01212					.01170			.01054	.00888	.00705	.00533	.00386	.00178	.00066	.00012	−.00012
10								.00924	.00879	.00764	.00631	.00501	.00382	.00199			

Example 2: Figure (2.13) shows a homogeneous half-space subjected to two circular loads, each 12 in. in diameter and spaced at 21 in. on centers. The pressure on the circular area is 100 psi. The half-space has elastic modulus 9600 psi and Poisson's ratio 0.35. Determine the vertical stress, strain, and deflection at point A, which is located as shown in figure.

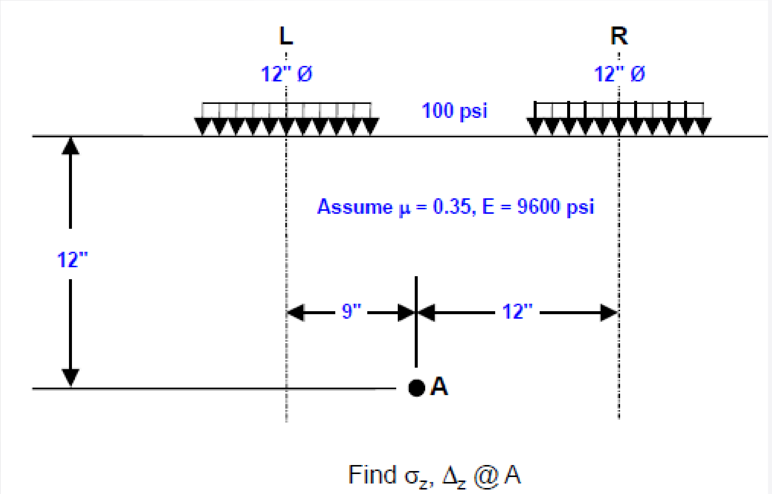


Figure 2.13. Example 2.

Solution:

For load (L): a= 6, z= 12, r= 9 → z/a = 2, r/a = 1.5 .
From table A = 0.06275, B =0.06371, C = -0.00782, D =0.05589,

H.W…. Using Foster and Ahlvin Charts

Equivalent Single Wheel load (ESWL)

- The study of ESWL for dual wheels was first initiated during World War II when the B-29 bombers were introduced into combat missions. Because the design criteria for flexible airport pavements then available were based on single-wheel loads, the advent of these dual-wheel planes required the development of new criteria for this type of loading.

- **Several theoretical studies have been develop for converting dual wheels to equivalent single wheel such as:**

1. **Criterion based on that the single wheel has the same contact radius of the dual wheels .**
   - ✓ **Equal Vertical Stress Criterion**
   - ✓ **Equal Vertical Deflection Criterion**
   - ✓ **Equal Tensile Strain Criterion**

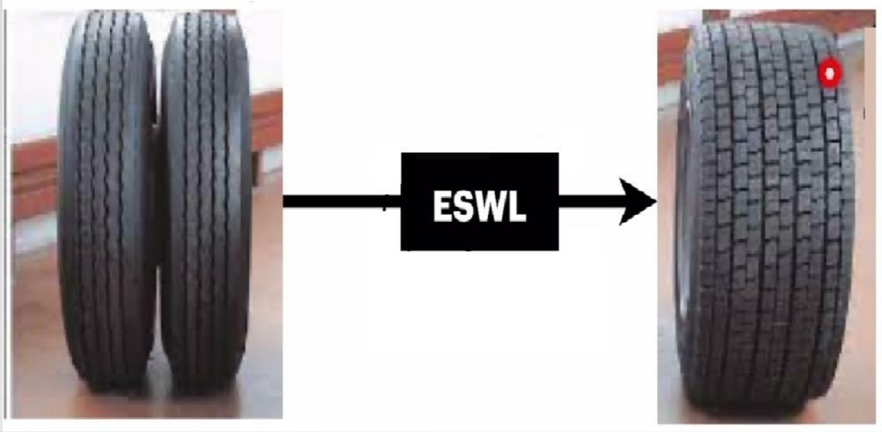


Figure 2.14. Dual wheel vs single wheel loads.

2. **Criterion Based on Equal Contact Pressure with different contact radius.**

3. **Criterion Based on Equivalent Contact Radius OR equivalent single-axle radius (ESAR) .**

Equivalent Single Wheel load (ESWL)

Based on **Equal Vertical Stress Criterion,** From Figure (2.15), the total load of the dual tire assembly is $2P_d$, with S_d being the center to center spacing and ***d*** being the clear distance between tire edges (***d*=S$_d$ – 2*a*$_c$**). It is assumed that for the pavement thickness (***t***) less than or equal to ***d/2 (t ≤ d/2)***, no stress overlap occurs. Thus, the stress depths is due to that of only one wheel of the dual (***P$_d$***). Likewise, at depth of approximately ***2S$_d$***, the effect of stress overlap is such that it is equivalent to the stress caused by the total load of the dual tire assembly (2***P$_d$***). For intermediate depth between ***d/2*** and 2***S$_d$***, the wheel load acting is linear when plotted on a ***log load versus log thickness diagram*** as shown in Figure (2.16). This relationship can be used to find the ESWL for the diagram.

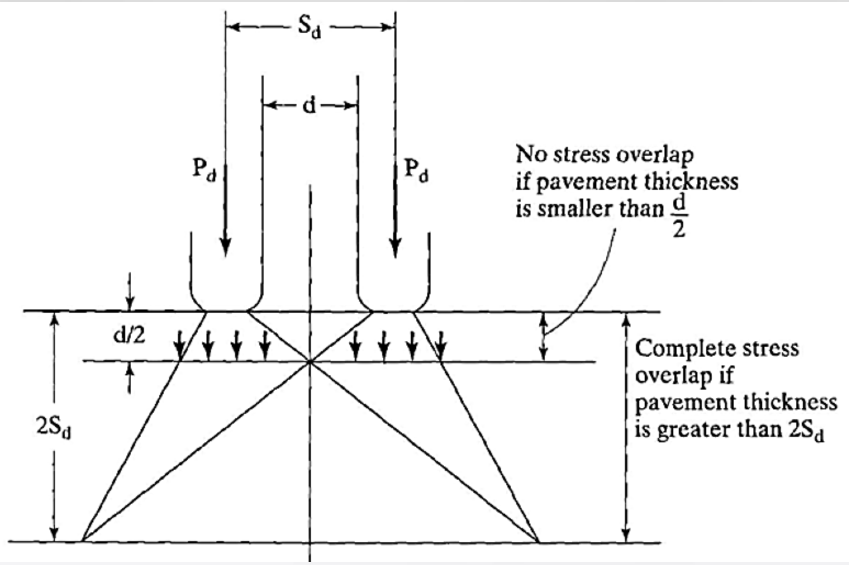


Figure 2.15. Influence of multiple wheels on stresses

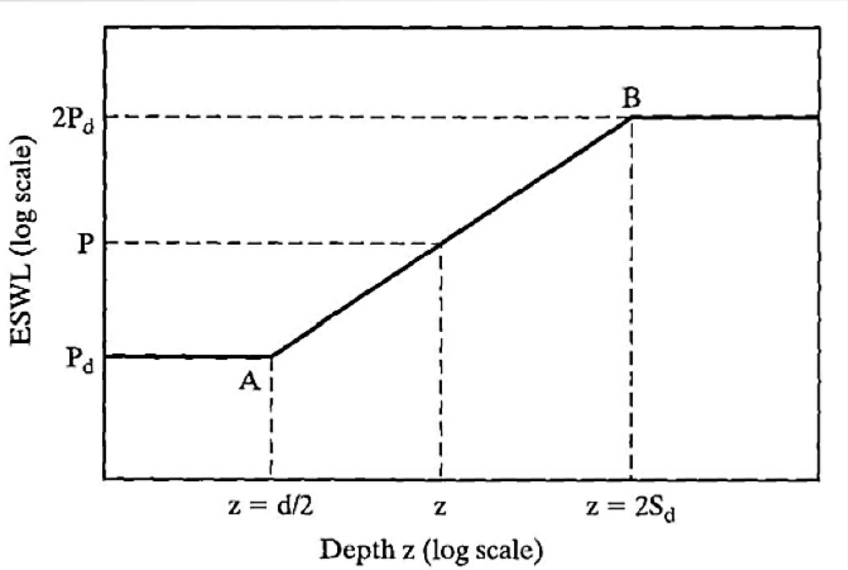
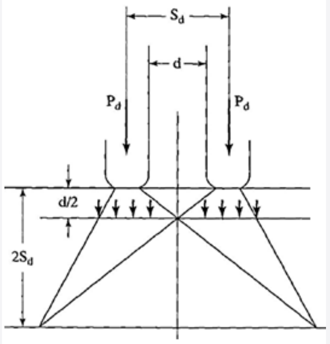


Figure 2.16. Method of determine ESWL for any dual wheel loads.

$$\log(\text{ESWL}) = \log P_d + \frac{0.301 \log(2z/d)}{\log(4S_d/d)}$$



Example 3: Find ESWL at depths of 5 cm, 20 cm and 40cm for a dual wheel carrying 2044 kN each. The center to center tire spacing is 20 cm and distance between the walls of the two tyres is 10 cm.

Solution

At depth z = 40cm, which is twice the tire spacing ($2S_d$), ESWL = $\boldsymbol{2P_d}$= 2 × 2044 = 4088 kN.

For depth, z = 5cm, which is half the distance between walls of the tire ($\boldsymbol{d/2}$),

ESWL = P = 2044 kN.

For z=20 cm, use the linear relationship: log (ESWL) = 3.511.

Therefore, ESWL = antilog(3.511)= 3244.49 kN

2.1.2. Layard Systems

Flexible pavements are layered systems with better materials on top and cannot be represented by a homogeneous mass. These layers are subjected to applied stress which is uniformly distributed over a circular area (radius ***a***) as shown in Figure (2.17). For this system, the following basic assumptions to be satisfied are :

1. Each layer is: homogeneous, isotropic, linearly elastic , and with an elastic modulus $\mathbf{E_i}$ and a Poisson ratio $\boldsymbol{\mu_i}$ **(where i for each layer).**
2. The material is weightless and infinite in the horizontal direction.
3. Each layer has a finite thickness **h**, except that for the lowest layer (subgrade) which has an infinite in thickness .
4. A uniform pressure ***q*** is applied on the surface over a circular area of radius ***a*** .
5. Continuity conditions are satisfied at the layer interfaces, as indicated by the same vertical stress, shear stress, vertical displacement, and radial displacement .

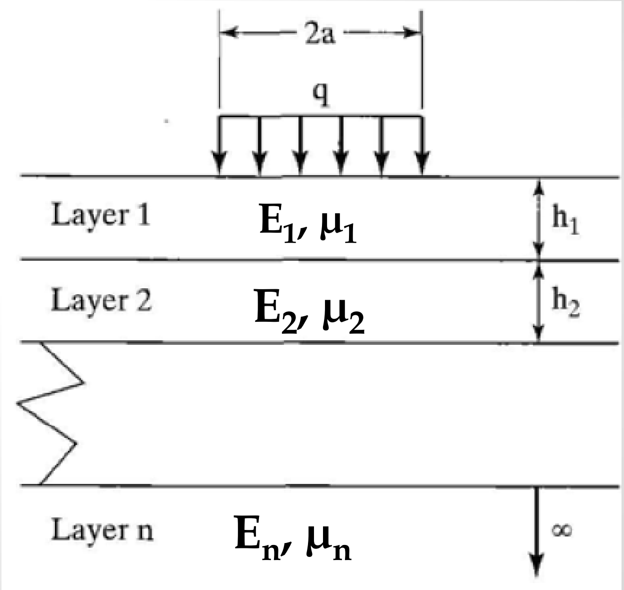


Figure 2.17. An n-layer system.

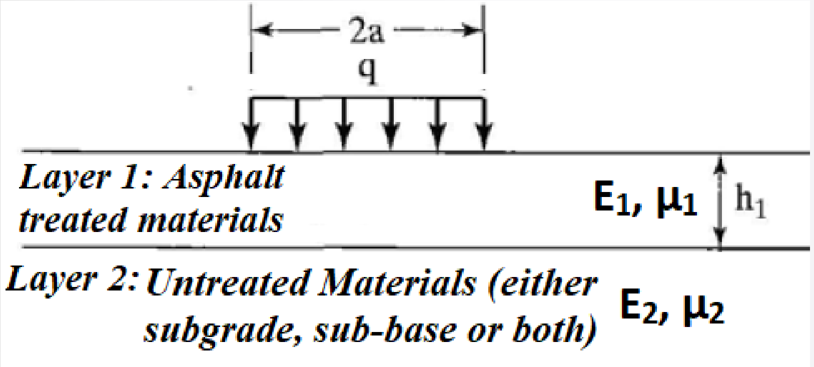


Figure 2.18. A two layers system

2.1.2.1.Two-Layer Systems

The two-layer system is a composed of: hot mix asphalt (HMA) layer which is consisted of surface, binder, and stabilized layers which are treated as a first layer with E_1 and the second layer consists of untreated layer (granular material such as base, sub-base, and subgrade) with E_2, as shown in Figure (2.18). (**Note: $E_1 > E_2$**)

Vertical Stress

- The vertical stress on the top of subgrade is an important factor in pavement design. The function of a pavement is to reduce the vertical stress on the subgrade so that detrimental pavement deformations will not occur. The allowable vertical stress on a given subgrade depends on the strength or modulus of the subgrade.

• The stresses in a two-layer system depend on the modulus ratio $\mathbf{E_1/E_2}$ and the thickness–radius ratio **h/a** . Figure 2.19 shows the effect of a pavement layer on the distribution of vertical stresses under the center of a circular loaded area . **The chart is applicable to the case when the thickness h_1 of the top layer is equal to the radius of contact area, or $h_1/a = 1$ and μ is assumed to be 0.5 for both layers**. It can be seen that the vertical stresses decrease significantly with the increase in modulus ratio. For example: at the pavement–subgrade interface (i.e. contact surface between layer 1 and 2), the vertical stress is about **68% of the applied pressure if E_1/E_2=1**, and when **E_1/E_2=100** the vertical stress distribution reduces to about **8%** of the applied pressure.

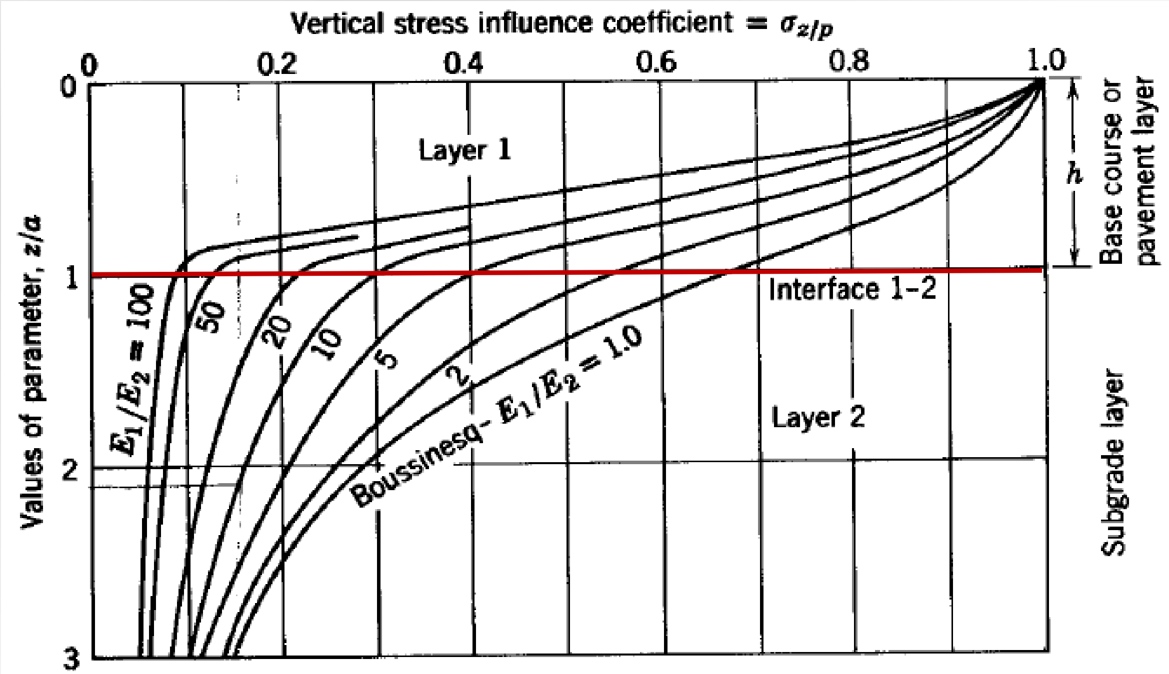


Figure 2.19. Vertical stress distribution in a two layers system.

Figure 2.20 shows the effect of pavement thickness and modulus ratio on the vertical stress $\boldsymbol{\sigma_c}$ at the pavement–subgrade interface under the center of a circular loaded area. For a given applied pressure **q**, the vertical stress increases with the increase in contact radius and decreases with the increase in thickness.

Example 4 : A circular load having radius 6 in. (and uniform pressure 80 psi (552 kPa) is applied on a two-layer system, as shown in Figure 2.21 .The subgrade has an elastic modulus 5000 psi (35 MPa) and can support a maximum vertical stress ($\boldsymbol{\sigma_c}$) of 8 psi. If the HMA has an elastic modulus 500,000 psi, what is the required thickness of a full-depth pavement? If a thin surface treatment is applied(instead of HMA) on a granular base with an elastic modulus 25,000 psi what is the thickness of base course required ?

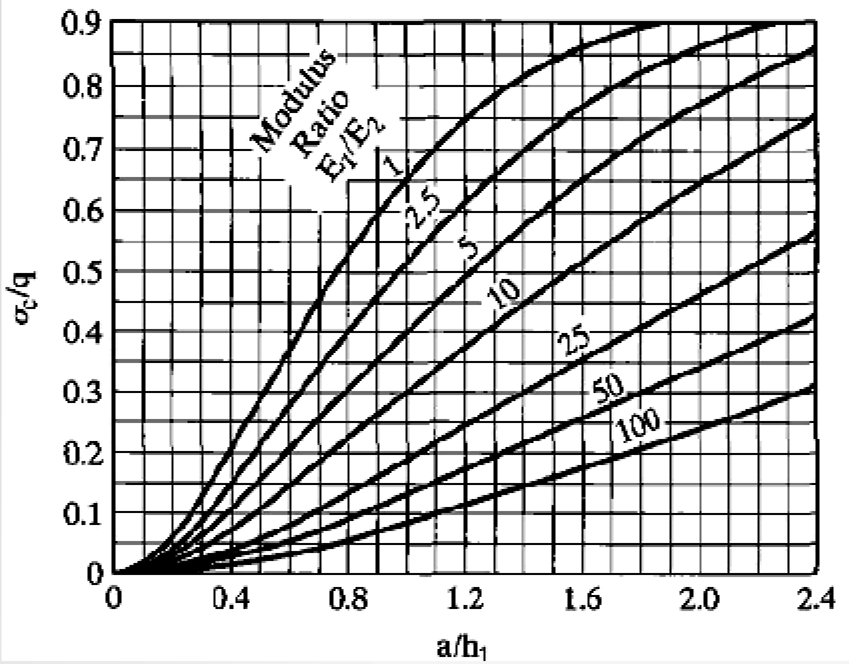


Figure 2.20. Vertical interface stresses for two-layer system

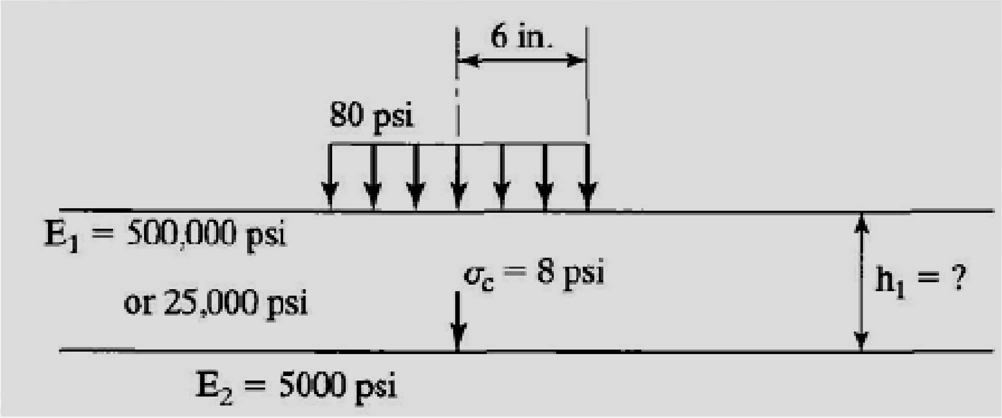


Figure 2.21. Example 4

Solution: a) Given $\mathbf{E_l/E_2}$ = 500,000/5000 = 100 , and $\boldsymbol{\sigma_c/q}$ = 8/80 = 0.1, from Figure 2.20, find $\mathbf{a/h_1}$ = 1.15, so the value of $\mathbf{h_l}$ = 6/1.15 = 5.2 in ., which represents the minimum thickness for full depth . **b)** Given $\mathbf{E_l/E_2}$= 25,000/5000 = 5 , and $\boldsymbol{\sigma_c/q}$ = 0.1, from Figure 2.20, for $\mathbf{a/h_l}$ = 0 .4, so the value of $\mathbf{h_1}$ = 6/0.4 = 15 in., which is the minimum thickness of granular base required. **Note: compare between the two values of $\mathbf{h_1}$**

- **The allowable vertical stress** should depend on the number of load repetitions ,using the Shell design criterion and the AASHTO equation, Huang et al. (1984b) developed the relationship: $\boldsymbol{N_d = 4.873 \times 10^{-5}\, \sigma_c{}^{-3.734} E_2{}^{3.583}}$

in which N_d is the allowable number of stress repetitions to limit permanent deformation, $\boldsymbol{\sigma_c}$ is the vertical compressive stress on the surface of the subgrade **in psi**, and E_2 is the elastic modulus of the subgrade **in psi**.

Example 5: Use the data in example 4 to find the allowable number of repetitions?
Solution: For a stress of 8 psi (5 kPa) and an elastic modulus of 5000 psi (35 MPa), the allowable number of repetitions is $\boldsymbol{N_d = 3.7 \times 10^5}$.

- **Vertical Surface Deflection:** Vertical surface deflections have been used as a criterion of pavement design. Figure 2.21 can be used to determine the surface deflections for two-layer systems. The deflection is expressed in terms of the deflection factor F_2 by :

$$w_o = \frac{1.5\, qa}{E_2} F_2 \qquad \text{.....................2.1}$$

The deflection factor is a function of $\mathbf{E_1/E_2}$ **and** $\mathbf{h_1/a}$. For a homogeneous half-space with $\mathbf{h_1/a} = 0$, $\mathbf{F_2 = 1}$, so Eq. 2.1 is identical to Equation for flexible plate when μ = 0 .5 . If the load is applied by a rigid plate, then, from Eq. 2.2. of rigid plate.

$$w_o = \frac{1.18\, qa}{E_2} F_2 \qquad \text{.....................2.2}$$

Example.5: A total load of 20,000 lb (89 kN) was applied on the surface of a two-layer system through a rigid plate 12 in. in diameter, as shown in Figure 2.22. Layer 1 has a thickness of 8 in. and layer 2 has an elastic modulus of 6400 psi (44.2 MPa). Both layers are incompressible with a Poisson ratio of 0.5. If the deflection of the plate is 0.1 in . (2.54 mm), determine the elastic modulus of layer 1.

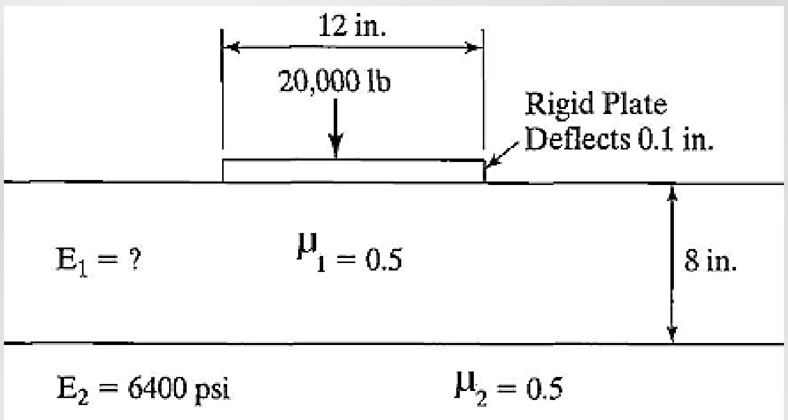


Figure 2.22. Example 5.

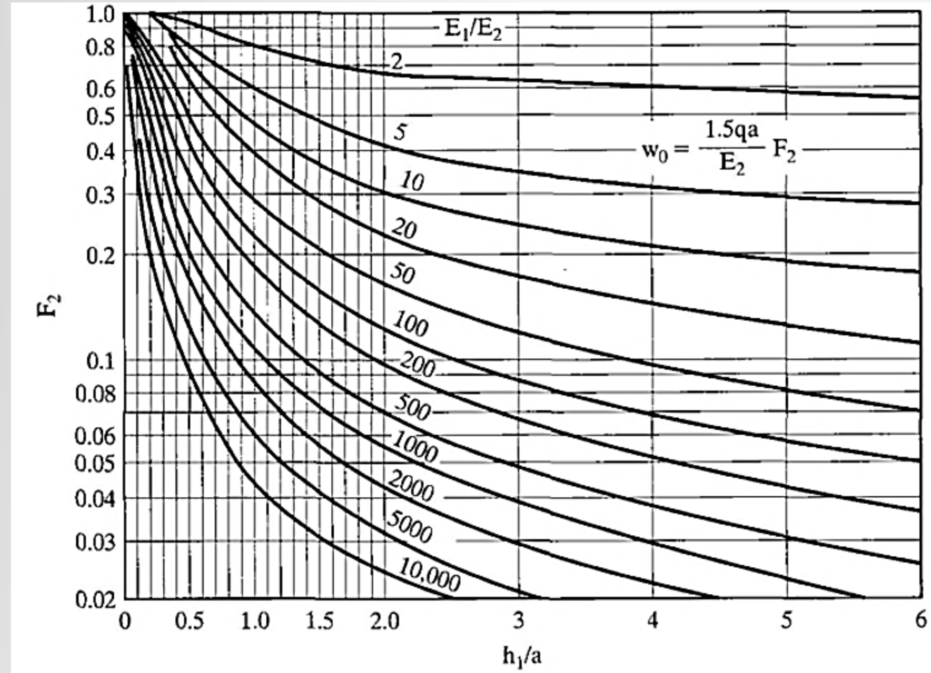


Figure 2.21. Vertical surface deflections for two-layer systems

Solution:

The average pressure on the plate is **q** = 20,000/(36π) = 176.8 psi (1.22 MPa). From Eq. 2.2, find the value of $\mathbf{F_2}$= 0.1 × 6400 / (1 .18 × 176.8 × 6) = 0.511. Given $\mathbf{h_1/a}$= 8/6 = 1.333, from Figure 2.20, $\mathbf{E_1/E_2}$ = 5, or $\mathbf{E_1}$ = 5 × 6400 = 32,000 psi (221 MPa) .

Critical Tensile Strain

The tensile strains at the bottom of asphalt layer have been used as a design criterion to prevent fatigue cracking. Two types of principal strains could be considered.

1. One is the overall principal strain based on all six components of normal and shear stresses.
2. The other, which is more popular and was used in KENLAYER, is the **horizontal principal strain** based on the **horizontal**, **normal and shear** stresses only.

Note: The overall principal strain is slightly greater than the horizontal principal strain, so the use of overall principal strain is on the safe side . The critical tensile strain is the overall strain and can be determined from Eq. 2.3.

$$e = \frac{q}{E_1} F_e \qquad \text{………………2.3}$$

where: ***e*** is the critical tensile strain and

F_e is the strain factor, which can be determined from the charts .

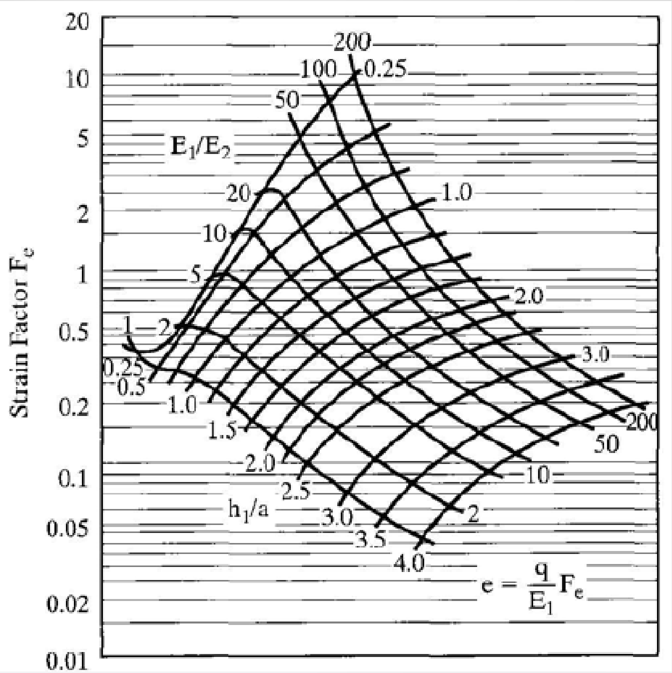
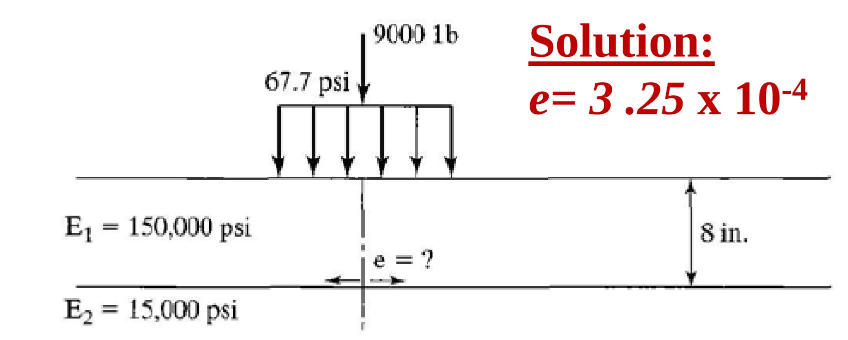


Figure 2 .23.**Single Wheel** chart for the strain factor of a two-layer system under a circular loaded area .

Example 6: Figure 2.24 shows a full-depth asphalt pavement 8 in. thick subjected to a single-wheel load of 9000 lb (40 kN) having contact pressure 67.7 psi. If the elastic modulus of the asphalt layer is 150,000 psi and that of the subgrade is 15,000 psi, determine the critical tensile strain in the asphalt layer .



Solution:

***e*= 3 .25 x 10^{-4}**

Figure 2 .24. Example 6.

The Figure 2.23 is used for single wheel, **in dual wheels**,

- The strain factor for dual wheels depends on the parameters: contact radius ***a*,** dual spacing S_d , S_d / a , E_1/E_2 ,and h_1/a.
- There are two charts one for dual wheels with S_d **= 24 in**. (610 mm) and ***a* = 3 in.** and the other for S_d **= 24 in.** (610 mm) and ***a* = 8 in.** to determine conversion factors: C_1 and C_2 as shown in Figure 2.24.

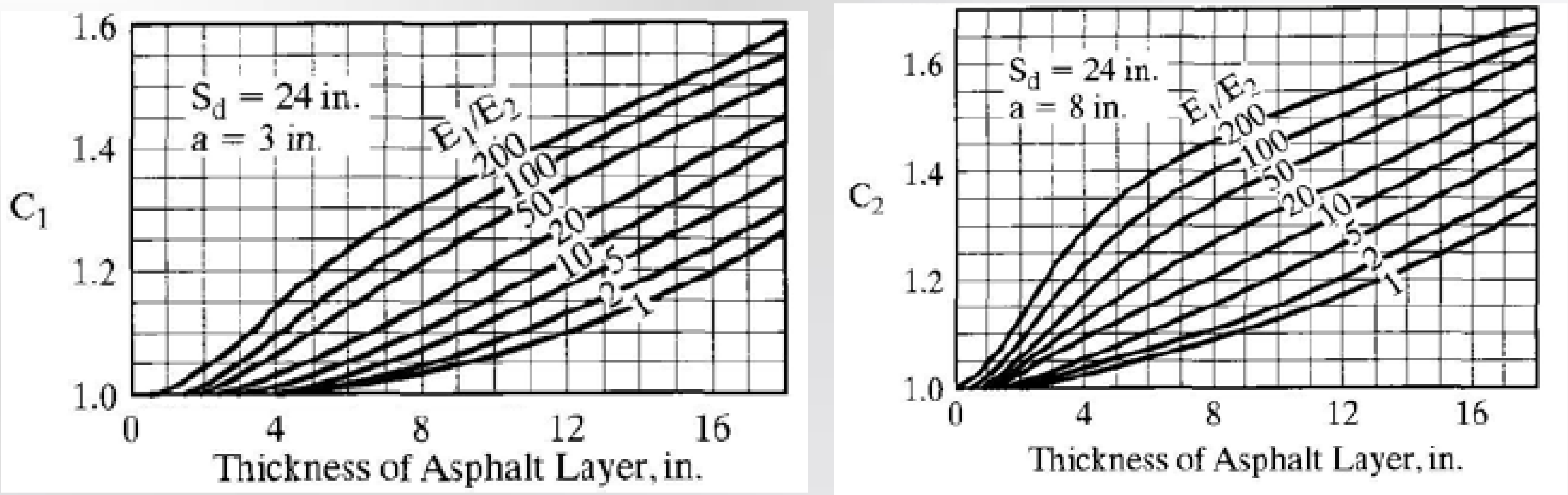


Figure 2.24. Conversion factors.

For any other different S_d and a values the following procedure can be used:

1. From the given S_d, h_1 , and **a**, determine the modified radius **a'** and the modified thickness h_1**'** :

$$a' = \frac{24}{S_d} a \quad \text{and} \quad h_1' = \frac{24}{S_d} h_1$$

2. Using h_1**'** as the pavement thickness, find conversion factors C_1 and C_2 from Figure 2.24.
3. Determine the conversion factor for **a'** by a straight-line interpolation between 3 and 8 in. or the formula.

$$C = C_1 + 0.2 \times (a' - 3) \times (C_2 - C_1)$$

Example 7:

For the same pavement as in Example 6, if the 9000-lb (40-kN) load is applied over a set of dual tires with a center-to-center spacing of 11.5 in. and a contact pressure of 67.7 psi ,as shown in Figure 2 .25, determine the critical tensile strain in the asphalt layer.

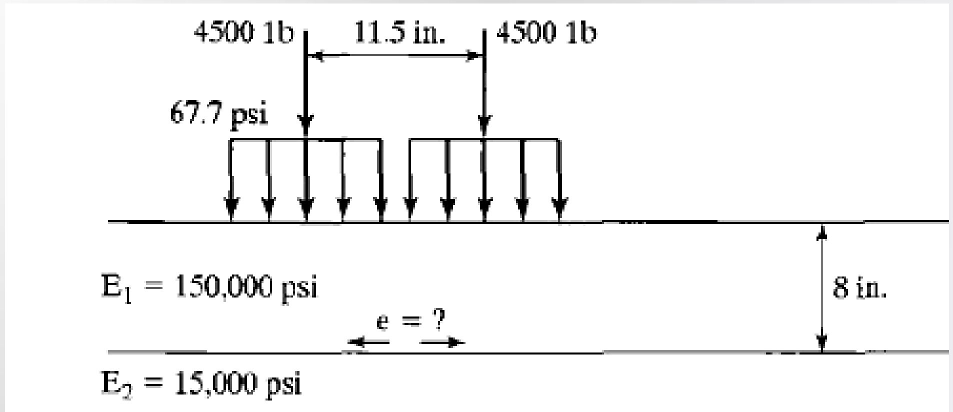


Figure 2 .25. Example 7.

Solution:

Compute a = 4.6 in., h_1 = 8 in. from $a' = \frac{24}{S_d} a$ and $h_1' = \frac{24}{S_d} h_1$

a' = 24 x 4.6/11 .5 = 9 .6 in. and h'_1 = 24 x 8/11.5 = 16.7 in., E_1/E_2 = 10 and an asphalt layer thickness of 16.7 in. from Figure 2.24, C_1 = 1.42 and C_2 = 1.46. From interpolation equation, C = 1 .42 + 0 .2 (9 .6 – 3) (1 .46 – 1 .42) = 1.473 (C is a modified factor to F_e which is found from Figure 2.22). From Figure 2 .22, the strain factor for a single wheel = 0 .47 (**E_1/E_2 = 10 & h_1/a= 1.74**) and that for dual wheels = 1.473 x 0 .47 = 0 .692, so the critical tensile strain is: **e = 67 .7 X 0.692/150,000 = 3 .12 x 10^{-4}**.

2.1.2.2. Three Layers System.

Figure 2.٢٦ shows a three-layer system and the stresses at the interfaces on the axis of symmetry. These stresses include vertical stress at interface 1, σ_{z1} , vertical stress at interface 2, σ_{z2} , radial stress at bottom of layer 1, σ_{r1} , radial stress at top of layer 2, σ'_{r1} , radial stress at bottom of layer 2, σ_{r2}, and radial stress at top of layer 3, σ'_{r2} . **Note that, on the axis of symmetry, $\sigma_r = \sigma_t$ and the sheer stress is equal to 0. When the Poisson ratio is 0.5, we have:**

$$\epsilon_z = \frac{1}{E}(\sigma_z - \sigma_r) \quad \text{.................} \ 2.4$$

$$\epsilon_r = \frac{1}{2E}(\sigma_r - \sigma_z) \quad \text{.................} \ 2.5$$

$$\epsilon_z = -2\epsilon_r \quad \text{........................} \ 2.6$$

Important note:
$\mu_1 = \mu_2 = \mu_3 = 0.5$

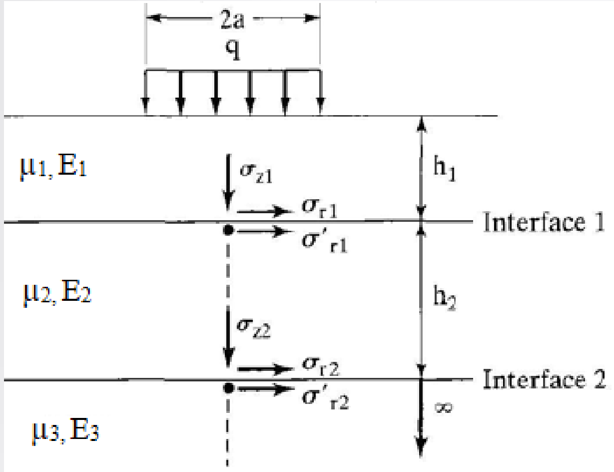


Figure 2.٢٦. Three layers system

<u>Note</u>:

- The horizontal strain is equal to one-half of the vertical strain
- To understand these Eqs. 2.4 to 2.6 go back to slides No. 3 and 5.

Solution Method for Three Layers System Using Jones' Tables (Figure 2.27).

The stresses in a three-layer system depend on the ratios ***k_1, k_2, A,*** and ***H***, defined as

$$k_1 = \frac{E_1}{E_2} \qquad k_2 = \frac{E_2}{E_3} \quad \text{..............................} \ 2.7$$

$$A = \frac{a}{h_2} \qquad H = \frac{h_1}{h_2} \quad \text{..............................} \ 2.8$$

$$\sigma_{z1} - \sigma'_{r1} = \frac{\sigma_{z1} - \sigma_{r1}}{k_1} \quad \text{..............................} \ 2.9$$

$$\sigma_{z2} - \sigma'_{r2} = \frac{\sigma_{z2} - \sigma_{r2}}{k_2} \quad \text{..............................} \ 2.10$$

$H = 0.12$

$k_1 = 0.2$

a_1	σ_{z_1}	$\sigma_{z_1}-\sigma_{r_1}$	$\sigma_{z_1}-\sigma_{r}$	σ_{z_2}	$\sigma_{z_2}-\sigma_{r_2}$	$\sigma_{z_2}-\sigma_{r_3}$
						$k_2 = 0.2$
0.1	0.66045	0.12438	0.62188	0.01557	0.00332	0.01659
0.2	0.90249	0.13546	0.67728	0.06027	0.01278	0.06391
0.4	0.95295	0.10428	0.52141	0.21282	0.04430	0.22150
0.8	0.99520	0.09011	0.45053	0.56395	0.10975	0.54877
1.6	1.00064	0.08777	0.43884	0.86258	0.13755	0.68777
3.2	0.99970	0.04129	0.20643	0.94143	0.10147	0.50736
						$k_2 = 2.0$
0.1	0.66048	0.12285	0.61424	0.00892	0.01693	0.00846
0.2	0.90157	0.12916	0.64582	0.03480	0.06558	0.03279
0.4	0.95120	0.08115	0.40576	0.12656	0.23257	0.11629
0.8	0.99235	0.01823	0.09113	0.37307	0.62863	0.31432
1.6	0.99918	-0.04136	-0.20680	0.74038	0.93754	0.49377
3.2	1.00032	-0.03804	-0.19075	0.97137	0.82102	0.41051
						$k_2 = 20.0$
0.1	0.66235	0.12032	0.60161	0.00256	0.03667	0.00183
0.2	0.90415	0.11787	0.58933	0.01011	0.14336	0.00717
0.4	0.95135	0.03474	0.17370	0.03838	0.52691	0.02635
0.8	0.98778	-0.14872	-0.74358	0.13049	1.61727	0.08086
1.6	0.99407	-0.50533	-2.52650	0.36442	3.58944	0.17947
3.2	0.99821	-0.80990	-4.05023	0.76669	5.15409	0.25770
						$k_2 = 200.0$
0.1	0.66266	0.11720	0.58599	0.00057	0.05413	0.00027
0.2	0.90370	0.10495	0.52477	0.00226	0.21314	0.00107
0.4	0.94719	-0.01709	-0.08543	0.00881	0.30400	0.00402
0.8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Figure 2.27. atypical Jones' Tables

- Jones developed a Tables to determine the stress factors for three-layer systems.

$$\sigma_{z1} = q\,(ZZ1) \quad \cdots\cdots\cdots 2.11 \qquad \sigma_{z2} = q\,(ZZ2) \quad \cdots\cdots\cdots 2.12$$

$$\sigma_{z1} - \sigma_{r1} = q\,(ZZ1 - RR1) \quad \cdots\cdots\cdots 2.13 \qquad \sigma_{z2} - \sigma_{r2} = q\,(ZZ2 - RR2) \quad \cdots\cdots\cdots 2.14$$

- Where ***q*** is the contact pressure (tire inflation in psi), ZZ1,ZZ2,---- etc. are factors found from Jones` tables.

- The sign convention is positive in compression and negative in tension . Four sets of stress factors,ZZ1, ZZ2, (ZZ1 – RR1), and (ZZ2 – RR2) are shown in tables. The product of the contact pressure and the stress factors gives the stresses. The tables presented by Jones consist of **four values of k1 and k_2 (0.2, 2, 20, and 200), so solutions for intermediate values of k_1 and k_2 can be obtained by interpolation.**

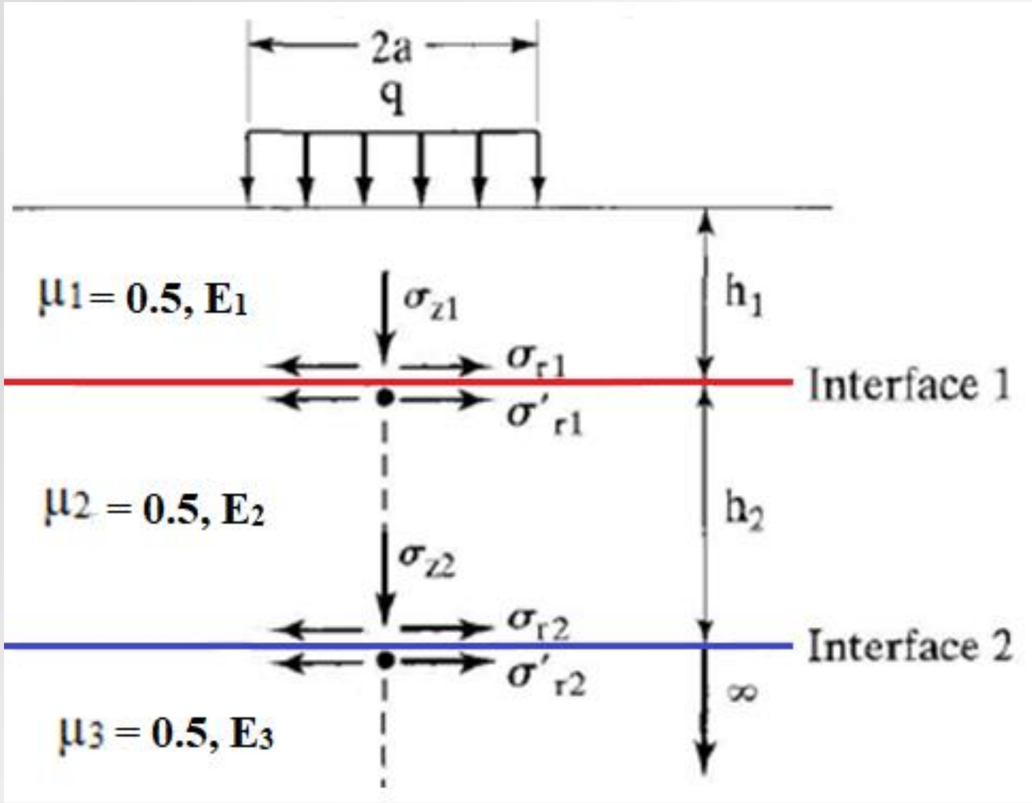


Figure 2 .28. Detailed stresses in three layers system.

From Figure 2.28. it can be observed, that the presence of friction has a significant influence on the radial (horizontal) stress at the bottom of the top layer especially at low values for the ratio E_1/E_2. We also note that the influence on the vertical stress is much smaller.
If there is full friction or full bond at the interface, the following conditions are satisfied:

- **The vertical stress just below and above the interface are equal because of equilibrium, so:**

σ_{z1} at the bottom of the top layer (1) = σ_{z1} at the top of the bottom layer (2) (interface 1)
σ_{z2} at the bottom of the top layer (2) = σ_{z2} at the top of the bottom layer (3) (interface 2)

- **The horizontal displacements just above and below the interface are the same because of full friction, so:**

ε_{r1} at the bottom of the top layer (1) = $\varepsilon`_{r1}$ at the top of the bottom layer (2) (interface 1)
($\sigma_{r1} = \sigma`_{r1}$ at interface 1)
ε_{r2} at the bottom of the top layer (2) = $\varepsilon`_{r2}$ at the top of the bottom layer (3) (interface 2)
($\sigma_{r2} = \sigma`_{r2}$ at interface 2)

- **The vertical displacements just above and below the interface are the same because of continuity, so:**

ε_{z1} at the bottom of the top layer (1) = ε_{z1}at the top of the bottom layer (2) (interface 1)
ε_{z2} at the bottom of the top layer (2)= ε_{z2}at the top of the bottom layer (3) (interface 2)

Example 8:

Given the three-layer system shown in Figure 2.29 with a = 122 mm, q = 828 kPa, h1 = 152 mm), h2 = 6 in. (203 mm), E1 = 400,000 psi (2.8 GPa), E2 = 20,000 psi (138 MPa), and E3 = 10,000 psi (69 MPa), determine all the stresses and strains at the two interfaces on the axis of symmetry .

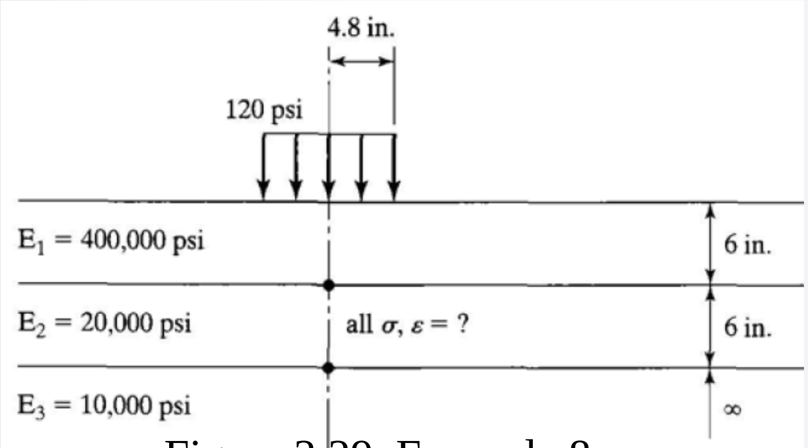


Figure 2.29. Example 8.

<u>Solution:</u>

Given $\mathbf{k_l}$ = 400,000/20,000 = 20, $\mathbf{k}_{2}$= 20,000/10,000 = 2, **A** = 4 .8/6 = 0 .8, and **H** = 6/6 = 1, from Table (….) Find the factors: , ZZ1 = 0 .12173, ZZ2 = 0 .05938, ZZ1 - RR1 = 1 .97428, and ZZ2 - RR2 = 0.09268 .

From Eq. 2 .11, σ_{z1} = q × ZZ1 = 120 × 0.12173 = 14.61 psi (101 kPa)

From Eq. 2 .12 σ_{z2} = q × ZZ2 = 120 × 0 .05938 = 7.12 psi (49.1 kPa)

From Eq. 2 .13 σ_{z1} - σ_{r1} = q × (ZZ1 – RR1) = 120 × 1 .97428 = 236.91 psi (1 .63 MPa), and σ_{r1} = 14.61-236.91 = -222.31 psi.

From Eq. 2 . 14 σ_{z2} - σ_{r2} = q × (ZZ2 – RR2) = 120 × 0.09268 = 11.12 psi,

σ_{r2} = 7.12 – 11.12 = -4.0 psi

From Equations 2.9 and 2.10.

$\sigma`_{r1}$ = 2.76 psi, $\sigma`_{r2}$ = 1.56 psi

$$\sigma_{z1} - \sigma'_{r1} = \frac{\sigma_{z1} - \sigma_{r1}}{k_1} \quad \text{.............................} \ 2.9$$

$$\sigma_{z2} - \sigma'_{r2} = \frac{\sigma_{z2} - \sigma_{r2}}{k_2} \quad \text{.............................} \ 2.10$$

$$\epsilon_z = \frac{1}{E}(\sigma_z - \sigma_r) \quad \text{………………..} \ 2.4$$

$$\epsilon_r = \frac{1}{2E}(\sigma_r - \sigma_z) \quad \text{………………..} \ 2.5$$

$H = 1.0$

$k_1 = 20.0$

a_1	σ_{z_1}	$\sigma_{z_1}-\sigma_{r_1}$	$\sigma_{z_1}-\sigma_{r_2}$	σ_{z_2}	$\sigma_{z_2}-\sigma_{r_2}$	$\sigma_{z_2}-\sigma_{r_3}$
						$k_2 = 0.2$
0.1	0.00417	0.04050	0.00202	0.00271	0.00039	0.00195
0.2	0.01641	0.15675	0.00784	0.01080	0.00155	0.00777
0.4	0.06210	0.55548	0.02777	0.04241	0.00606	0.03028
0.8	0.21057	1.53667	0.07683	0.15303	0.02198	0.10991
1.6	0.58218	2.77359	0.13868	0.49705	0.06327	0.31635
3.2	1.06296	2.55195	0.12760	1.00217	0.09906	0.49525
						$k_2 = 2.0$
0.1	0.00263	0.04751	0.00238	0.00100	0.00160	0.00080
0.2	0.01029	0.18481	0.00924	0.00397	0.00637	0.00319
0.4	0.03810	0.66727	0.03336	0.01565	0.02408	0.01240
0.8	0.12173	1.97428	0.09871	0.05038	0.09268	0.04634
1.6	0.31575	4.37407	0.21870	0.20098	0.29253	0.14626
3.2	0.66041	6.97695	0.34885	0.53398	0.65446	0.32723
						$k_2 = 20.0$
0.1	0.00193	0.05737	0.00287	0.00024	0.00322	0.00016
0.2	0.00751	0.22418	0.01121	0.00098	0.01283	0.00064
0.4	0.02713	0.82430	0.04121	0.00387	0.05063	0.00253
0.8	0.08027	2.59672	0.12984	0.01507	0.19267	0.00963
1.6	0.17961	6.77014	0.33851	0.05549	0.66326	0.03316
3.2	0.34355	15.23252	0.76163	0.18344	1.88634	0.09432
						$k_2 = 200.0$
0.1	0.00176	0.06733	0.00337	0.00006	0.00478	0.00002
0.2	0.00683	0.26401	0.01320	0.00022	0.01908	0.00010
0.4	0.02443	0.98346	0.04917	0.00088	0.07557	0.00038
0.8	0.06983	3.23164	0.16158	0.00348	0.29194	0.00146
1.6	0.14191	9.28148	0.46407	0.01339	1.05385	0.00527
3.2	0.22655	24.85236	1.24262	0.04911	3.37605	0.01688

Solution:

At bottom of layer 1:

To calculate the strains at the bottom of layer 1 use Equations 2.4 and 2.5.

$\varepsilon_{z1} = (\sigma_{z1} - \sigma_{r1}) / E_1 = 236.91 \ / \ 400000 = 5.92 \times 10^{-4}$

$\varepsilon_{r1} = (\sigma_{r1} - \sigma_{z1}) / 2 E_1 = -236.91 \ / \ 2 \times 400000 = -2.96 \times 10^{-4}$ (or directly, using equation 2.6 for find ε_r)

$$\epsilon_z = -2\epsilon_r \quad \text{..........................} \ 2.6$$

At top of layer 2:

To calculate the strains at the top of layer 2 use Equations 2.4 and 2.5

$\varepsilon_{z1} = (\sigma_{z1} - \sigma`_{r1}) / E_2 = (14.61-2.76) / \ 20000 = 5.92 \times 10^{-4} = \varepsilon_z$ at bottom of layer 1

$\varepsilon`_{r1} = (\sigma`_{r1} - \sigma_{z1}) / 2 E_2 = (2.76-14.61) / \ 2 \times 20000 = -2.96 \times 10^{-4} = \varepsilon_{r1}$ at bottom of layer 1

At bottom of layer 2 :

To calculate the strains at the bottom of layer 2 use Equations 2.4 and 2.5.

$\varepsilon_{z2} = (\sigma_{z2} - \sigma_{r2}) / E_2 = 11.12 \ / \ 20000 = 5.56 \times 10^{-4}$

$\varepsilon_{r2} = (\sigma_{r2} - \sigma_{z2}) / 2 E_2 = -11.12 \ / \ 2 \times 20000 = -2.78 \times 10^{-4}$

At top of layer 3:

To calculate the strains at the top of layer 3 use Equations 2.4 and 2.5

$\varepsilon_{z3} = (\sigma_{z2} - \sigma`_{r2}) / E_3 = 5.56 \ / \ 10000 = 5.56 \times 10^{-4} = \varepsilon_{z2}$ At bottom of layer 2

$\varepsilon_{r3} = (\sigma`_{r2} - \sigma_{z2}) / 2 E_3 = -5.56 \ / \ 2 \times 10000 = -2.78 \times 10^{-4}$ At bottom of layer 2

2.2. Equivalent Thickness Method (OdeMark's Concept)

Odemark's equivalent-layer-thickness (ELT) concept is often used as a simple method of approximation in pavement structural analysis, since it permits the conversion of a multilayered system into a single layer with equivalent thickness. It is based on the principle that the equivalent layer has the same stiffness as the original layer, so as to give the same pressure distribution beneath the layer as shown in Figures 2.30 to 2.32.

Stiffness of layer 1 = $\dfrac{E_1 I_1}{1-\mu_1^2}$

Stiffness of layer 2 = $\dfrac{E_2 I_2}{1-\mu_2^2}$

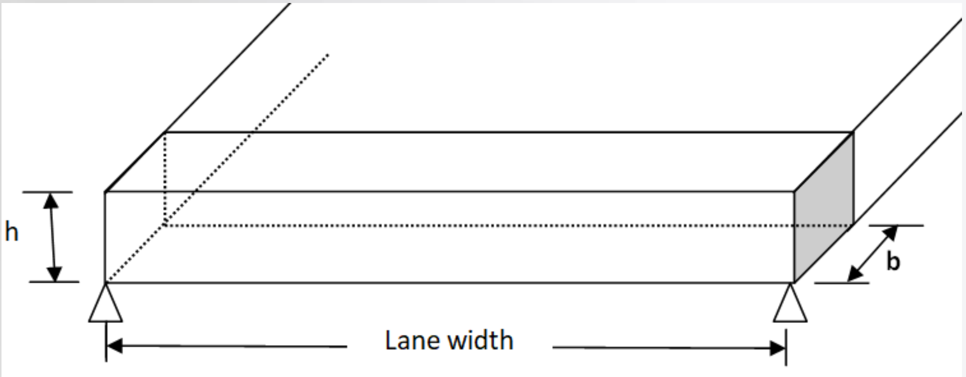


Figure 2.30. Moment of inertia of simply supported beam.

$$I_{beam} = \frac{bh^3}{12}$$

$$I_1 = \frac{b_1 h_1^3}{12}, \quad I_2 = \frac{b_2 h_2^3}{12}$$

for $b_1 = b_2 = 1$ m

<u>According to Odemark`s theory:</u>

Stiffness of layer 1 = Stiffness of layer 2

If $\mu_1 = \mu_2 = 0.5$

$$\frac{E_1 I_1}{1-\mu_1^2} = \frac{E_2 I_2}{1-\mu_2^2}$$

$$for\ layer\ 1 \quad h_e = \sqrt[3]{\frac{E_1}{E_e}}\ h_1 \qquad for\ layer\ 2 \quad h_e = \sqrt[3]{\frac{E_2}{E_e}}\ h_2$$

$$for\ layer\ i \quad h_e = \sqrt[3]{\frac{E_i}{E_e}}\ h_i$$

Equivelent thickness (h_e) of multy layers

$$h_e = \sqrt[3]{\frac{E_1}{E_e}}\ h_1 + \sqrt[3]{\frac{E_2}{E_e}}\ h_2 + \cdots + \sqrt[3]{\frac{E_i}{E_e}}\ h_i$$

$$The\ general\ formula: \quad h_e = f \sum_{i=1}^{n-1} \sqrt[3]{\frac{E_i}{E_e}}\ h_i$$

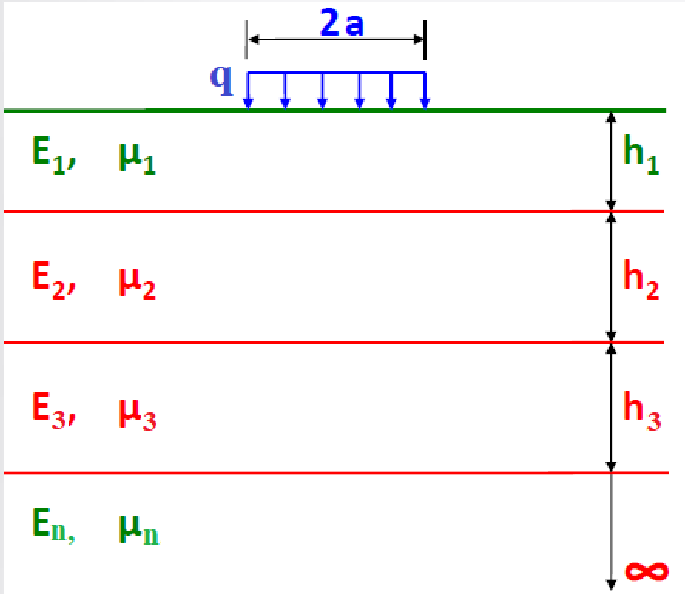


Figure 2 .31. Multilayer system.

<u>For *f* value:</u>

- ➢ In a 2-layer pavement system, use ***f* = *0.9*** to convert the upper layer.
- ➢ In a multi-layer pavement system, use ***f* = *0.8*** to convert the rest of the layers.

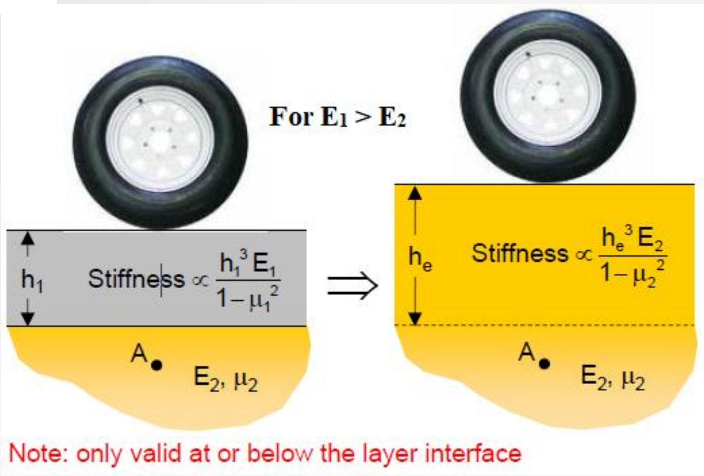


Figure 2 .32. Odemark`s concept.

Example 9:

The structure as shown in Figure 2.33 represents a multilayer pavement system?. By using Odemark`s concept, find the equivalent thickness of the structure?.

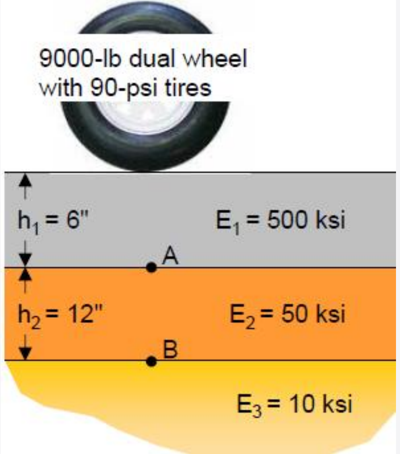
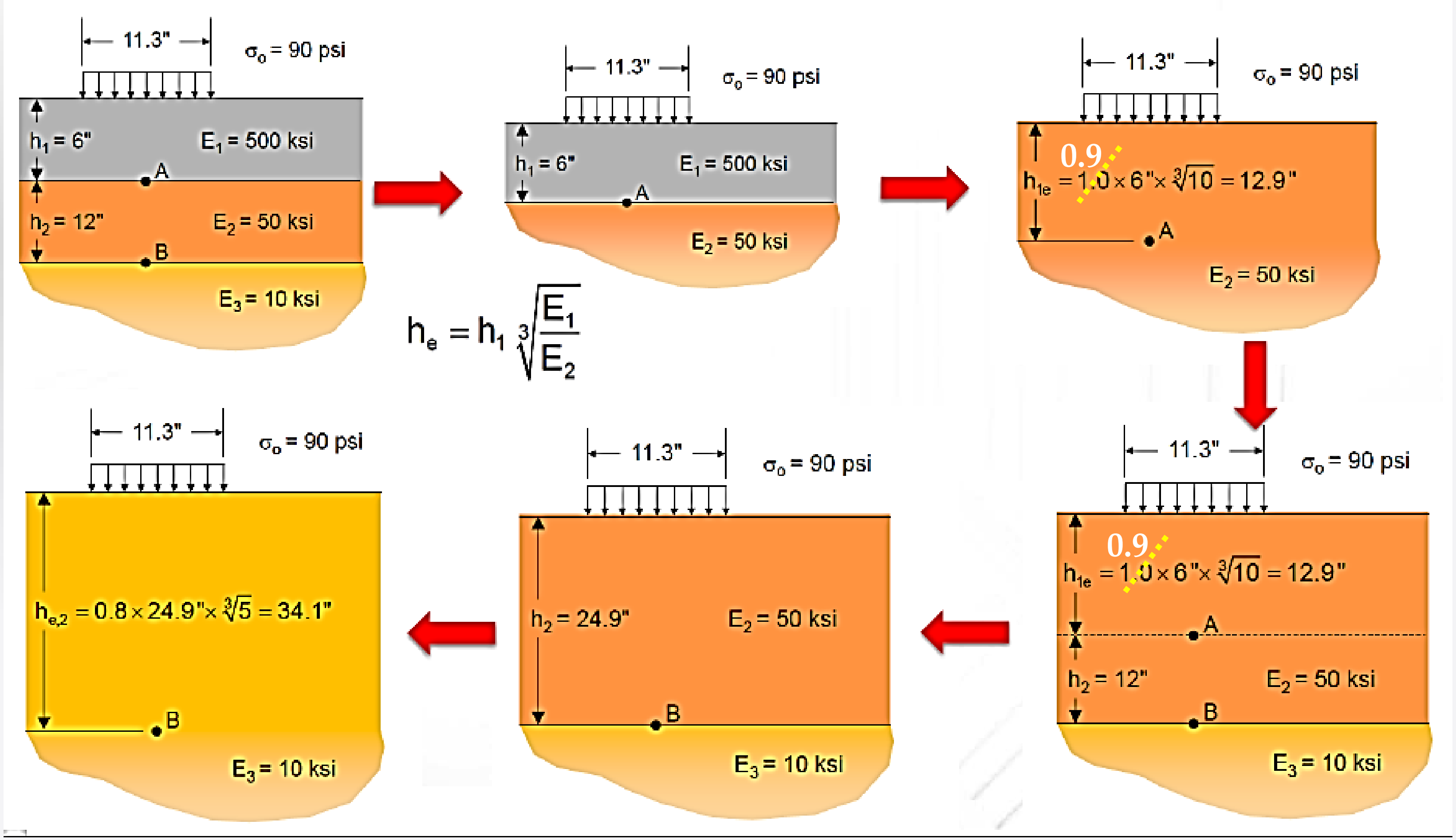


Figure 2 .33. Example 9.

Solution:

As detailed in the Figures



Example 10:

Compute the stresses at the bottom of a flexible pavement surface layer 0.3 m thick resting on a semi-infinite subgrade layer. The load consists of a circular tire with a 0.1 m radius carrying a uniform pressure of 700 kPa. The stresses are to be computed under the centerline of the load. The layer moduli are 1400 MPa and 140 MPa, respectively, and μ is 0.5 for both layers.

Solution:

Using equivalent thickness equation gives the equivalent thickness of the top layer in terms of the modulus of the bottom layer as:

$$h_e = 0.9\left(\frac{E_1}{E_2}\right)^{\frac{1}{3}} h$$

$$h_e = 0.9\left(\frac{1400}{140}\right)^{\frac{1}{3}} 0.3 = 0.582 \text{ m}$$

Using one layer system formula to compute the stresses in the subgrade. At the bottom of the top layer, they are:

$$\sigma_z = q\left[1 - \frac{z^3}{(a^2+z^2)^{3/2}}\right] \quad , \quad \tau_{zr} = 0$$

$$\sigma_r = \sigma_t = \frac{q}{2}\left[(1+2\mu) - \frac{2(1+\mu)z}{\sqrt{a^2+z^2}} + \frac{z^3}{(a^2+z^2)^{3/2}}\right]$$

$$\sigma_z = 700\left[-1 + \frac{0.582^3}{(0.1^2+0.582^2)^{3/2}}\right] = -29.9 \text{ kPa}$$

$$\sigma_r = \sigma_\theta = \frac{700}{2}\left[-(1+2\,0.5) + \frac{2(1+0.5)0.582}{\sqrt{0.1^2+0.582^2}} - \frac{0.582^3}{(0.1^2+0.582^2)^{3/2}}\right] = -0.218 \text{ kPa}$$

2.3. Viscoelastic Solutions

The previous discussion assumed elastic material behavior; however, asphalt concretes exhibit viscoelastic behavior, hence their response is time-dependent. Their response to a time-dependent (e.g., moving) load is simulated through two general methods for characterizing viscoelastic materials: one by a creep-compliance model, the other by a mechanical model:

2.3.1. Creep-Compliance Model

Creep Compliance (D(t)) can characterize viscoelastic materials at various times, D(t), defined as:

$$D(t) = \frac{\epsilon(t)}{\sigma} \quad \text{....(2-11)}$$

Boltzmann's superposition principle, assuming linear viscoelastic behavior. In the time domain, this is expressed by the following convolution integral:

where, ε(t) is the strain at time t, D(t − ξ') is the creep compliance of the asphalt concrete layer after a lapsed time of (t − ξ'), and σ (ξ) is the stress history as a function of time

$$\varepsilon(t) = \int_0^t D(t - \xi') \frac{\partial \sigma(\xi')}{\partial \xi'} d\xi' \quad \text{....(2-12)}$$

2.3.2. Mechanical Model

There are various mechanical models for characterizing viscoelastic materials which are formed of two basic elements : **a spring and a dashpot** as shown in Figure 2.34.

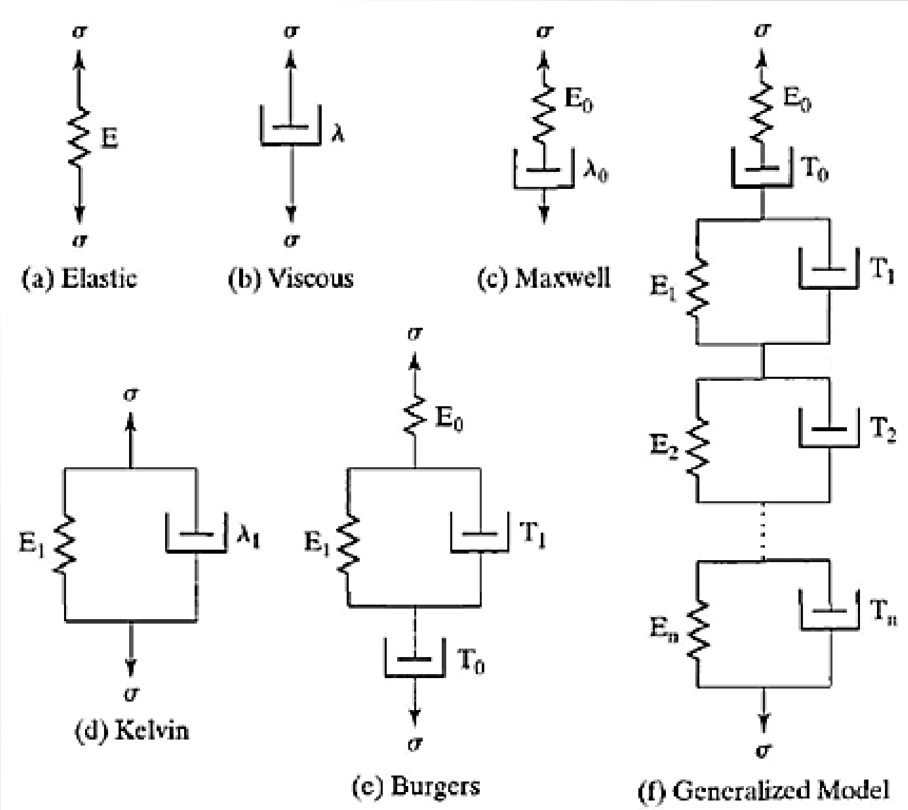


Figure 2 .34. Mechanical Model for Viscoelastic Materials.

2.3.2.1. Basic Models An elastic material is characterized by a spring (obeys Hooke's law, Equation 2.13).

$$\boldsymbol{\sigma = E\varepsilon} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ (\mathbf{2.13})$$

While the viscous material is characterized by a dashpot (obeys Newton's law, Equation 2.14), according to which stress is proportional to the time rate of strain :

$$\boldsymbol{\sigma = \lambda \frac{\partial \varepsilon}{\partial t}} \ \text{(integration)}\ldots\ldots\ldots$$

$$\boldsymbol{\varepsilon = \frac{\sigma t}{\lambda}} \quad \ldots\ldots (\mathbf{2.14})$$

Where: $\boldsymbol{\lambda}$ is viscosity and t is time .

2.3.2.2.Maxwell Model

A Maxwell model is a combination of spring and dashpot in series, Under a constant stress, the total strain is the sum of the strains of both spring and dashpot, Equation (2.15).

$$\epsilon = \frac{\sigma}{E_0} + \frac{\sigma t}{\lambda_0} = \frac{\sigma}{E_0}\left(1 + \frac{t}{T_0}\right) \ldots\ldots \ (\mathbf{2.15})$$

Where: $\mathbf{T_o = \lambda_o / E_o}$ = relaxation time

2.3.2.3. Kelvin Model

A Kelvin model is a combination of spring and dashpot in parallel, where both have the same strain, but the total stress is the sum of the two stresses, Equation (2.16).

$$\sigma = E_1\epsilon + \lambda_1 \frac{\partial \epsilon}{\partial t} \quad \textbf{by integration} \quad \epsilon = \frac{\sigma}{E_1}\left[1 - \exp\left(-\frac{t}{T_1}\right)\right] \quad \text{..... (2.16)}$$

2.3.2.4. Burger Model

$$\epsilon = \frac{\sigma}{E_0}\left(1 + \frac{t}{T_0}\right) + \frac{\sigma}{E_1}\left[1 - \exp\left(-\frac{t}{T_1}\right)\right] \quad \text{... (2.17)}$$

A Burgers model is a combination of Maxwell and Kelvin models in series Under a constant stress, the combinations of both models (Kelvin and Maxwell) to form the final calculations (Equation 2.17). The physical meaning of the model terms is illustrated in Figure 2.35.

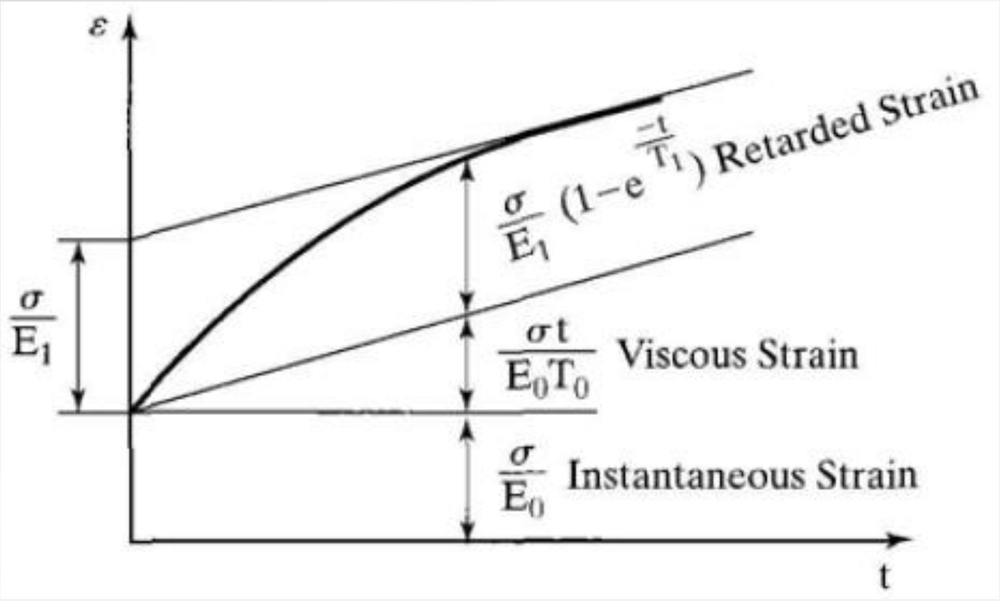


Figure 2.35. Physical meaning of the Burger model.

2.3.2.5. Generalized Model

- Generalized Model is that can be used to characterize any viscoelastic material. Generalized model (Figure 2.36) can be written as: Where n is the number of Kelvin models

$$\epsilon = \frac{\sigma}{E_0}\left(1 + \frac{t}{T_0}\right) + \sum_{i=1}^{n} \frac{\sigma}{E_i}\left[1 - \exp\left(-\frac{t}{T_i}\right)\right] \quad \text{... (2.18)}$$

- Determination the viscoelastic constants, E_0, T_0, E_i and T_i for a generalized model, the creep compliances at various times can be computed from Equation. 2.18.

Note: This model explains the effect of load duration on pavement responses. Under a single load application, the instantaneous and the retarded elastic strains predominate, and the viscous strain is negligible. However, under a large number of load repetitions, the accumulation of viscous strains is the cause of permanent deformation .

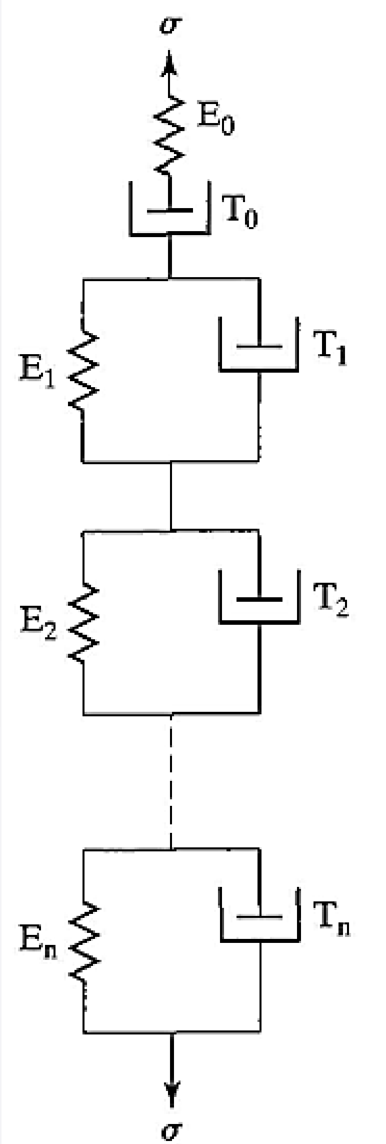


Figure 2.36: Generalized model.

Example 11:

A viscoelastic material is characterized by one Maxwell model and three Kelvin models connected in series with the viscoelastic constants shown in Figure 2.37. Determine the creep compliance at various times, and plot the creep-compliance curve .

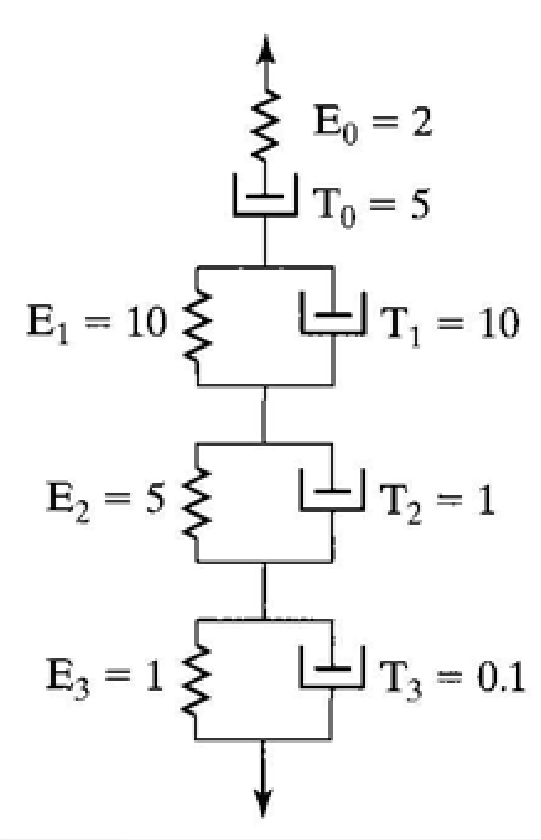


Figure 2.37: Example 11.

Solution:

Note: All constants are without units. If E is in lb/in^2, then the creep compliance is in $in.^2/lb$. If E is in kN/m^2, then the creep compliance is in m^2/kN.

From Eq. 2.18

$$\epsilon = \frac{\sigma}{E_0}\left(1 + \frac{t}{T_0}\right) + \sum_{i=1}^{n} \frac{\sigma}{E_i}\left[1 - \exp\left(-\frac{t}{T_i}\right)\right]$$

When: t=0, D=1/E°=1/2=0.5; and

when t = 0.1; **D = 0.5 (1+0.1/5)+0.1(1–e$^{-0.01}$)+0.2(1 — *e$^{-0.1}$)* + (1 — e^{-1}) = 1 .162.**

For any farther times D(t) can be calculated using Equation 2.18. So the creep compliances at various times are tabulated in Table (1) below and plotted in Figure 2.38. It can be seen that, after t = 5, all the retarded strains have nearly completed and only the viscous strains exist, as indicated by a straight line . **If the retarded strain lasts much longer, more Kelvin models with longer retardation times will be needed** .

Table 2.1: Creep complain data calculated based on Equation 2.18.

Creep Compliance at Various Times			
Time	Creep compliance	Time	Creep compliance
0	0.500	2	1.891
0.05	0.909	3	2.016
0.1	1.162	4	2.129
0.2	1.423	5	2.238
0.4	1.592	10	2.763
0.6	1.654	20	3.786
0.8	1.697	30	4.795
1.0	1.736	40	5.798
1.5	1.819	50	6.799

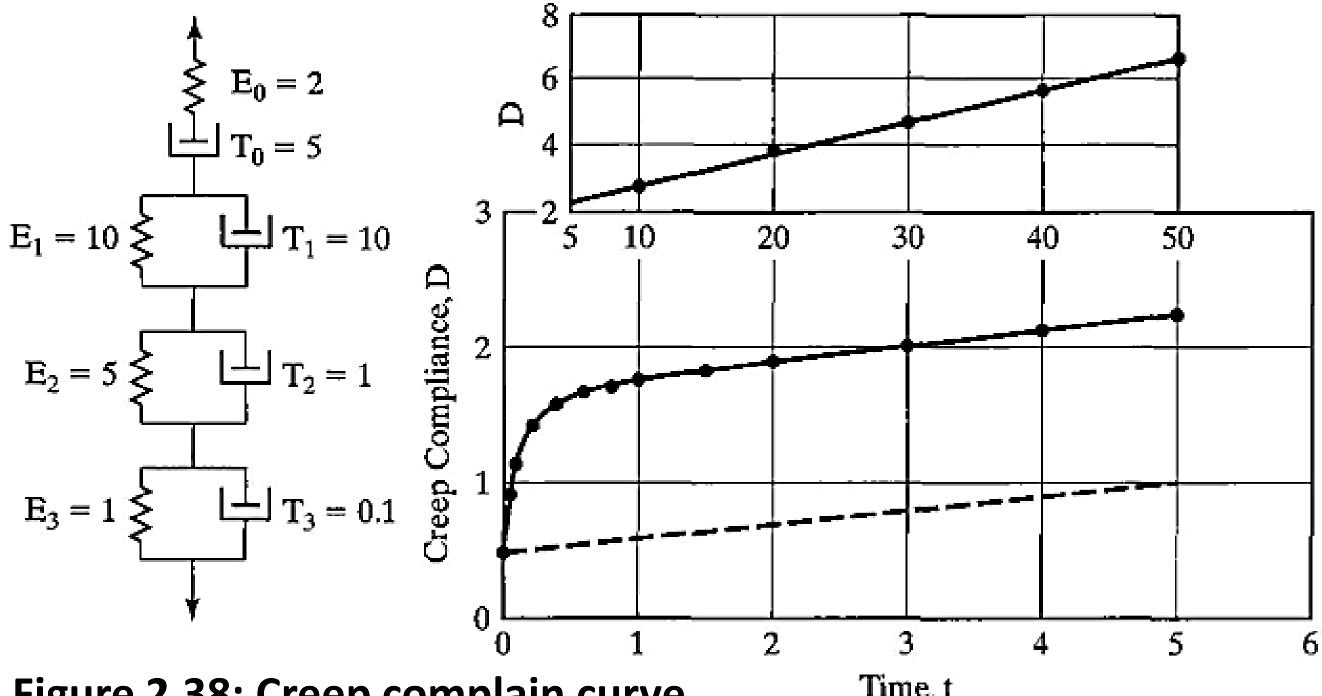


Figure 2.38: Creep complain curve.

Determination Generalized Model Constant

Generalized model consist of Maxwell model and n of Kelvin model as shown in Figure 2.36, Equation 2.18. To solve this model, several parameters, i.e. viscoelastic constants. Ei, Ti and n, are required to be identified; if a creep compliance curve (D(t), t) is given, the viscoelastic constants of a generalized model can be determined by several methods:

$$\epsilon = \frac{\sigma}{E_0}\left(1 + \frac{t}{T_0}\right) + \sum_{i=1}^{n} \frac{\sigma}{E_i}\left[1 - \exp\left(-\frac{t}{T_i}\right)\right] \quad \text{....... (2.18)}$$

1. Successive Residuals Method

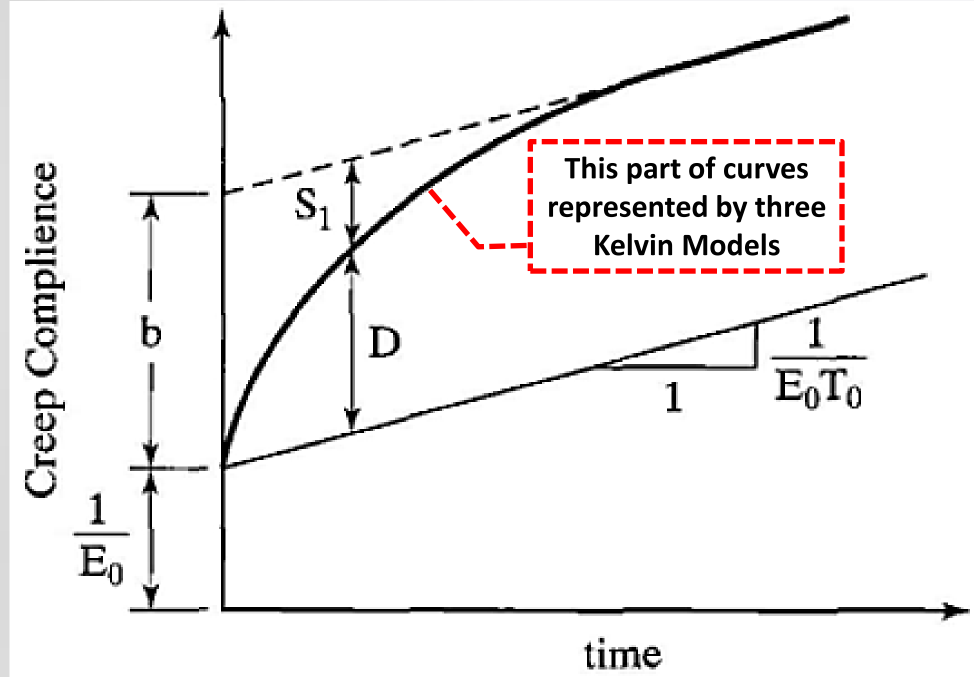
This method is an approximate method of **collocation method**, it is used to determine the constants *Ei* and *Ti* of a viscoelastic material directly from the creep curve,

1. The creep compliances D due to retarded strains (Equation 2.19) are determined by deducting the instantaneous and viscous strains from the total strains, as shown in Figure 2.39.

$$D = \frac{1}{E_1}\left[1 - \exp\left(-\frac{t}{T_1}\right)\right] + \frac{1}{E_2}\left[1 - \exp\left(-\frac{t}{T_2}\right)\right] + \frac{1}{E_3}\left[1 - \exp\left(-\frac{t}{T_3}\right)\right] \quad \text{.... (2.19)}$$

2. The actual number of Kelvin models required is not known at this time but can be determined later . For illustration, it is assumed that three Kelvin models are needed to describe retarded strains.

Figure 2.39: Separation of creep compliances .



Let:

$$b = \frac{1}{E_1} + \frac{1}{E_2} + \frac{1}{E_3} \ldots\ldots\ (2.20)$$

Where b is the intercept of retarded strain as shown in Figure 2.39, or it can be said as:

$$\boldsymbol{b = D_1 + D_2 + D_3} \ \ldots\ldots(2.21)$$

if three Kelvin models are used, so Equation 2.19 can be written as:

$$S_1 = b - D = \frac{1}{E_1}\exp\left(-\frac{t}{T_1}\right) + \frac{1}{E_2}\exp\left(-\frac{t}{T_2}\right) + \frac{1}{E_3}\exp\left(-\frac{t}{T_3}\right) \ldots\ldots\ldots\ (2.21)$$

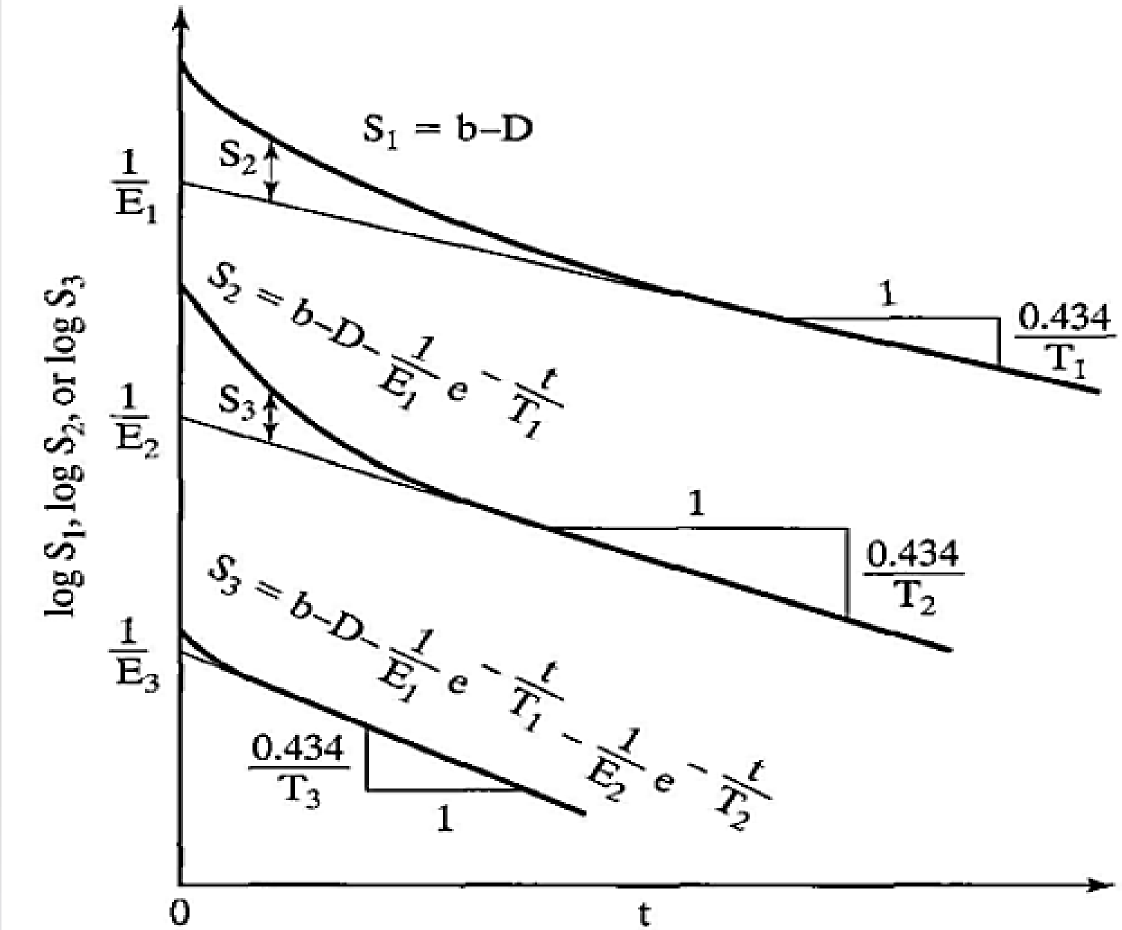
Notes:

1. If T_1 is much greater than T_2 and T_3 , then, after a sufficient period of time, the last two terms on the right side of Equation 2.21 vanish to be Equation 2.22. This equation shows that a plot of log S1 versus (t) results in a straight line, as indicated by Equation 2.23 and Figure 2.40.

$$S_1 = \frac{1}{E_1}\exp\left(-\frac{t}{T_1}\right) \ldots\ldots\ (2.22)$$

$$\log S_1 = \log\left(\frac{1}{E_1}\right) - \frac{0.434t}{T_1} \ \ldots\ (2.23)$$

Figure 2.40: Method of successive residuals.



2. The slope of the straight line can be used to determine T_1, and the intercept at t = 0 can be used to determine E_1. After E_l and T_1 are found, Equation 2.21 can be written as:

$$S_2 = b - D - \frac{1}{E_1}\exp\left(-\frac{t}{T_1}\right) = \frac{1}{E_2}\exp\left(-\frac{t}{T_2}\right) + \frac{1}{E_3}\exp\left(-\frac{t}{T_3}\right) \quad .... \ (2.24)$$

3. in which S_2 is the vertical intercept between the curve and the straight line. If T_2 is much greater than T_3, a plot of log S_2 versus (t) should also finally become a straight line, so T_2 and E_2 can be determined. The process is continued until the intercept becomes negligibly small .

Example 12:

The creep compliances of a viscoelastic material are shown in Table 2.1. of Example 11. Develop a mechanical model and determine its viscoelastic constants.

Solution:

- The generalized model is represented by Eq. 2.18. When $t = 0$, $D = 1/E_o$.
- From Table *2.1, D = 0.5* when $t = 0$, so $\mathbf{E_o = (1/D) = 2}$.
- At long loading times, only the viscous strains exist (as detailed in Figure 2.35).
- The rate of change in compliance due to viscous strains *is* $1/(E_oT_o)$, as can be seen from Eq. 2.15 or 2.18. At $t = 40$, $D = 5.798$ and at $\boldsymbol{t} = 50$, $D = 6.799$, so the change in compliance per unit time is:
  [(6 .799 - 5.798)/10 = 0.1], and $E_oT_o = 10$, or $\mathbf{T_o = 5}$.

Table 2.2 shows the procedure for computing successive residuals.

- Column 2 is the compliance of the dashed line shown in Figure 2.41 and can be computed by [6.799 - (50 - t) x 0.1].
- Column 3 is given in Table 2.1.
- Column 4 is the difference between Columns 2 and 3.
- A plot of log S_l versus (t) is shown in Figure 2.41 and results in a straight line. The slope of the straight line is (0.0455) or $T_l = 9.54$.

$$\frac{0.434}{T_1} = \frac{\log 0.1 - \log 0.01}{22} = 0.0455$$

- The intercept at t = 0 is $1/E_l = 0.1$, or $E_l = 10$.
- Column 5 can be calculated by [0.1 exp(—t ×19 .54)].
- Column 6 is the difference between columns 4 and 5.

- A plot of S_2 versus (t) results in a straight line. The slope of the straight line is (0.426) or $T_2 = 1.02$.

$$\frac{0.434}{T_2} = \frac{\log 0.2 - \log 0.0015}{5} = 0.426$$

- The intercept at t = 0 is $1/E_2 = 0.2$, or $E_2 = 5$.
- Column 7 can be calculated by [0 .2×exp(—t/1.02)].
- Column 8 is the difference between columns 6 and 7
- A plot of S_3 versus (t) results in a straight line. The slope of the straight line is (4.424) or T3 = 0.098.

$$\frac{0.434}{T_3} = \frac{\log 1 - \log 0.017}{0.4} = 4.424$$

- The intercept at t = 0 is $1/E_3 = 1$ or $E_3 = 1$.
- Because all points on S_3 lie practically on a straight line, three Kelvin models are sufficient to describe the creep-compliance curve.
- The equation for predicting the creep compliance is

$$D(t) = \frac{1}{2}\left(1 + \frac{t}{5}\right) + \frac{1}{10}\left[1 - \exp\left(-\frac{t}{9.54}\right)\right] + \frac{1}{5}\left[+\left[1 - \exp\left(-\frac{t}{0.098}\right) + \left[1 - \exp\left(-\frac{t}{0.098}\right)\right]\right.\right.$$

Note:

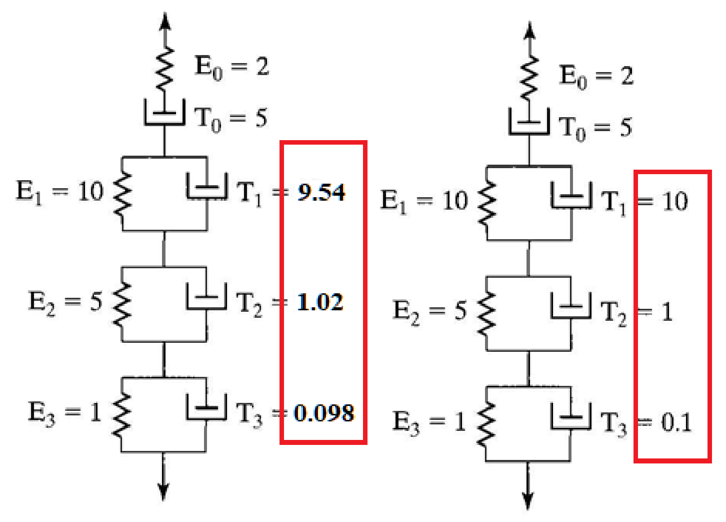
- ✓ **The values of E are the same as the original model shown in Figure 2.38, but the values of T are slightly different, as a result of plotting error (Figure 2.42)**
- ✓ **It can be seen that the stress—strain relationship of viscoelastic material can be characterized by a mechanical model or a creep curve . When one is known, the other can be determined .**

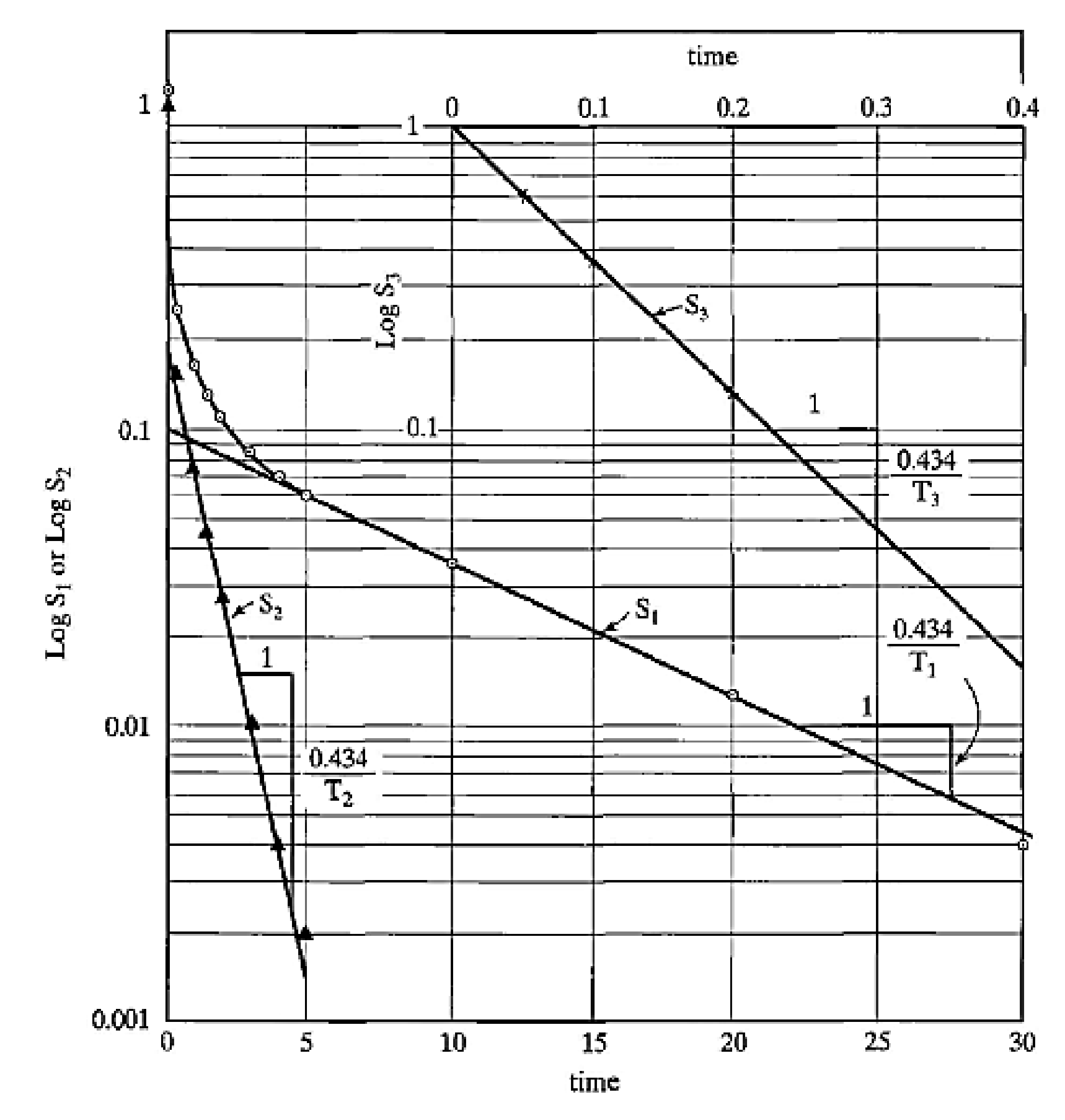
Table 2.2. Computation of Successive Residuals

	Compliance						
Time (1)	Dashed line (2)	Total (3)	S_1 (4)	$\frac{\exp(-t/T_1)}{E_1}$ (5)	S_2 (6)	$\frac{\exp(-t/T_2)}{E_2}$ (7)	S_3 (8)
0	1.799	0.500	1.299	0.100	1.199	0.200	0.999
0.05	1.804	0.909	0.895	0.099	0.796	0.190	0.606
0.1	1.809	1.162	0.647	0.099	0.548	0.181	0.367
0.2	1.819	1.423	0.396	0.098	0.298	0.164	0.134
0.4	1.839	1.592	0.247	0.096	0.151	0.135	0.016
0.6	1.859	1.654	0.205	0.094	0.111	0.111	0.000
0.8	1.879	1.697	0.182	0.092	0.090		
1.0	1.899	1.736	0.163	0.090	0.073		
1.5	1.949	1.819	0.130	0.085	0.045		
2	1.999	1.891	0.108	0.081	0.027		
3	2.099	2.016	0.083	0.073	0.010		
4	2.199	2.129	0.070	0.066	0.004		
5	2.299	2.238	0.061	0.059	0.002		
10	2.799	2.763	0.036	0.035	0.001		
20	3.799	3.786	0.013				
30	4.799	4.795	0.004				
40	5.799	5.798	0.001				
50	6.799	6.799	0.000				

Figure 2.41: Example 12.

Figure 2.42: differences in the viscoelastic parameters.





Example 13 :

Figure 2.43 shows a viscoelastic two-layer system under a circular loaded area having radius 10 in. (254 mm) and uniform pressure 100 psi (690 kPa). The thickness of layer 1 is 10 in. (254 mm), and both layers are incompressible, with Poisson ratio 0.5. The creep compliances of the two materials at five different times are tabulated in Table 2.1. Determine the surface deflection under the center of the loaded area at the given times.

TABLE 2.3. Creep Compliances and Surface Deflections

Time (s)	0.01	0.1	1	10	100
Layer 1 $D(t)$ (10^{-6}/psi)	1.021	1.205	2.683	9.273	18.320
Layer 2 $D(t)$ (10^{-6}/psi)	1.052	7.316	19.520	73.210	110.000

Note. 1 psi = 6.9 kPa, 1 in. = 25.4 mm.

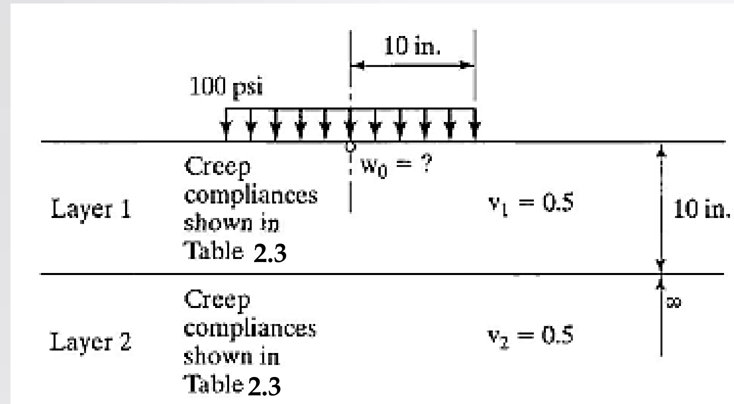


Figure 2.43. Example 13

Solution:

If the modulus ratio is greater than 1, the surface deflection w_o at any given time can be determined from Figure 2.21.

Take t = 1 s, for example . The elastic modulus is the reciprocal of creep compliance.

For layer 1, $E_l = 1/D_1 = 1/(2.683 \text{ X } 10\text{-}6) = 3.727 \text{ X } 10^5$ psi (2.57 GPa) and;

for layer 2, $E_2 = 1/D_2 = 1/(19.52 \text{ X } 10\text{-}6) = 5.123 \text{ X } 10^4$ psi (353 MPa).

So $E_1/E_2 = 3.727 \text{ X } 10^5 / (5.123 \text{ X } 10^4) = 7.27$.

From Figure 2.21, $F_2 = 0.54$, so $w_0 = 1.5 \text{ X } 100 \text{ X } 10 \text{ X } 0.54/(5.123 \text{ X} 10^4) = 0.016$ in. (4.1 mm).

The same procedure can be applied to other time durations and the results are shown in Table 2.4 .

TABLE 2.3. Creep Compliance and Surface Deflection

Time (s)	0.01	0.1	1	10	100
Layer 1 $D(t)$ (10^{-6}/psi)	1.021	1.205	2.683	9.273	18.320
Layer 2 $D(t)$ (10^{-6}/psi)	1.052	7.316	19.520	73.210	110.000
Deflection w_0 (in.)	0.0016	0.0064	0.016	0.059	0.096

Note. 1 psi = 6.9 kPa, 1 in. = 25.4 mm.

End of Chapter II