Power Flow Equation :

For the network shown in fig.

Transmission lines are represent

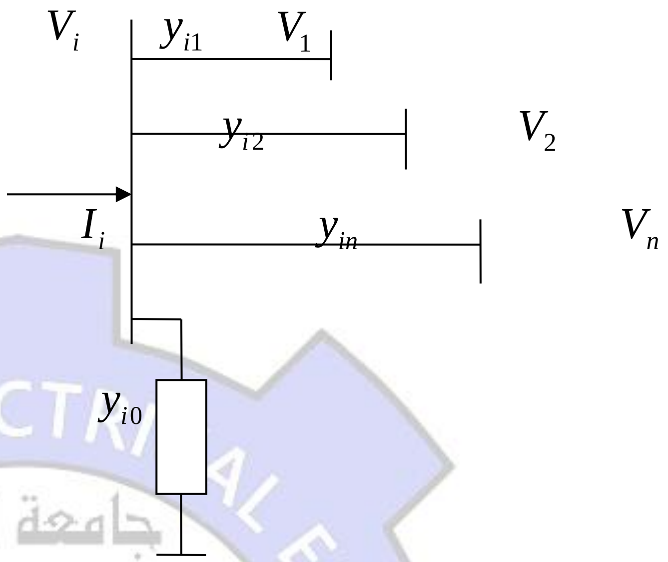
by their equivalent π models

where impedances have been

converted to per unit

admittances on a common

MVA base .



Application of KCL to this bus results in :

$$I_i = y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \ldots\ldots + y_{in}(V_i - V_n)$$

$$= (y_{i0} + y_{i1} + y_{i2} + \ldots.. y_{in})\,V_i - y_{i1}V_1 - y_{i2}V_2 - \ldots\ldots - y_{in}V_n$$

or $$I_i = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij}V_j \qquad j \neq i \qquad \ldots\ldots.(10)$$

The real and reactive power at bus *i* is :

$$P_i + jQ_i = V_i I_i^*$$

$$I_i^* = \frac{P_i + jQ_i}{V_i} \qquad ; \qquad I_i = \left(\frac{P_i + jQ_i}{V_i}\right)^* = \frac{P_i - jQ_i}{V_i^*}$$

Substituting for I_i in eq. (10) yields :

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i \quad .....(11)$$

The above equation is algebraic nonlinear equation which must be solved by iterative techniques .

Gauss-Seidel Power Flow Solution :

In the Gauss-Seidel method , the equation (11) is solved for V_i and the iterative sequence becomes :

$$V_i^{(k+1)} = \frac{\dfrac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} + \sum_{j=1}^{n} y_{ij} V_j^{(k)}}{\sum_{j=0}^{n} y_{ij}} \qquad j \neq i \quad ....(12)$$

Where y_{ij} is the actual admittance in per unit .

P_i^{sch} and Q_i^{sch} are the net real and reactive power expressed in per unit .

If equation (11) is solved for P_i and Q_i we have :

$$P_i^{(k+1)} = \mathrm{Re}\{V_i^{*^{(K)}}[V_i^{(k)} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{(k)}]\} \quad j \neq i$$

$$Q_i^{(k+1)} = -\mathrm{Im}\{V_i^{*^{(K)}}[V_i^{(k)} \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j^{(k)}]\} \quad j \neq i \qquad \text{...(13)}$$

The power flow equation is usually expressed in terms of the elements of the bus admittanc matrix (Y_{bus}) . Since the off-diagonal elements of Y_{bus} shown by Y_{ij} (i.e. $Y_{ij} = -y_{ij}$) an the diagonal elements are $Y_{ii} = \sum_{j=0}^{n} y_{ij}$,since equation (11) becomes :

$$\frac{P_i - jQ_i}{V_i^*} = V_i \, Y_{ii} + \sum_{j=1}^{n} Y_{ij} V_j \qquad j \neq i \quad \text{........(14)}$$

And equations (12) and (13) becomes :

$$V_i^{(k+1)} = \frac{\dfrac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j^{(k)}}{Y_{ii}} \qquad \text{.........................(15)}$$

$$P_i^{(k+1)} = \mathrm{Re}\{V_i^{*^{(K)}}[V_i^{(k)} \; Y_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j^{(k)}]\} \quad j \neq i$$

$$Q_i^{(k+1)} = -\operatorname{Im}\{V_i^{*(K)}[V_i^{(k)}\ Y_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} Y_{ij} V_j^{(k)}]\} \quad j \neq i \qquad ..(16)$$

Y_{ii} includes the admittance to ground of line charging susceptance and any other fixed admittanc to ground . Also which includes the effect of transformer tap setting .

For the Gauss-Seidel method , an initial voltage estimate of $1.0 + j0.0$ for unknown voltage i satisfactory , and the converged solution correlates with the actual operation states .

For *P*-*Q* buses , the real and reactive powers P_i^{sch} and Q_i^{sch} are known . Starting with an initia estimate , eq. (15) is solved for the real and imaginary components of voltage .

For *P*-*V* buses where P_i^{sch} and $|V_i|$ are specified , first eq.(16) is solved for $Q_i^{(k+1)}$, and then i used in eq (15) to solve for $V_i^{(k+1)}$. However , since $|V_i|$ is specified , only the imaginary part o $V_i^{(k+1)}$

is retained , and its real part is selected in order to satisfy .

$$|V_i|^2 = (e_i^{(k+1)})^2 + (f_i^{(k+1)})^2 \quad \text{......................(17)}$$

$$or \quad e_i^{(k+1)} = \sqrt{|V_i|^2 - (f_i^{(k+1)})^2} \quad \text{....................(18)}$$

Where $e_i^{(k+1)}$ and $f_i^{(k+1)}$ are the real and imaginary components of the voltage $V_i^{(k+1)}$ in th iterative sequence .

By applying an acceleration factor can be increased the rate of convergence .

$$V_i^{(k+1)} = V_i^{(k)} + \alpha\,(V_{i\,cal}^{(k)} - V_i^{(k)}) \quad \text{..............(19)}$$

Where α is the acceleration factor , the range of its value is (1.3 to 1.7) .

The process is continued until changes in the real and imaginary components of bus voltage between successive iterations are with a specified accuracy , i.e. :

$$\left| e_i^{(k+1)} - e_i^{(k)} \right| \le \varepsilon$$

$$\left| f_i^{(k+1)} - f_i^{(k)} \right| \le \varepsilon \quad \text{...................(20)}$$

A voltage accuracy (ε) in the range of 0.00001 to 0.00005 pu is satisfactory . In practic , th method for determining the completion of a solution is based on an accuracy index set up on th power mismatch . The iteration continues until the magnitude of the largest element in the ΔP an ΔQ columns is less than the specified value . A typical power mismatch accuracy is 0.001 pu .

After a solution is converged , the real and reactive powers at the slack bus are computed from e (16)